Fatigue assessment of old existing masonry arch bridges: critical review of research and application to a case study

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ABSTRACT
The conservation of old masonry arch bridges is nowadays a very interesting engineering challenge since mostly of them are still in service and are subjected, differently from the past, to higher and more frequent traffic loads. Therefore, it is very important to properly predict the actual behavior under cyclic loadings taking into account the fatigue strength of the masonry and its resistance degradation due to repeated loads. In this paper modeling criteria and assessment procedures of masonry arch bridges are being critically reviewed. A particular attention is given to the fatigue behavior under cyclic loads and to residual life evaluation. The paper concludes with some numerical investigations on an existing multi-span Italian masonry bridge with backfill above the arches in order to assess the fatigue bridge capacity.

1 INTRODUCTION

One of the actual engineering challenge is the conservation of the old masonry arch bridges. Very often they were built in last centuries for lightweight and sporadic traffic loads, while nowadays we find them along strategic roads where, differently from the past, transit heavy and frequent traffic loads. Moreover, the bridge bearing capacity strictly is influenced by owners maintenance, that in many cases is not adequate for contrasting material deterioration and worn away due to the environmental conditions. For these bridges it is important to know, in addition to the ultimate vertical load, the maximum service load, that is the traffic load beyond the which the accumulated fatigue damage level into the elements becomes not acceptable and, consequently, the bridge results out of service. For example, the BD 21/93 (Design manual for roads and bridges, 1993) sets out, as the serviceability fatigue limit for the bridges, the limit of 50% of the vertical ultimate capacity in according to the value proposed by Choo et al. (1995). Melbourne et al. 2004 suggest to further reduce the limit for better predicting the fatigue limit of bridges.

It is evident, therefore, that in general the fatigue limit may significantly vary and, for this reason, is necessary to develop a behavioural model for evaluating the masonry elements response with respect to the cyclic vertical loads. In this way the fatigue response may be realistically predicted estimating, consequently, also the residual service life of a bridge. This information will be used by the owners to identify the priorities and to plan an interventions list. Similarly to the approach used for steel elements reported in the Italian Code (NTC-08, 2008) and Eurocode3 (EC3, 2003), to quantify the masonry fatigue response some stress-life curves (also indicated usually as S-N curves) have been proposed into the published literature. They provide, for a given amplitude of the normal stress S the number of cycles to failure N. In these curves, moreover, also the endurance limit may be individuated, representing the limit below which the masonry does not fail and can be infinitely cycled.

This paper shows a review of the state of art on the evaluation of the masonry fatigue resistance. At first, the proposed stress-life curves are being examined and compared among them. Then, the fatigue assessment of an old multi-span Italian masonry bridge with backfill above the arches still in service is presented and discussed. Whereas, in Laterza et al. 2015 the results of seismic assessment of the bridge may be found.

2 STATE OF THE ART OF THE FATIGUE BEHAVIOR OF MASONRY ELEMENTS

To date few experimental investigations on the behavior of masonry elements under repeated normal stresses and on the fatigue limit evaluation are available in literature. All the investigations have been intended to establish some stress-life curves (S-N curves) in which is represented the alternating normal stress amplitude (S_a) or the maximum normal stress (S_{max}) as function of number of cycles (N) to masonry failure. As regards instead the fatigue limit under shear loadings very few informations are available. Hereinafter a literature review of the experimental results and stress-life curves proposed by different authors are presented.

Clark (1994) performed a series of laboratory tests on prisms to study the behavior under cyclic loading of the masonry, in dry and wet conditions. The tests were carried out subjecting five course masonry prisms to a central vertical load up to 5 million loads cycle at a frequency of 5 Hz. Stress-life curves were derived by applying in all tests a minimum stress of 0,7 MPa (about the 5% of the ultimate compressive strength of the masonry in all test specimens). In these tests the cyclic response was investigated taking into account only the maximum stress and not considering the mean stress or the stress range.

Ronca et al. (2004) performed a series of experimental tests on a number of small-scale masonry specimens. The specimens were cast with a mortar of M4 class, and using brick blocks with the average strength of 48,86 N/mm^2. The resulting masonry had an average ultimate strength ranged between 10 and 13 MPa. The brickwork prisms were tested under very high vertical loads (65-80% of the ultimate compressive strength) axially applied, and imposing a small variation of the alternating load with three different frequencies: 1, 5, and 10 Hz.

