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Optimization of olive-oil extraction using nonlinear H-infinity control

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Abstract: Olive-oil extraction is a process carried out by thousands of small industrial units in the Mediterranean countries (olive-oil mills), often in an empirical manner and using heuristic parametrization. The article proposes a new nonlinear H-infinity control approach for optimizing olive-oil extraction. The state-space model of the olive-oil extraction process is formulated after taking into account time-delays between the control inputs and its outputs. This state-space model undergoes approximate linearization around a temporary operating point which is re-computed at each step of the control method. The linearization is based on Taylor series expansion and on the computation of the model's Jacobian matrices. For the approximately linearized description of the olive-oil extraction system an optimal (H-infinity) feedback controller is designed. The computation of the controller's feedback gain requires the solution of an algebraic Riccati equation which also takes place at each iteration of the control method. The stability properties of the H-infinity control scheme are proven through Lyapunov analysis.

Keywords: olive-oil extraction, olive-oil mills control, nonlinear H-infinity control, approximate linearization, Lyapunov function, asymptotic stability.

1. INTRODUCTION

By optimizing the olive-oil extraction process, olive-oil mills will arrive at producing larger quantities of oil and at better quality. The problem of the optimization of the functioning of olive-oil mills has been studied in Baez-Gonzalez et al. (2016), Altieri et al. (2015). In particular Model Predictive Control has been a popular approach in the pursuit of optimization of the olive-oil extraction process Bordons and Cueli (2004), Bordons and Nunez-Reyez (2008), Nunez-Reyes and Bordons (2003). However, this approach is not of ascertained stability in the case of the nonlinear dynamics of the aforementioned process. On the other side there have been several artificial intelligence-based methods targeting to the optimization of the olive-oil extraction process Cano Marchal et al. (2015), Sanchez-Martin and Rayon-Duran (2013), Farfari et al. (2007). These approaches remain to a great extent dependent on heuristics. One can finally note results on the modelling of the dynamics of the olive-oil extraction process Tamborrino et al. (2014), Cano-Marchal et al. (2014). These can be used for developing model-based control schemes.

In this article a new approach for the problem of nonlinear optimal control of the olive-oil extraction process is developed. First the time delays between inputs (olive paste flow, thermomixer's temperature and water inflow to the decanter) and the outputs of the process (outflow of extracted olive-oil) are taken into account in the process's state-space description. The presence of time-delays causes the appearance of zeros dynamics, that is the process's output is also affected by the time-derivatives of the initial control inputs. By defining additional state variables (dynamic extension) which are associated with the time derivatives of the control inputs a state-space model of higher dimensionality is obtained Rigatos (2015), Rigatos (2016).

The design of the nonlinear optimal controller advances by obtaining a linearized model of the process, around a temporary operating point (equilibrium) which is recomputed at each iteration of the control method Rigatos and Siano (2015), Rigatos et al. (2015a), Rigatos et al. (2015b). The equilibrium comprises the present value of the system's state vector, and the last value of the control inputs vector that was exerted on it. The linearization procedure is based on Taylor series expansion and on the computation of

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Jacobian matrices Rigatos and Tzafestas (2007), Basseville and Nikiforov (1993), Rigatos and Zhang (2009). The modelling error which is due to the truncation of higher order terms in the Taylor series expansion, is considered to be a disturbance that is compensated by the robustness of the control algorithm.

For the approximately linearized model of the process an H-infinity feedback controller is developed. Actually, the H-infinity controller stands for the solution of the optimal control problem under model uncertainty and external perturbations. Such a control scheme represents a mini-max differential game taking place between the controller which tries to minimize a quadratic cost function that comprises a quadratic term of the state vector's tracking error, and between the disturbance inputs which try to minimize this cost function Toussaint et al. (2000), Lublin and Athans (1995). The selection of the controller's feedback gain is performed at each time instant through the solution of an algebraic Riccati equation.

