Technical Note

Influence of Bed Roughness and Cross Section Geometry on Medium and Maximum Velocity Ratio in Open-Channel Flow

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Abstract: This paper deals with the analytic-theoretical derivation of the relationships between the entropic quantity $\Phi(M)$, representing the ratio between the mean and maximum flow velocities, and the relative submergence and aspect ratios, using classical open-channel flow equations. $\Phi(M)$ is found to be highly dependent on the relative submergence when large or intermediate roughness scales occur, whereas it might be assumed to be almost constant for a small roughness scale. Furthermore, considering the hydraulic geometry relationships, an attempt is made to relate the relative submergence to the aspect ratio of flow through a log relationship whose coefficients depend on the local bed slope, with an important implication for hydrological practices. Then a practical relation between $\Phi(M)$ and the aspect ratio is proposed and validated in an operative chain for discharge assessment that shows high robustness and stability. The proposed model has been applied to a set of experimental velocity data collected at gauged river sites with different geometric and hydraulic characteristics as well as low, medium, and high flows. **DOI: 10.1061/(ASCE)HY.1943-7900.0001064.** *© 2015 American Society of Civil Engineers.*

Author keywords: Entropy velocity profile; Relative submergence; Roughness; Aspect ratio; Water discharge.

Introduction

Knowledge of the flow field in terms of velocity distribution in natural channels has been and continues to be one of the most relevant challenges for hydraulic engineers. Sediment transport processes and pollutant diffusion are classic hydraulic examples in which knowledge of the flow dynamics plays a fundamental role for prediction and design purposes. It is of considerable interest to identify a simple velocity law that gives suitable results using few parameters that are easy to measure or derive.

Recent studies outline the opportunity to relate the local energy budget to the informational content held into the point velocity measurements through a velocity distribution profile derived from an entropy-probabilistic approach. In particular, Chiu (1987) showed the high correlation between the mean, U_m, and maximum, $U_{\max}$, flow velocities through the parameter $\Phi(M) = [e^M/(e^M - 1)] - (1/M)$. Considering the important implication that this finding could have on the monitoring of high flows in rivers, many researchers have investigated the reliability of this relationship using field data (e.g., Xia 1997; Moramarco et al. 2004; Mirauda et al. 2011; Greco and Mirauda 2015). Overall, they found $\Phi(M)$ and, hence, M to be constant at a river site and unaffected by the magnitude of flood. Therefore, M might represent an intrinsic parameter of a gauged site; this insight led Moramarco and Singh (2010) to explore the dependence of M on the hydraulic and geometric characteristics of a river site. This analysis was able to explain that M is not dependent on the dynamics of a flood, as is expressed by the energy or water surface slope, S_f, and to identify a formula expressing M as a function of the hydraulic radius, Manning's roughness, and the location, y_0, where the horizontal velocity is hypothetically equal to zero. Considering that y_0 is not simple to assess and therefore might have high uncertainty, the assessment of M should be addressed using hydraulic and geometric variables that are easy to measure mainly for ungauged river sites. This insight might be achieved considering the relative submergence D/d (where D is the average water depth and d is the characteristic dimension of the roughness elements). In natural rivers, indeed, the velocity distribution is affected by the channel geometry, vegetation, and bank roughness, and usually the velocity can be assumed to be monotonically increasing from 0 at y_0, near the channel bed, to the maximum value at the water surface. Moreover, in the case of channels that are not very wide, in addition to the boundary effect, the velocity varies even along the transverse direction, and the maximum velocity occurs at or below the water surface, which is known as the *dip phenomenon*. Furthermore, the location of the maximum velocity in the flow area depends on the aspect ratio B/D (Yang et al. 2004), with B the channel width, through a relationship that is easily assessed if the channel geometry is given. Therefore, at the global scale, the analysis of the effect of bed roughness and cross-sectional geometry on the $\Phi(M)$ assessment is relevant in the field of open-channel flow.

On this basis, the present paper deals with the relationship between $\Phi(M)$ and the geometrical flow ratios D/d and B/D using classical open-channel flow and regime theory equations. The relationship $\Phi(M) = f(B/D)$ is developed and applied to a set of experimental velocity data collected in gauged river sites with different geometric and hydraulic characteristics. Finally, the relationship is validated in an operative chain for water discharge assessment and showed high robustness and stability.