Tests details and results published by Ronca et al. (2004) are reported in Table 1, where for each specimen is indicated the number of failure cycles, or alternatively the maximum number of cycles registered after 37 hours of test. The Table 1 reports the ratios S_m/S_u and S_a/S_u originally indicated by the authors, being S_m the stress induced due to the average sustained load, S_a the stress induced to the alternate load, and S_u the ultimate compressive strength of masonry. In addition, in the same Table 1 are reported some derived details of all tests such as: S_{max} and S_{min} respectively the maximum and the minimum stress in the cycle, the difference ΔS between S_{max} and S_{min}, the stress ratio R of S_{min} to S_{max}, and the ratio S_{max}/S_u. In Figure 1 are plotted the experimental stress-life curves in terms of log N versus S_a/S_u as published by the authors, and in the derived form *logN* versus S_{max}/S_u. By comparing the results obtained in the two graphs it is possible to note that in the alternative form the experimental data may be better represented with a linear regression.

In Roberts et al. (2006) a series of fatigue tests on the masonry brick specimens were carried out with the aim of establishing the link between the fatigue strength in brick masonry and the fatigue capacity of the masonry arch bridges. They investigated the influence of stress gradient and of saturation degree on the quasi-static and high cycle fatigue strength of brick masonry.

Table 1a. Experimental results published by Ronca et al. (2004).

Sample	f (Hz)	Failure cycle (N)	Maximum cycle (N)	S_m/S_u	S_a/S_u
1	1	27	-	0.8	0.1
2	1	57	-	0.8	0.1
3	1	351	-	0.8	0.1
4	1	4838	-	0.8	0.075
5	1	221	-	0.8	0.075
6	1	39	-	0.8	0.075
7	1	135414	-	0.8	0.05
8	10	-	2500000	0.8	0.05
9	10	-	1000000	0.8	0.05
10	1	-	60600	0.65	0.1
11	5	-	65000	0.65	0.1
12	5	156007	-	0.65	0.1
13	10	-	220000	0.65	0.05

Table 1b. Experimental results published by Ronca et al. (2004).

Sample	S_{max} (MPa)	S_{min} (MPa)	$R=S_{min}/S_{max}$	$S=S_{max}/S_u$
1	11.56	8.99	0.78	0.90
2	11.56	8.99	0.78	0.90
3	11.56	8.99	0.78	0.90
4	11.24	9.31	0.83	0.88
5	11.24	9.31	0.83	0.88
6	11.24	9.31	0.83	0.88
7	10.91	9.63	0.88	0.85
8	10.91	9.63	0.88	0.85
9	10.91	9.63	0.88	0.85
10	9.63	7.06	0.73	0.75
11	9.63	7.06	0.73	0.75
12	9.63	7.06	0.73	0.75
13	8.99	7.70	0.86	0.70

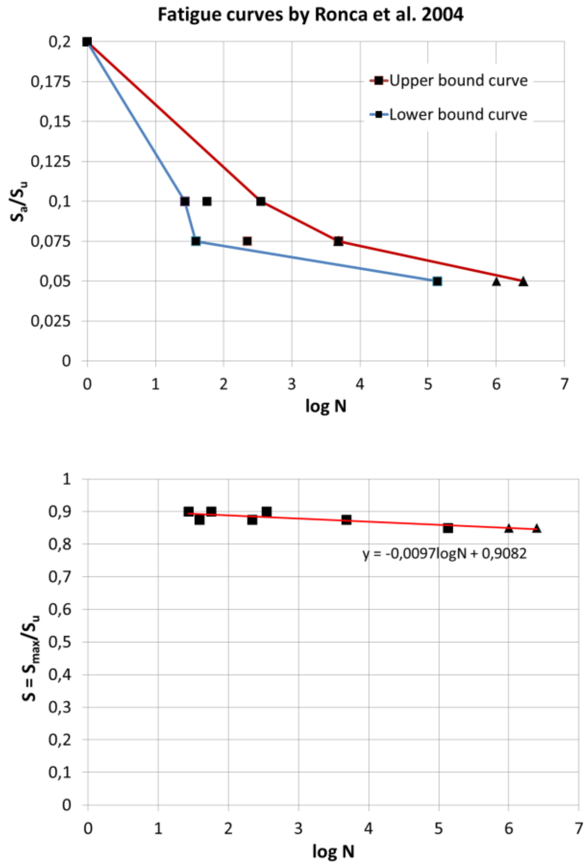


Figure 1. Ronca et al (2004): experimental results on masonry specimens subjected to cyclic loads. A) data published by authors; b) derived graph in the form of log N versus Smax/ Su.