The stability of the control method is proven through Lyapunov analysis. First, it is shown that the control loop satisfies the H-infinity tracking performance criterion and this signifies elevated robustness against model uncertainty and exogenous disturbances. Moreover, under moderate conditions, it is proven that the control scheme is also globally asymptotically stable. Finally, to implement state estimation-based control of the olive-oil extraction process, through the processing of measurements coming from a small number of state variables the H-infinity Kalman Filter is used as a robust state estimator.

2. DYNAMIC MODEL OF THE OLIVE-OIL EXTRACTION PROCESS

As mentioned above, the olive-oil extraction problem is a multivariable one, having as output the flow of the extracted olive-oil and receiving as inputs the following parameters: (i) the paste flow to the decanter, (ii) the thermomixer's temperature, (iii) the flow of the additional water entering the decanter, where decanter stands for the last part of the olive-oil mill.

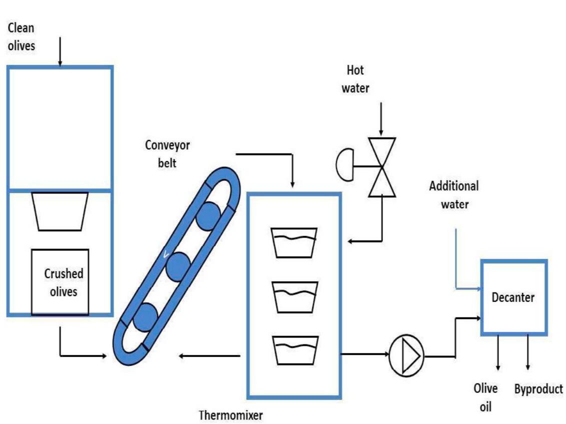


Fig. 1. Diagram of the olive-oil mill

The model can be expressed initially in the s-frequency domain. By denoting the system;s output as y_1 the system's inputs as u_1, u_2 and u_3 one obtains

$$y_1(s) = \frac{k_{11}e^{-T_{11}s}}{\tau_{11}s+1}u_1(s) + \frac{k_{12}e^{-T_{12}s}}{\tau_{12}s+1}u_2(s) + \frac{k_{13}e^{-T_{13}s}}{\tau_{13}s+1}u_3(s) \tag{1}$$

The previous equation signifies that the measured output of the olive-oil extraction system responds to its individual control inputs under time-delays Bordons and Cueli (2004),Bordons and Nunez-Reyez (2008). From Eq. (1) one obtains

$$\begin{array}{c}(\tau_{11}s+1)(\tau_{12}s+1)(\tau_{13}s+1)y_1(s)e^{(T_{11}+T_{12}+T_{13})s} = \\ k_{11}(\tau_{12}s+1)(\tau_{13}s+1)e^{(T_{12}+T_{13})s}u_1(s)+ \\ k_{12}(\tau_{11}s+1)(\tau_{13}s+1)e^{(T_{11}+T_{13})s}u_2(s)+ \\ k_{13}(\tau_{11}s+1)(\tau_{12}s+1)e^{(T_{11}+T_{12})s}u_3(s)\end{array} \tag{2}$$

Using Taylor series expansion for the exponential terms one has that $e^{(T_{11}+T_{12}+T_{13})s} \simeq 1 + (T_{11} + T_{12} + T_{13})s$, $e^{(T_{12}+T_{13})s} \simeq 1 + (T_{12} + T_{13})s$, $e^{(T_{11}+T_{13})s} \simeq 1 + (T_{11} + T_{13})s$, $e^{(T_{11}+T_{12})s} \simeq 1 + (T_{11} + T_{12})s$.