Entropy Velocity Profile and Geometric Ratios in Open-Channel Flow

Shannon (1948) formulated the concept of entropy as a measure of information or uncertainty associated with a random variable or its

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probability distribution. Chiu (1987, 1991) applied the principle of maximum entropy (POME) to open-channel flows, and modeled the velocity distribution, shear stress, and sediment concentration. Analysis of the velocity distribution in the probability domain has the advantage of determining the cross-sectional mean flow velocity and the momentum and energy coefficients without dealing directly with the geometrical shape of cross sections, which tends to be extremely complex in natural channels (Chiu 1991; Luo and Singh 2011; Cui and Singh 2013).

Thus, Chiu (1987), by using POME, inferred the two-dimensional velocity distribution, obtaining

$$u(\xi) = \frac{U_{\max}}{M} \ln \left[1 + (e^M - 1)\frac{\xi - \xi_0}{\xi_{\max} - \xi_0}\right] \tag{1}$$

where u = velocity; $(\xi - \xi_0)/(\xi_{\max} - \xi_0)$ = cumulative probability distribution function, in which ξ is a function of the spatial coordinates in the physical space; $\xi_{\max} = \xi$ at the point where $U_{\max}$ occurs; and $\xi_0 = \xi$ at the channel bed where $u = 0$. Under such circumstances, u monotonically increases from ξ_0 to $\xi_{\max}$. Therefore, M can be used as a measure of the uniformity of probability and velocity distributions, and, as shown by Chiu (1987), its value can be determined by the mean, U_m, and maximum velocity values as