The tests were conducted on three types of test specimens for simulating more closely the masonry arch barrel, and considering both dry and saturated conditions. They were applied different vertical load eccentricity ratios e/d from 0 to 0,256 (where e is the vertical load eccentricity and d the specimen depth). During all tests the frequency was typically kept constant to five cycles per second until the specimen failure. As far as the materials details are concerned, the bricks had a mean compressive strength of 18,6 N/mm^2 in dry condition, and of 18,5 N/mm^2 in saturated conditions. Whereas, the mortar was mixed in order to reproduce the representative mortar used for old brick masonry arches. It showed a compressive strength measured at 28 days ranged between 0,45 MPa and 2,78 MPa. The masonry compressive strength determined by assuming a linear stress distribution along the specimens varied between 6 and 14 N/mm^2. More details on the specimens and tests set-up may be found in Roberts et al. (2006). The authors proposed the following induced stresses function F(S) for describing the fatigue resistance of masonry:

$$F(S) = \frac{(\Delta S S_{max})^{0,5}}{S_u} \tag{1}$$

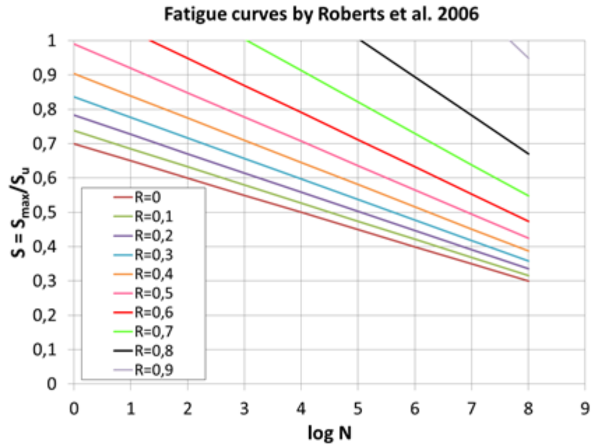


Figure 2. Fatigue curves derived from experimental results of Roberts et al. (2006).

where ΔS is the induced stress range calculated as the difference between maximum and minimum induced stress, S_{max} is the maximum induced stress, and S_u the quasi-static compressive strength of masonry. Starting from the experimental results the following fatigue curve for dry, wet and submerged brick masonry was derived:

$$F(S) = \frac{(\Delta S S_{max})^{0,5}}{S_u} = 0.7 - 0.05 \log N \tag{2}$$

that represents the lower bound curve of all fatigue strengths experimentally evaluated.

By introducing the stress ratio R= S_{min}/S_{max} of the minimum to the maximum stress induced in the cycles, the experimental fatigue curve [Eq. (2)] proposed by Roberts et al. (2006) may be rewritten in the more familiar form S_{max}/S_u – logN, where S_{max} is the maximum induced vertical stress:

$$S = \frac{S_{max}}{S_u} = \frac{1 - 0,05 \log N}{\sqrt{1-R}} \tag{3}$$

In Figure 2 are drawn the fatigue curves derived from the Eq. (3) by varying the stress ratio R.

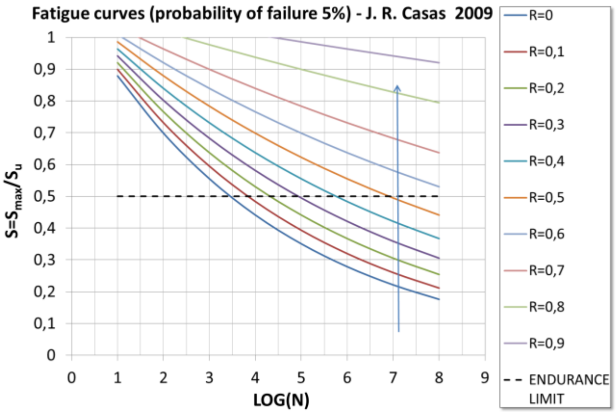


Figure 3. Stress-life curves proposed by Casas (2009) starting from the experimental data of Roberts et al. (2006).