By substituting the previous Taylor series expansions in Eq. (2) one obtains

$$\begin{array}{l}(\tau_{11}s+1)(\tau_{12}s+1)(\tau_{13}s+1)[1+(T_{11}+T_{12}+T_{13})s]\cdot \\ \cdot y_1(s) = k_{11}(\tau_{12}s+1)(\tau_{13}s+1)[1+(T_{12}+T_{13})s]u_1(s) \\ \quad +k_{12}(\tau_{11}s+1)(\tau_{13}s+1)[1+(T_{11}+T_{13})s]u_2(s) \\ \quad +k_{13}(\tau_{11}s+1)(\tau_{12}s+1)[1+(T_{11}+T_{12})s]u_3(s)\end{array} \tag{3}$$

After intermediate operations and by grouping terms of the same order one obtains the following form for Eq. (3)

$\{[(\tau_{11}\tau_{12}\tau_{13})(T_{11} + T_{12} + T_{13})]s^4 + [(\tau_{11}\tau_{12} + \tau_{11}\tau_{13} + \tau_{12}\tau_{13})(T_{11} + T_{12} + T_{13}) + \tau_{11}\tau_{12}\tau_{13}]s^3 + [(\tau_{11} + \tau_{12} + \tau_{13})(T_{11} + T_{12} + T_{13}) + (\tau_{11}\tau_{12} + \tau_{11}\tau_{13} + \tau_{12}\tau_{13})]s^2 + [(\tau_{11}+\tau_{12}+\tau_{13})]s+1\}y_1(s) = k_{11}\{[\tau_{12}\tau_{13}(T_{12}+T_{13})]s^3 + [(\tau_{12} + \tau_{13})(T_{12} + T_{13}) + \tau_{12}\tau_{13}]s^2 + [(T_{12} + T_{13}) + (\tau_{12} + \tau_{13})]s + 1\}u_1(s) + k_{12}\{[\tau_{11}\tau_{13}(T_{11} + T_{13})]s^3 + [(\tau_{11} + \tau_{13})(T_{11} + T_{13}) + \tau_{11}\tau_{13}]s^2 + [(T_{11} + T_{13}) + (\tau_{11} + \tau_{13})]s + 1\}u_2(s)+k_{13}\{[\tau_{11}\tau_{12}(T_{11}+T_{12})]s^3+[(\tau_{11}+\tau_{12})(T_{11}+T_{12}) + \tau_{11}\tau_{12}]s^2 + [(T_{11} + T_{12}) + (\tau_{11} + \tau_{12})]s + 1\}u_3(s)$

Next, the following coefficients are defined for the above equation:

$a_{1,1} = [(\tau_{11}\tau_{12}\tau_{13})(T_{11} + T_{12} + T_{13})]$, $a_{2,1} = [(\tau_{11}\tau_{12} + \tau_{11}\tau_{13}+\tau_{12}\tau_{13})(T_{11}+T_{12}+T_{13})+\tau_{11}\tau_{12}\tau_{13}]$, $a_{3,1} = [(\tau_{11} + \tau_{12} + \tau_{13})(T_{11} + T_{12} + T_{13}) + (\tau_{11}\tau_{12} + \tau_{11}\tau_{13} + \tau_{12}\tau_{13})]$, $a_{4,1} = [(\tau_{11} + \tau_{12} + \tau_{13})]$, $a_{5,1} = 1$

$b_{1,1} = k_{11}[\tau_{12}\tau_{13}(T_{12} + T_{13})]$, $b_{2,1} = k_{11}[(\tau_{12} + \tau_{13})(T_{12} + T_{13}) + \tau_{12}\tau_{13}]$, $b_{3,1} = k_{11}[(T_{12} + T_{13}) + (\tau_{12} + \tau_{13})]$, $b_{4,1} = k_{11}1$, $b_{5,1} = k_{12}[\tau_{11}\tau_{13}(T_{11} + T_{13})]$, $b_{6,1} = k_{12}[(\tau_{11} + \tau_{13})(T_{11}+T_{13})+\tau_{11}\tau_{13}]$, $b_{7,1} = k_{12}[(T_{11}+T_{13})+(\tau_{11}+\tau_{13})]$, $b_{8,1} = k_{12}1$, $b_{9,1} = k_{13}[\tau_{11}\tau_{12}(T_{11}+T_{12})]$, $b_{10,1} = k_{13}[(\tau_{11} + \tau_{12})(T_{11}+T_{12})+\tau_{11}\tau_{12}]$, $b_{11,1} = k_{13}[(T_{11}+T_{12})+(\tau_{11}+\tau_{12})]$, $b_{12,1} = k_{13}1$.