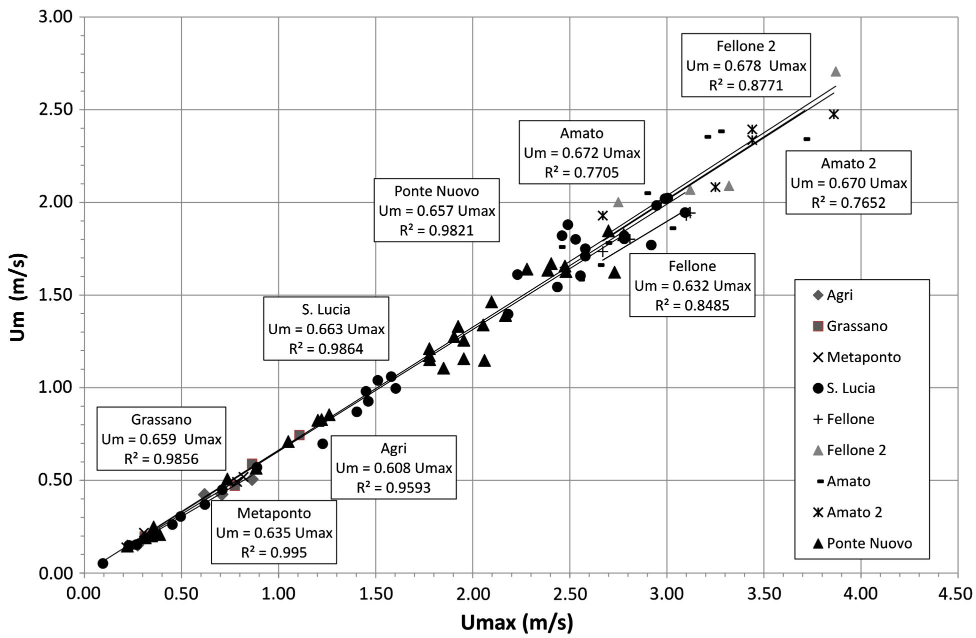


Fig. 1. Mean and maximum velocities observed in field

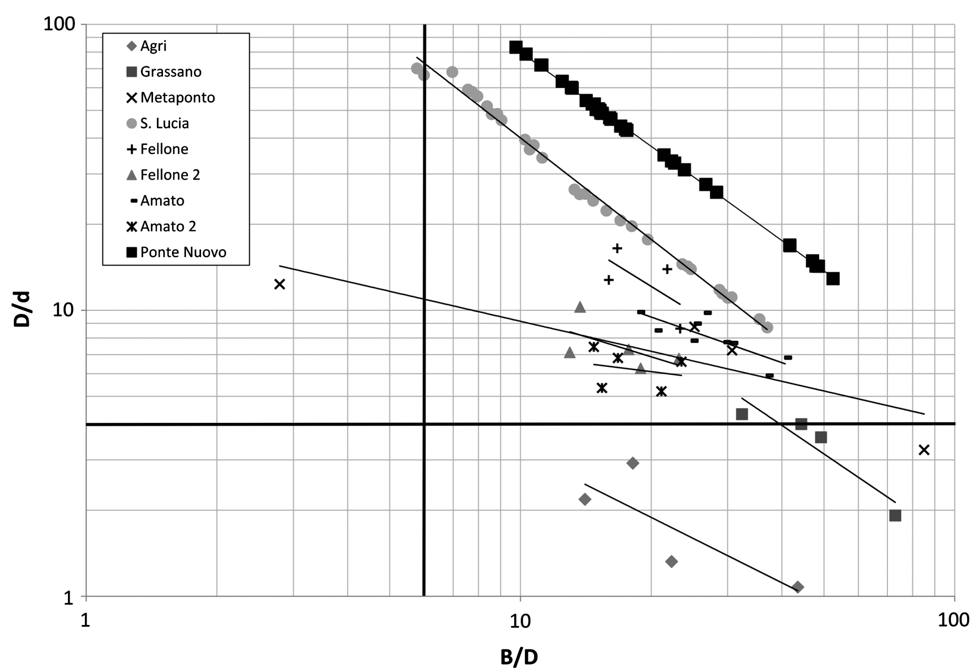


Fig. 2. Observed relationship between aspect ratio and relative submergence

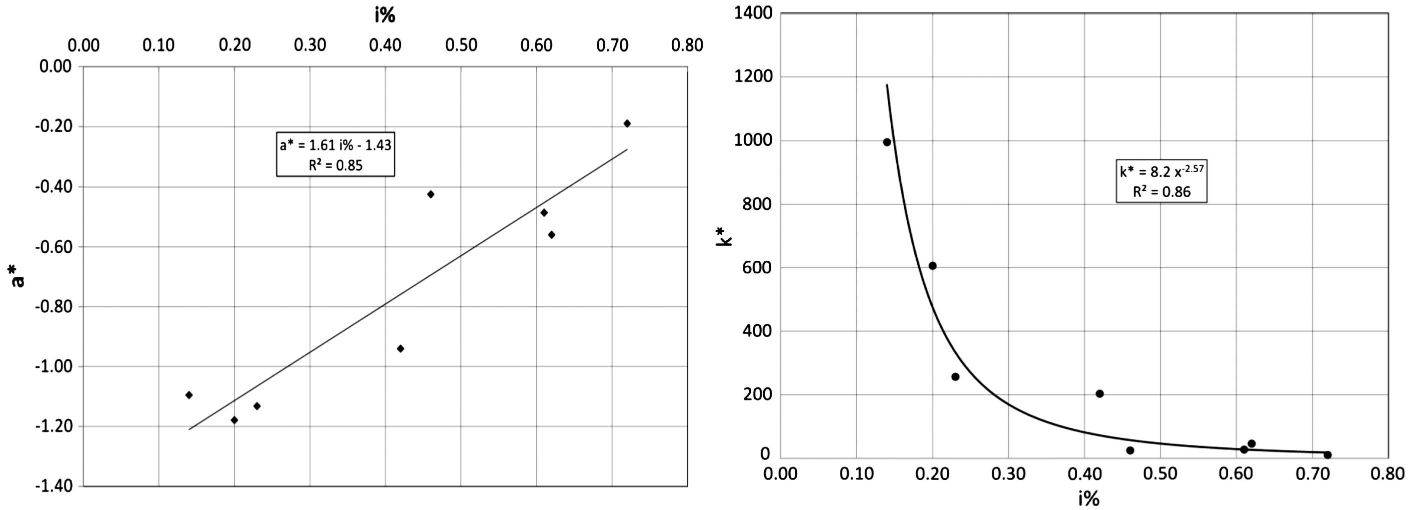


Fig. 3. Observed correlations between parameters k^* and a^* and the local bed slope

$$\Phi(M) = \frac{U_m}{U_{\max}} = \left(\frac{e^M}{e^M - 1} - \frac{1}{M}\right) \tag{2}$$

Eq. (2) was verified by several authors by collecting velocity data in gauged river sites worldwide (Xia 1997; Moramarco et al. 2004; Greco et al. 2014; Ammari and Remini 2010); it represents the fundamental relationship, from a practical point of view, for estimating $\Phi(M)$, which is known as the *velocity ratio*, using the ratio between mean and maximum velocities recorded at gauged sites.