Starting from the experimental data obtained by Roberts et al. (2006), Casas (2009) suggested new fatigue stress-life curves derived with a probabilistic approach. By using the Weibull distribution function for describing the progressive fatigue deterioration of brick masonry, has widely used for the analysis of metals and extended for concrete elements, the following fatigue equation was proposed:

$$S = \frac{S_{\max}}{S_u} = A \times N^{-B(1-R)}, \quad S > 0{,}5 \tag{4}$$

where the coefficients A and B are the fatigue parameters depending on the survival probability, and S=0.5 is the endurance limit. In the case of a survival probability of 95% (i.e. 5% of probability of failure) for masonry in any condition (dry, wet, and submerged) the previous equation becomes:

$$S = \frac{S_{\max}}{S_u} = 1{,}106 N^{-0{,}1034(1-R)}, \quad S > 0{,}5 \tag{5}$$

In Figure 3 the stress-life curves derived from the Eq. (5) are shown for different values of the stress ratio *R*. In some fatigue tests carried out, an endurance limit can be observed for *S = 0,5*.

3 COMPARISONS AMONG THE FATIGUE MODELS CONSIDERED

In Figure 4 are illustrated the comparisons among the stress – life curves proposed by the authors so far mentioned. All of three models considered are compared in the semi-logarithmic plane *LogN* - S_{max}/S_u for different values of the ratio *R* considered.

In the case of Casas fatigue model the stress-life curves are drawn by referring to a survival probability of 95%. Whereas, the Ronca curve is reported in the graph of *R=0.8,* that is approximately the mean value of R imposed in the Ronca experimental tests. Finally, in all graphs the endurance limit *S=0.5* is also indicated.

By comparing the different stress-life curves it is clear to note that the scatter between the Casas and Roberts curves significantly reduces for the R ratios of 0,4 and 0,6. Moreover, the Ronca stress-life curve provides values of the same order of the Roberts and Casas models.

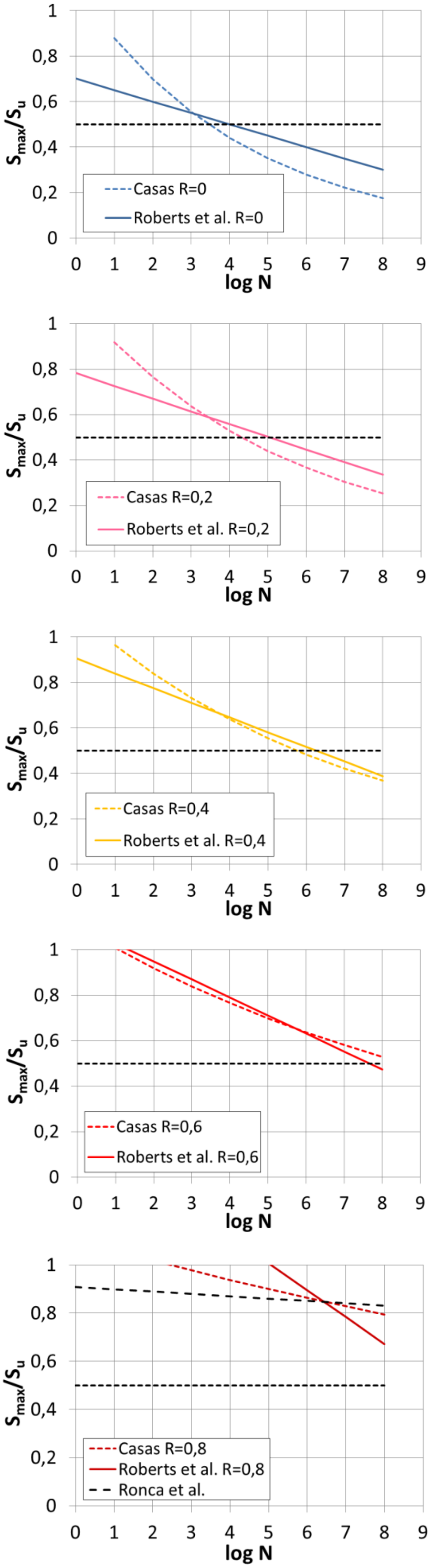


Figure 4. Comparisons among the different stress-life curves considered.