First, using the previous notation of coefficients, the dynamics of the olive-oil extraction control system becomes

$$\begin{aligned} a_{1,1}y_1^{(4)} + a_{2,1}y_1^{(3)} + a_{3,1}\ddot{y}_1 + a_{4,1}\dot{y}_1 + a_{5,1}y_1 = \\ b_{1,1}u_1^{(3)} + b_{(2,1}\ddot{u}_1 + b_{3,1}\dot{u}_1 + b_{4,1}u_1 + \\ b_{5,1}u_2^{(3)} + b_{6,1}\ddot{u}_2 + b_{7,1}\dot{u}_2 + b_{8,1}u_2 + \\ b_{9,1}u_3^{(3)} + b_{10,1}\ddot{u}_3 + b_{11,1}\dot{u}_3 + b_{12,1}u_3 \end{aligned} \tag{4}$$

Equivalently, one obtains the following description about the dynamics of the control system of Eq. (4)

$$\begin{aligned} y_1^{(4)} = -\tfrac{a_{2,1}}{a_{1,1}}y_1^3 - \tfrac{a_{3,1}}{a_{1,1}}\ddot{y}_1 - \tfrac{a_{4,1}}{a_{1,1}}\dot{y}_1 - \tfrac{a_{5,1}}{a_{1,1}}y_1 \\ +\tfrac{b_{1,1}}{a_{1,1}}u_1^{(3)} + \tfrac{b_{(2,1}}{a_{1,1}}\ddot{u}_1 + \tfrac{b_{3,1}}{a_{1,1}}\dot{u}_1 + \tfrac{b_{4,1}}{a_{1,1}}u_1 + \\ \tfrac{b_{5,1}}{a_{1,1}}u_2^{(3)} + \tfrac{b_{6,1}}{a_{1,1}}\ddot{u}_2 + \tfrac{b_{7,1}}{a_{1,1}}\dot{u}_2 + \tfrac{b_{8,1}}{a_{1,1}}u_2 + \\ \tfrac{b_{9,1}}{a_{1,1}}u_3^{(3)} + \tfrac{b_{10,1}}{a_{1,1}}\ddot{u}_3 + \tfrac{b_{11,1}}{a_{1,1}}\dot{u}_3 + \tfrac{b_{12,1}}{a_{1,1}}u_3 \end{aligned} \tag{5}$$

Next, the following state variables are defined: $z_1 = y_1$, $z_2 = \dot{y}_1$, $z_3 = \ddot{y}_1$, $z_4 = y_1^{(3)}$, while as additional state variables the control inputs and their derivatives are considered: $z_5 = u_1$, $z_6 = \dot{u}_1$, $z_7 = \ddot{u}_1$, $z_8 = u_2$, $z_9 = \dot{u}_2$, $z_{10} = \ddot{u}_2$, $z_{11} = u_3$, $z_{12} = \dot{u}_3$, $z_{13} = \ddot{u}_3$. Moreover, as new control inputs, the following variables are defined: $v_1 = u_1^{(3)}$, $v_2 = u_2^{(3)}$, and $v_3 = u_3^{(3)}$. Additionally the following functions are defined:

$$\begin{aligned} f_1 = -\tfrac{a_{2,1}}{a_{1,1}}z_4 - \tfrac{a_{3,1}}{a_{1,1}}z_3 - \tfrac{a_{4,1}}{a_{1,1}}z_2 - \tfrac{a_{5,1}}{a_{1,1}}z_1 \\ +\tfrac{b_{2,1}}{a_{1,1}}z_5 + \tfrac{b_{3,1}}{a_{1,1}}z_6 + \tfrac{b_{4,1}}{a_{1,1}}z_7 + \\ +\tfrac{b_{6,1}}{a_{1,1}}z_8 + \tfrac{b_{7,1}}{a_{1,1}}z_9 + \tfrac{b_{8,1}}{a_{1,1}}z_{10} + \\ +\tfrac{b_{10,1}}{a_{1,1}}z_{11} + \tfrac{b_{11,1}}{a_{1,1}}z_{12} + \tfrac{b_{12,1}}{a_{1,1}}z_{13} \end{aligned} \tag{6}$$

$$g_{11} = \tfrac{b_{1,1}}{a_{1,1}} \quad g_{12} = \tfrac{b_{5,1}}{a_{1,1}} \quad g_{13} = \tfrac{b_{9,1}}{a_{1,1}} \tag{7}$$

Thus, one arrives at

$$\dot{z}_4 = f_1 + g_{11}v_1 + g_{12}v_2 + g_{13}v_3 \tag{8}$$

The dynamics of the olive-oil extraction process under input-output time delays can be also written in the following matrix form:

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \\ \dot{z}_7 \\ \dot{z}_8 \\ \dot{z}_9 \\ \dot{z}_{10} \\ \dot{z}_{11} \\ \dot{z}_{12} \\ \dot{z}_{13} \end{pmatrix} = \begin{pmatrix} z_2 \\ z_3 \\ z_4 \\ f_1 \\ z_6 \\ z_7 \\ 0 \\ z_9 \\ z_{10} \\ 0 \\ z_{12} \\ z_{13} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{11} & g_{12} & g_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \tag{9}$$

Using, the previous notation, the state-space description of the olive-oil extraction system takes the form

$$\dot{z} = \tilde{f}(z) + \tilde{g}(z)v \tag{10}$$

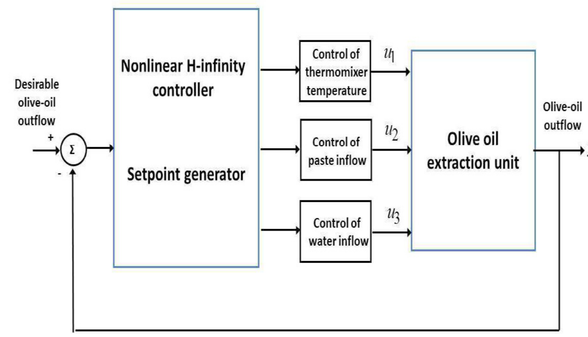


Fig. 2. Control loop of the olive-oil extraction process

3. APPROXIMATE LINEARIZATION OF THE OIL EXTRACTION PROCESS

Using again the notation x for the system's state vector in place of z, the state-space description of the olive-oil extraction process, given in Eq. (10), becomes

$$\dot{x} = \tilde{f}(x) + \tilde{g}(x)v \tag{11}$$

Next, this model undergoes approximate linearization. The linearization procedure for the dynamic model of olive-oil extraction is based on Taylor series expansion around a temporary operating point (x^*, u^*), where x^* is the present value of the system's state vector and u^* is the last value of the control inputs vector that was exerted on the system. The linearization requires the computation of the system's Jacobian matrices. The state-space description of the approximately linearized model of the system is

$$\dot{x} = Ax + Bu + \tilde{d} \tag{12}$$

where

$$\begin{aligned} A = \nabla_x[\tilde{f}(x) + \tilde{g}(x)u]|_{(x^*,u^*)} \Rightarrow \\ A = \nabla_x \tilde{f}(x)|_{(x^*,u^*)} \end{aligned} \tag{13}$$

$$\begin{aligned} B = \nabla_u[\tilde{f}(x) + \tilde{g}(x)u]|_{(x^*,u^*)} \Rightarrow \\ B = g(x)|_{(x^*,u^*)} \end{aligned} \tag{14}$$