To identify the dependence of M on the hydraulic and geometric characteristics of channels, i.e., the relative submergence and aspect ratio, respectively, the formulation proposed by Greco (2015) for U_m is considered

$$\frac{U_m}{u_*} = \frac{1}{k}\ln\frac{D}{d} + \frac{1}{k}\ln C_0 \tag{3}$$

in which u_* = shear velocity; d = characteristic bottom roughness height (i.e., d_{50} or d_{84}); k = von Karman constant; and C_0 = dimensionless coefficient.

The maximum velocity conveys important information about channel flow because it defines the range of the velocity distribution. In the flow area, the location of maximum velocity from the river bottom, $y_{\max}$, is of interest, because the maximum velocity does not always occur at the water surface but at some distance below it. Overall, this phenomenon, known as a *velocity dip*, may be induced by several factors, one of which is the *secondary currents* (Nezu and Nakagawa 1993), which refer to the circulation in a transverse channel cross section, while the longitudinal flow component is called the *primary flow*.

In this context, Moramarco and Singh (2010) identified the ratio between $U_{\max}$ and u_* as

$$\frac{U_{\max}}{u_*} = \frac{1}{k}\ln\left[\frac{D}{y_0(1+\alpha)}\right] + \frac{\alpha}{k}\ln\left(\frac{\alpha}{1+\alpha}\right) \tag{4}$$

with $\alpha = (D/y_{\max} - 1)$.

y_0 can be assumed to be proportional to the characteristic bottom roughness height, d, as suggested by Rouse (1965) *through the* experimental parameter $C_\xi = y_0/d$. Therefore, Eq. (4) becomes

$$\frac{U_{\max}}{u_*} = \frac{1}{k}\ln\left(\frac{D}{d}\right) + \frac{1}{k}\ln\left[\frac{\alpha^\alpha}{C_\xi(1+\alpha)^{1+\alpha}}\right] \tag{5}$$

Unlike in Moramarco and Singh (2010), here the ratio between Eqs. (3) and (5), based on logarithmic properties, explicitly proposes $\Phi(M)$ as a function of the relative submergence D/d

$$\Phi(M) = \frac{U_m}{U_{\max}} = \frac{\ln\left(\frac{C_0 D}{d}\right)}{\ln\left[\frac{D}{d}\frac{\alpha^\alpha}{C_\xi(1+\alpha)^{1+\alpha}}\right]} \cong A_\Phi \ln\frac{D}{d} + B_\Phi \tag{6}$$

where A_Φ and B_Φ are simple coefficients. Eq. (6) was derived under the assumption that the variability of $\ln(D/d)$ ranges from 1 to 10 and the corresponding ratio between the two coefficients, $\ln(C_0)/\ln[(\alpha^\alpha)/C_\xi(1+\alpha)^{1+\alpha}]$, is less than 2, as is generally found in field data. In fact, under these constraints, the relation