4 FATIGUE ASSESSMENT OF CAVONE BRIDGE

The chosen case study is an old multispan masonry arch bridge built in Italy before the Second World War and actually still in service. The “Cavone Bridge” takes the name from the crossing river and consists of seven arches of bricks masonry having an overall length of 140 m, and a width of 5.6 m. More in details, the bridge has four secondary arches of 10 m span length, and three main arches of 22 m span length. The three main arches are supported by two piers falling into the riverbed, with a total height from the foundation plane of about 24 m, of which 14 m are outside the riverbed. The bridge has been interested by some in situ tests addressed to identify the typology and the thickness of all elements. The tests have highlighted that the piers are made by an external layer of regular stone blocks containing a core of cohesive backfill, while the live load is distributed from the road to the arches through an incoherent backfill. Moreover, the abutments and the spandrel walls are in regular stone blocks. From figure 5 to 7 are reported some photos of the bridge and its geometrical schematization. No in-situ test to date has been performed for estimating the material strength. Therefore, in this study for the bridge masonry are used the values indicated into the Italian Code NTC-08 Instructions (2009).

Figure 5. Some photos of the Cavone Bridge.

Figure 6. A secondary arch (on the left) and a main arch (on the right) of the Cavone Bridge.

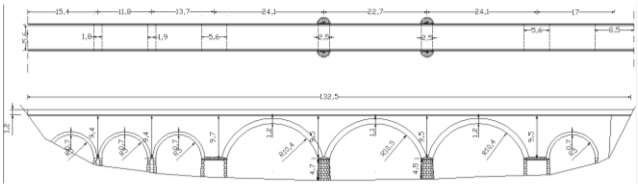


Figure 7. Geometrical schematization of the Cavone Bridge.

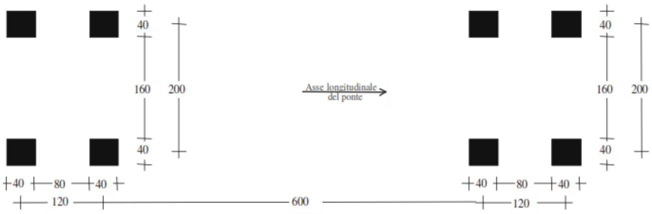


Figure 8. Scheme of Fatigue Load Model 3 adopted for the fatigue assessment of the Cavone Bridge.

The fatigue assessment is focused on the two masonry bricks arches of the bridge. It is known that the stress spectrum is depending on the vehicles geometry, axle loads, vehicle spacing, composition of the traffic and its dynamic effects. In according to the Italian code NTC-08 (2008) and Eurocode1 (EC1, 2003), the *Fatigue Load Model 3* is considered (Figure 8) for assessing the fatigue life by using the fatigue strength curves. The *Fatigue Load Model 3* is schematized through four axles, each of them having two identical wheels. The weight of each axle is equal to 120 kN, and each wheel has a square contact surface of 0.40 m x 0.40 m dimensions. Each couple of axes has a distance of 1.20 m, while the distance between the two internal axes is 6.00 m. As suggested in the EC1 this fatigue model is more appropriate for typical heavy traffic on European main roads or motorways.

The analyses are performed with "*Arco*" v. 1.2, a software completely dedicated to masonry arches. The program finds, through an iterative approach, the thrust line along the arch by neglecting the slips among the mortar joints of arch, and by assuming that the masonry has infinite strength in compression and no strength in tension.