4. DESIGN OF AN H-INFINITY NONLINEAR FEEDBACK CONTROLLER

4.1 Equivalent linearized dynamics of olive-oil extraction

After linearization round its current operating point, the dynamic model of olive-oil extraction process is written as

$$\dot{x} = Ax + Bu + d_1 \tag{15}$$

Parameter d_1 stands for the linearization error in the dynamic model of the olive-oil extraction process appearing in Eq. (15). The reference setpoints for the state vector of the olive-oil extraction process are denoted by $\mathbf{x_d} = [x_1^d, \cdots, x_6^d]$. Tracking of this trajectory is succeeded after applying the control input u^*. At every time instant the control input u^* is assumed to differ from the control input u appearing in Eq. (15) by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \tag{16}$$

The dynamics of the controlled system described in Eq. (15) can be also written as

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \tag{17}$$

and by denoting $d_3 = -Bu^* + d_1$ as an aggregate disturbance term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \tag{18}$$

By subtracting Eq. (16) from Eq. (18) one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \tag{19}$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$, the tracking error dynamics becomes

$$\dot{e} = Ae + Bu + \tilde{d} \tag{20}$$

4.2 The nonlinear H-infinity control

The initial nonlinear model of the olive-oil extraction process is in the form

$$\dot{x} = \tilde{f}(x, u) \quad x \in R^n, \ u \in R^m \tag{21}$$

Linearization of the system (olive-oil extraction process) is performed at each iteration of the control algorithm round its present operating point $(x^*, u^*) = (x(t), u(t - T_s))$, where T_s is the sampling period. The linearized equivalent model of the system is described by

$$\begin{aligned} \dot{x} = Ax + Bu + L\tilde{d} \quad x \in R^n, \ u \in R^m, \ \tilde{d} \in R^q \\ y = Cx \end{aligned} \tag{22}$$

where matrices A and B are obtained from the computation of the system's Jacobians and vector $\tilde{d}$ denotes disturbance terms due to linearization errors. The problem of disturbance rejection for the linearized model that is described by Eq. (22), cannot be handled efficiently if the classical LQR control scheme is applied. This is because of the existence of the perturbation term $\tilde{d}$.The disturbances' effect are incorporated in the following quadratic cost function:

$$\begin{aligned} J(t) = \tfrac{1}{2}\int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \\ -\rho^2 \tilde{d}^T(t)\tilde{d}(t)]dt, \quad r, \rho > 0 \end{aligned} \tag{23}$$

The significance of the negative sign in the cost function's term that is associated with the perturbation variable $\tilde{d}(t)$ is that the disturbance tries to maximize the cost function $J(t)$ while the control signal $u(t)$ tries to minimize it. This problem of mini-max optimization can be written as $min_u max_{\tilde{d}} J(u, \tilde{d})$.

4.3 Computation of the feedback control gains

For the linearized system given by Eq. (22) the cost function of Eq. (23) is defined, where the coefficient r determines the penalization of the control input and the weight coefficient ρ determines the reward of the disturbances' effects. It is assumed that (i) The energy that is transferred from the disturbances signal $\tilde{d}(t)$ is bounded, that is $\int_0^\infty \tilde{d}^T(t)\tilde{d}(t)dt < \infty$, (ii) the matrices $[A, B]$ and $[A, L]$ are stabilizable, (iii) the matrix $[A, C]$ is detectable. Then, the optimal feedback control law is given by

$$u(t) = -Kx(t) \tag{24}$$

with $K = \frac{1}{r}B^T P$, where P is a positive semi-definite symmetric matrix which is obtained from the solution of the Riccati equation

$$A^T P + PA + Q - P(\tfrac{1}{r}BB^T - \tfrac{1}{2\rho^2}LL^T)P = 0 \tag{25}$$

where Q is also a positive definite symmetric matrix. The worst case disturbance is given by $\tilde{d}(t) = \frac{1}{\rho^2}L^T Px(t)$. The diagram of the considered control loop is shown in Fig. 3.