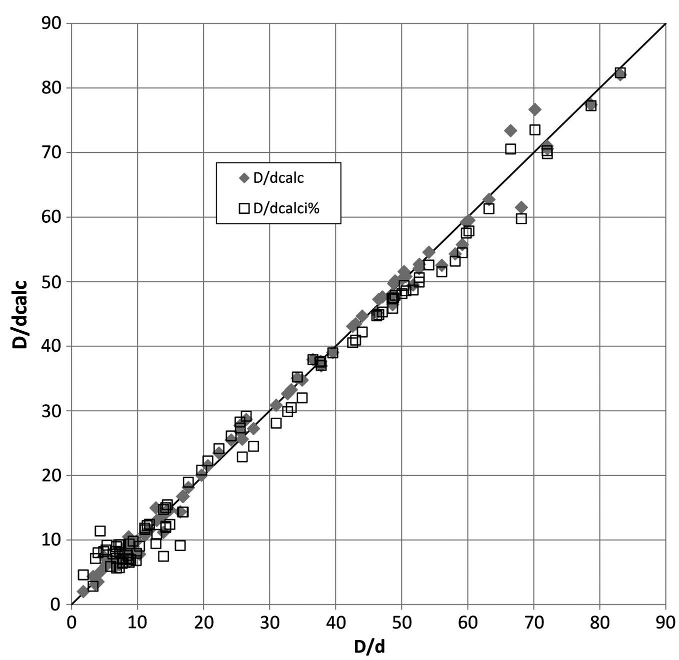


Fig. 4. Observed relative submergence versus computed one

$[\ln(C_0 D/d)/\ln[D/d \times (\alpha^\alpha/C_\xi(1+\alpha)^{1+\alpha})]; \ln(D/d)]$ can be linearly interpolated with a high correlation of determination ($R^2 \approx 0.9$).

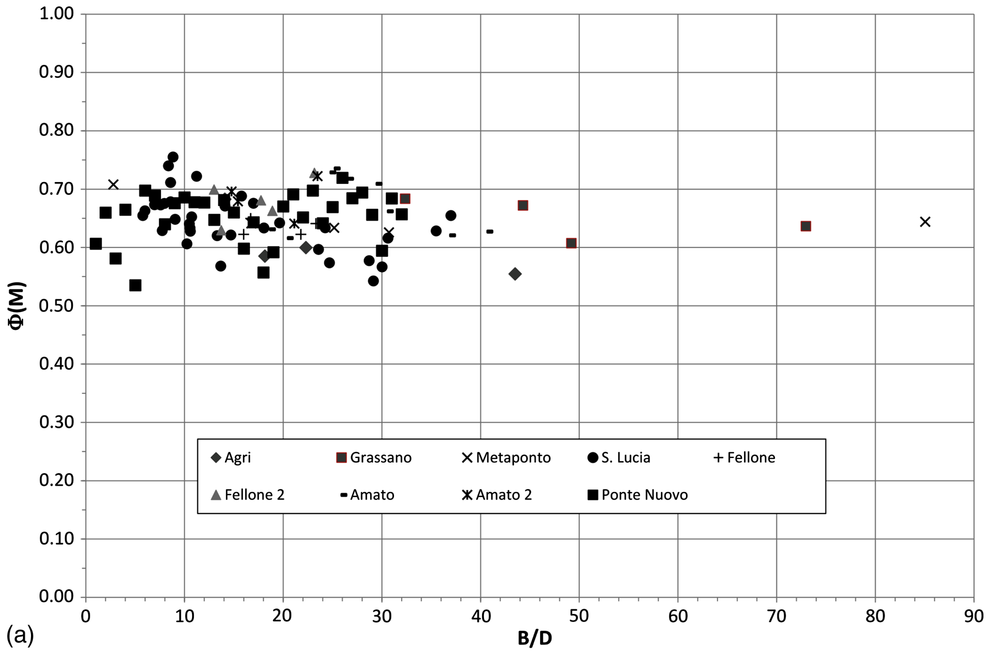
Eq. (6) explains the possible effects of bed roughness on the entropy velocity distribution in open-channel flow depending on the scale of large, intermediate, and small roughness (Bathurst 1985). Further, invoking the hydraulic geometry relationships, it is always possible to express average flow width, B, and depth, as well as the ratio D/d, as functions of the water discharge, Q (Leopold and Maddock 1953; Leopold et al. 1964; Griffiths 1980)

$$B = \alpha Q^a; \qquad D = \beta Q^b; \qquad \frac{D}{d} = \frac{\gamma}{i\%^j} Q^c \tag{7}$$

in which $i\%$ represents the local bed slope, and a, b, c, α, β, γ, and j are numerical coefficients. After a little algebra, the relative submergence can be reported in terms of the aspect ratio, B/D, obtaining:

$$\frac{D}{d} = \frac{\gamma}{i\%^j}\left[\frac{\beta}{\alpha}\cdot\frac{B}{D}\right]^{c/(a-b)} = k^*\left(\frac{B}{D}\right)^{a^*} \tag{8}$$

where k^* and a^* are coefficients. Finally, Eq. (6) can be reformulated taking into account Eq. (8), and the velocity ratio $\Phi(M)$ can be derived through the aspect ratio as follows:



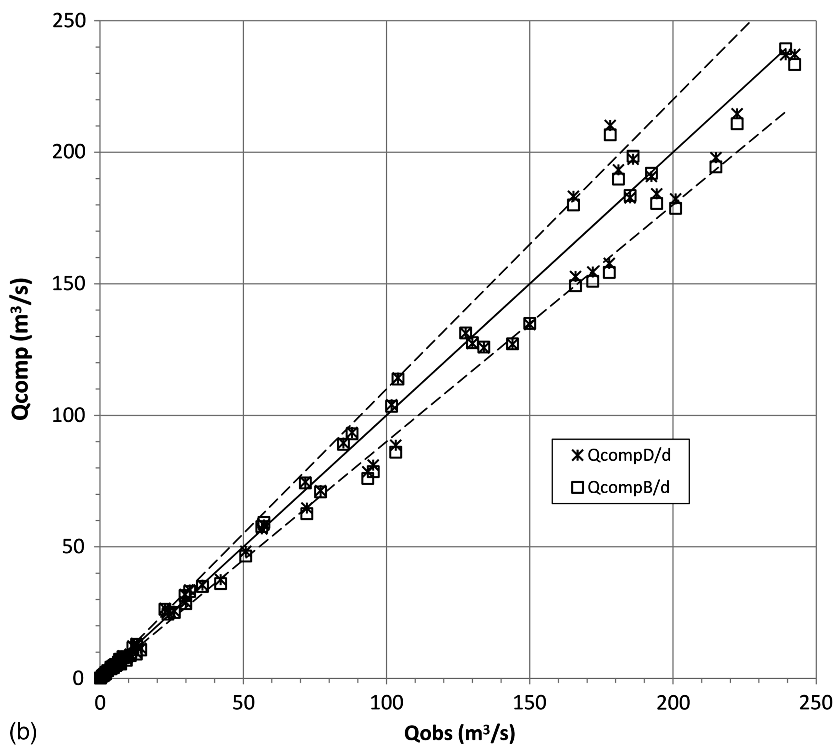


Fig. 5. (a) $\Phi(M)$ versus aspect ratio, B/D; (b) observed discharge versus computed ones by coupling Eqs. (2) and (9)

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$$\Phi(M) = A_B \ln\frac{B}{D} + B_B \tag{9}$$

in which A_B and B_B are appropriate coefficients.

Greco (2015) *specified* the coefficient assumed in Eq. (6) as a function of D/d, yielding

$$\begin{cases} A_\Phi = 0.11 \\ B_\Phi = 0.51 \end{cases} \Rightarrow \frac{D}{d} < 4 \quad \text{and} \quad \begin{cases} A_\Phi = 0 \\ B_\Phi = 0.66 \end{cases} \Rightarrow \frac{D}{d} > 4 \tag{10}$$

A_B and B_B in Eq. (9) can be easily inferred.

Field Investigation and Data Discussion

The aforementioned dependence between the entropy parameter, M, through ratio $\Phi(M)$, the relative submergence, D/d, and the aspect ratio, B/D, has been investigated based on a large volume of data collected at several gauged river sites along different Italian rivers located in Southern Italy, i.e., the Fellone and Amato Rivers in the Calabria region, Basento, Sinni, Agri, and Cavone Rivers in the Basilicata region, and the Tiber River in the Umbria region in central Italy. Such a database is extremely significant because it covers a relevant interval of water discharge, from a few liters up to hundreds of cubic meters per second (0.07–240 m^3/s), a bed slope in the range 0.1–1%, flow depth (0.07–5, 28 m), and mean sediment diameter, d_{50}, ranging from 3 to 7 cm. Furthermore, assuming d_{50} to be representative of roughness, the relative submergence (D/d_{50}) ranges from 1.15 to 70.