In this study the fatigue assessment procedure for steel elements proposed in Italian Code (NTC-08, 2008) and Eurocode3 (EC3, 2003) is followed, because of no particular indication to date is present in the examined codes for masonry elements. More in details, in the steel fatigue assessment the stress range $\Delta\sigma_i$ (due to the stress fluctuation resulting from the transit of the model along the arch) is amplified by γ_{Ff}, while the fatigue strength is divided by γ_{Mf} for obtaining from the factored stress-life curve the endurance value N_{Ri}. The entire cumulated damage during the design life may be calculated from:

$$D = \sum_{i}^{n} \frac{n_{Ei}}{N_{Ri}} \tag{6}$$

where n_{Ei} is the number of cycles associated with the stress range $\gamma_{Ff}\Delta\sigma_i$ for band i in the factored spectrum expected in certain period (for example the service life); N_{Ri} is the fatigue endurance (in cycles) obtained from the factored stress-life curve $\Delta\sigma_c / \gamma_{Mf}$ - N_R for a stress range of $\gamma_{Ff}\Delta\sigma_i$. The partial factor for fatigue strength γ_{Mf} takes into account the consequence of the failure and the design assessment used, while γ_{Ff} is a partial factor for equivalent constant amplitude stress range. In this study is assumed that the high consequence of failure arises when the fatigue strength is reached, in conjunction with the fact the masonry arches are not damage tolerant. Therefore, γ_{Mf} is assumed equal to 1.35. The γ_{Ff} partial factor is equal to 1.0.

The Eq. (6) represents the cumulative damage summation, known as Palmgren-Miner rule, and expresses the entire damage cumulated in the examined element when repeated vehicles load transit along the bridge. At the end of service life the fatigue verification is satisfied if the cumulative damage summation D is less or equal to 1. On the contrary, if D is greater than 1 then the residual life, related to the remaining cycles up to fatigue failure, may be estimated.

The fatigue strength of the secondary and main arches are investigated in this work. The stress range $\Delta\sigma_i$ due to the *Fatigue Load Model 3* is evaluated starting from the permanent load condition (self-weight of all elements). The fatigue verification is conducted for the most unfavourable position of the moving load for the stresses in compression along each arch. It has been supposed that the problem is plane and the wheels concentrated load spreads with an angle of 45° through the fillment up to the arches extrados. In Figure 9 are reported the spreading schemes of the fatigue load referred to the most unfavourable positions for both the arches considered. Moreover, in the same figure is also indicated the derived equivalent distributed load q calculated from:

$$q = \frac{4 \times W_{axle}}{B \times L} \tag{7}$$

where W_{axle} is the load of each axle, B is the bridge width, and L is spreading area length.

Whereas, in Figure 10 and Figure 11 are reported the thrust-line and the stresses along the two arches.

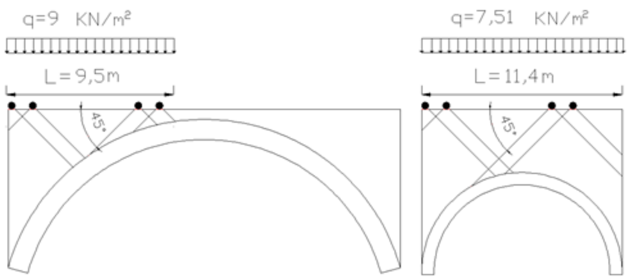
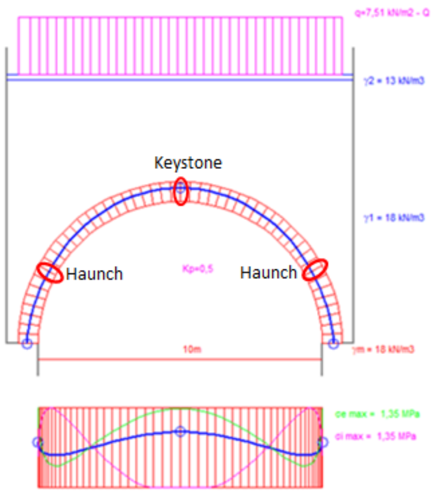
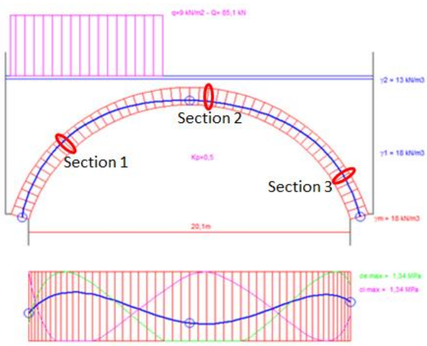


Figure 9. Fatigue load spreading scheme of secondary and main arch referred to the most unfavourable position of the moving load.