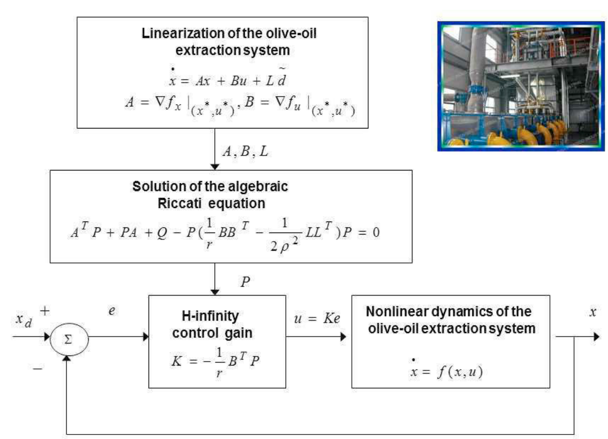


Fig. 3. Diagram of the control scheme for olive-oil extraction

5. LYAPUNOV STABILITY ANALYSIS

The tracking error dynamics for the olive-oil extraction process is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \tag{26}$$

where $L = I \in R^n$ with I being the identity matrix. Variable $\tilde{d}$ denotes model uncertainties and external disturbances of the olive-oil extraction model. The following Lyapunov equation is considered

$$V = \frac{1}{2}e^T P e \tag{27}$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\begin{array}{c}\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} \Rightarrow \\ \dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P e + \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow\end{array} \tag{28}$$

$$\begin{array}{c}\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T] P e + \\ + \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow\end{array} \tag{29}$$

$$\begin{array}{c}\dot{V} = \frac{1}{2}e^T A^T P e + \frac{1}{2}u^T B^T P e + \frac{1}{2}\tilde{d}^T L^T P e + \\ \frac{1}{2}e^T P A e + \frac{1}{2}e^T P B u + \frac{1}{2}e^T P L \tilde{d}\end{array} \tag{30}$$

The previous equation is rewritten as

$$\begin{array}{c}\dot{V} = \frac{1}{2}e^T (A^T P + PA) e + (\frac{1}{2}u^T B^T P e + \frac{1}{2}e^T P B u) + \\ + (\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d})\end{array} \tag{31}$$

Assumption: For given positive definite matrix Q and coefficients r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^T P + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \tag{32}$$

Moreover, the following feedback control law is applied to the system

$$u = -\frac{1}{r}B^T P e \tag{33}$$

By substituting Eq. (32) and Eq. (33) one obtains

$$\begin{array}{c}\dot{V} = \frac{1}{2}e^T[-Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P]e + \\ + e^T P B(-\frac{1}{r}B^T P e) + e^T P L \tilde{d} \Rightarrow\end{array} \tag{34}$$

$$\begin{array}{c}\dot{V} = -\frac{1}{2}e^T Q e + \frac{1}{r}e^T P B B^T P e - \frac{1}{2\rho^2}e^T P L L^T P e \\ -\frac{1}{r}e^T P B B^T P e + e^T P L \tilde{d}\end{array} \tag{35}$$

which after intermediate operations gives

$$\dot{V} = -\frac{1}{2}e^T Q e - \frac{1}{2\rho^2}e^T P L L^T P e + e^T P L \tilde{d} \tag{36}$$

or, equivalently

$$\begin{array}{c}\dot{V} = -\frac{1}{2}e^T Q e - \frac{1}{2\rho^2}e^T P L L^T P e + \\ + \frac{1}{2}e^T P L \tilde{d} + \frac{1}{2}\tilde{d}^T L^T P e\end{array} \tag{37}$$

Lemma: The following inequality holds

$$\frac{1}{2}e^T P L \tilde{d} + \frac{1}{2}\tilde{d} L^T P e - \frac{1}{2\rho^2}e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \tag{38}$$