The observed domain (U_m, U_{max}) is shown in Fig. 1. In the same figure, the linear regressions, differentiated among data sets in the bulk of field data, are reported, and quite similar values of $\Phi(M)$ ranging from 0.608 to 0.678 are identified.

Considering Eq. (8), the theoretical relationship derived from the regime theory between the relative submergence and the aspect ratio is reported in Fig. 2 for each gauged site. Although a few cross sections were well populated (i.e., S. Lucia, Ponte Nuovo, Grassano, Amato), the log dependence between the aspect ratio and the relative submergence seems to be consistent. Fig. 2 shows an existing robust relationship between D/d and B/D, differentiated among the cross-section data set. This insight supports the idea of a correlation between coefficients k^* and a^* of Eq. (8) and the local geometric characteristics of the flow. Thus, the dependence of parameters k^* and a^* on the local bed slope has been investigated for the nine gauge sites, as reported in Fig. 3, obtaining for k^* and a^* the relationship

$$k^* = 8.2 i\%^{-2.57} (R^2 = 0.86) \quad \text{and}$$
$$a^* = 1.61 i\% - 1.43 (R^2 = 0.85) \tag{11}$$

Based on the available data set, Fig. 4 depicts a comparison between the relative submergence, D/d, computed by Eq. (8), using k^* and a^* expressed through the observed correlation of each cross section (Fig. 2), and D/d, also given by Eq. (8), but where k^* and a^* are estimated as a function of $i\%$ using Eq. (11). As can be seen, the results are quite satisfactory and the relationship between the relative submergence and the aspect ratio encourages the use of such a relationship during operational activities. Indeed, if the river geometry is known, B/D and, as a consequence, D/d can be explicitly estimated using Eq. (8), where k^* and a^* are given by Eq. (11). Therefore, $\Phi(M)$ can be quickly computed using Eq. (9), and, as shown in Fig. 5(a), the estimated values represent the observed ones fairly well at the investigated gauged sites. Therefore, once the roughness condition and the aspect ratio are estimated, velocity measurements can be collected in the cross section at all stage levels—low, medium, and high—obtaining for each one U_{max}, which is the maximum value sampled among all velocity points in the flow area. Note that U_{max} occurs in the upper portion of the flow area and can be easily sampled even during high flooding. The U_{max} value can be used in Eq. (2), together with the value of $\Phi(M)$, corresponding to the observed relative submergence or aspect ratio, independently using Eq. (6) or Eq. (9), obtaining, *de facto*, the mean flow velocity U_m and, hence, the discharge. Fig. 5(b) compares the discharges computed applying Eqs. (2) and (9) versus the observed ones, with reference to the available data set, with a resulting percentage error of less than 20%.

Conclusion

An analytic-theoretical derivation of the relationships between the medium and maximum velocity ratio, $\Phi(M)$, and the geometrical flow ratios, like relative submergence and aspect ratio, was proposed using entropy theory and hydraulic geometry relationships. The analysis was based on the experimental evidence that $\Phi(M)$ is dependent on the relative submergence in the case of high-intermediate roughness flow ($D/d < 4$), while such relationships can be assumed to be negligible once low roughness flow occurs ($D/d > 4$). On this basis, the relationship between $\Phi(M)$, D/d, and B/D is found to be robust, and this is of paramount importance in identifying a practical formulation relating the velocity ratio $\Phi(M)$ to the ratio flow width/flow depth, avoiding the need to assess the bottom roughness height, d.

The application of the proposed methodology to a set of experimental velocity data collected in gauged river sites with different geometric and hydraulic characteristics, as well as low, medium, and high stages, has shown that the approach can be conveniently used to estimate $\Phi(M)$ at a river site. Finally, discharge can be estimated with an error of ±20% once the maximum velocity and geometry are known. Further investigations are, however, needed for natural channels wider than those investigated.

Notation

The following symbols are used in this paper:

B = flow width;
B/D = aspect ratio;
D = water depth;
d = characteristic bottom roughness height;
D/d = relative submergence;
$i\%$ = bed slope;
M = entropy parameter;
Q = water discharge;
U_m = medium cross velocity;
U_{max} = maximum cross velocity;
y_0 = location where the horizontal velocity is zero; and
$\Phi(M)$ = medium and maximum velocity ratio.

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