Proof: The binomial $(\rho a - \frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\begin{array}{c}\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow \\ ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2\end{array} \tag{39}$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T P L$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d} - \frac{1}{2\rho^2}e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \tag{40}$$

Eq. (40) is substituted in Eq. (37) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \tag{41}$$

Eq. (41) shows that the H_∞ tracking performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\begin{array}{c}\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T ||\tilde{d}||^2 dt \Rightarrow \\ 2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||\tilde{d}||^2 dt\end{array} \tag{42}$$

Moreover, if there exists a positive constant $M_d > 0$ such that $\int_0^\infty ||\tilde{d}||^2 dt \leq M_d$, then one gets

$$\int_0^\infty ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d \tag{43}$$

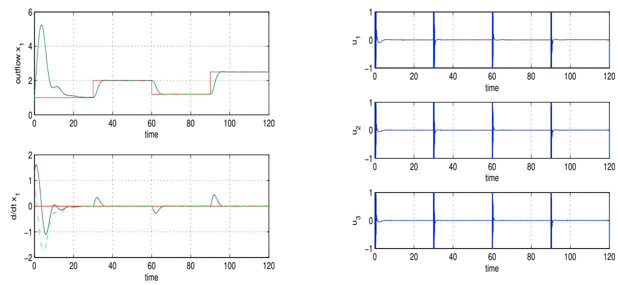
Thus, the integral $\int_0^\infty ||e||_Q^2 dt$ is bounded. Moreover, $V(T)$ is bounded and from the definition of the Lyapunov function V in Eq. (27) it becomes clear that $e(t)$ will be also bounded since $e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}$. According to the above and with the use of Barbalat's Lemma one obtains $lim_{t \to \infty} e(t) = 0$.

6. SIMULATION TESTS

The performance of the proposed H-infinity control scheme for the model of the olive-oil extraction process has been confirmed through simulation experiments. The computation of the feedback gain of the H-infinity controller required the solution of the algebraic Riccati equation of Eq. (32) at each step of the control method. The obtained results are depicted in Fig. 4 to Fig. 5 It can be noticed that the H-infinity control method achieved fast and accurate tracking of the reference setpoints while it also resulted into moderate variations of the control inputs.

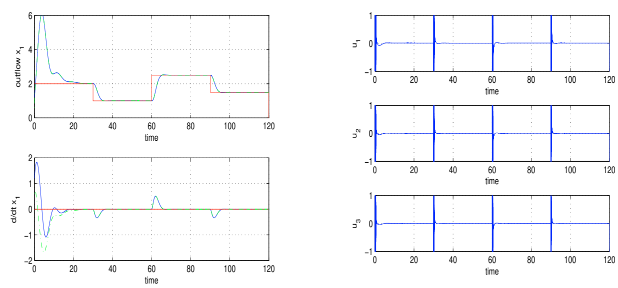
7. CONCLUSIONS

The article has proposed a nonlinear H-infinity control approach for the optimization of the olive-oil extraction process. First, the state-space model of the process has been formulated after taking into account time delays between its control inputs (olive paste flow, temperature of the thermomixer and inflow of water to the decanter) and its output (flow rate of extracted oil). Next, for this state-space description approximate linearization has been performed around a temporary operating point which was recomputed at each iteration of the control algorithm. The linearization has been based on Taylor series expansion



(a) **(b)**

Fig. 4. Test case 4: (a) tracking of reference setpoints by the state variables x_1 (olive oil outflow) and x_2 (outflow's derivative) of the olive-oil extraction system (b) control inputs v_1 to u_3



(a) **(b)**

Fig. 5. Test case 5: (a) tracking of reference setpoints by the state variables x_1 (olive oil outflow) and x_2 (outflow's derivative) of the olive-oil extraction system (b) control inputs v_1 to u_3

and on the computation of the associated Jacobian matrices. For the approximately linearized model of the olive-oil extraction process an H-infinity (optimal) controller has been designed. The global asymptotic stability properties of the control loop were proven through Lyapunov analysis.

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