Normal stresses	keystone	haunches
S_{max} (MPa)	1,35	1,35
S_{min} (MPa)	1,21	1,21

Figure 10. Secondary arch numerical analysis considering the diffusion of the Fatigue Load Model 3 on the extrados of secondary arches: adopted for the fatigue assessment of the Cavone Bridge.



Normal stresses	Section 1	Section 2	Section 3
S_{max} (MPa)	1,34	1,34	1,34
S_{min} (MPa)	1,10	1,07	1,07

Figure 11. Main arch numerical analysis considering the diffusion of the Fatigue Load Model 3 on the extrados of secondary arches: adopted for the fatigue assessment of the Cavone Bridge.

Since no material test has been conducted up to date on the arches, for the fatigue assessment are being used in this study the values of the compression strength indicated into the Italian Code NTC-08 Instructions (2009) in the case of the masonry bricks. In particular are considered two different values of compressive strength, associated to the following knowledge levels:

– limited knowledge level KL1. In this case the mean value of masonry compressive strength assigned to arches is equal to 2.40 MPa (the minimum value of the interval of strength indicated). The confidence factor is CF =1.35 for reducing the strength considered;

– full knowledge level KL3. The compressive strength assigned to arches is supposed to be equal to 3.20 MPa, corresponding to the mean value of the interval of strength for bricks masonry. The confidence factor is CF=1.00.

In the Table 2 and Table 3 are reported the results of the analyses performed for the main and secondary arches, whereas in Figure 12 are reported the ratios S_{max}/S_u obtained by varying the knowledge level of the two arches.

Table 2. Main arch: maximum and minimum normal stresses

	Section 1	Section 2 and 3
S_{max} (MPa)	1,34	1,34
S_{min} (MPa)	1,10	1,07

$R=S_{max}/S_{min}$	0,82	0,80

KL	S_u (MPa)	$S=S_{max}/S_u$ (section 1)	$S=S_{max}/S_u$ (section 2 and 3)
LC1	1,78	0,75	0,75
LC3	3,20	0,42	0,42

Table 3. Secondary arch: maximum and minimum stresses

	Section 1,2,3
S_{max} (MPa)	1,35
S_{min} (MPa)	1,21
$R=S_{max}/S_{min}$	0,90

KL	S_u (MPa)	$S=S_{max}/S_u$ (section 1,2,3)
LC1	1,78	0,76
LC3	3,20	0,42

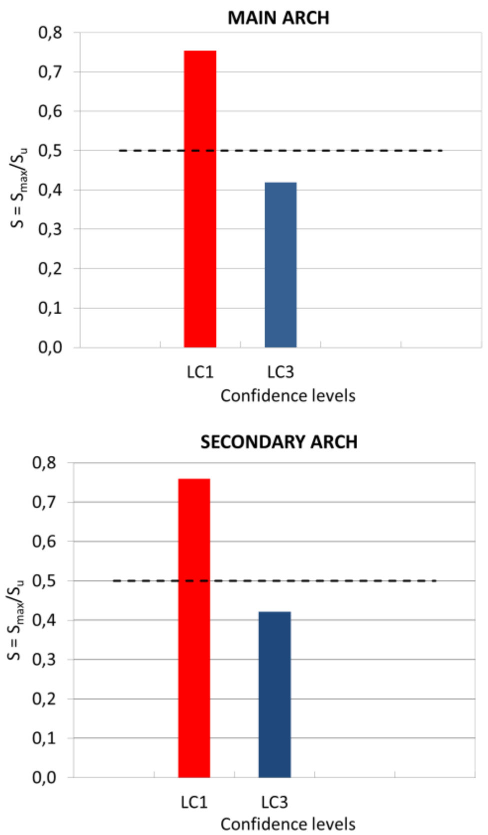


Figure 12. S_{max}/S_u ratio obtained by varying the knowledge level of the two arches analyzed.

The fatigue assessment of the two arches is performed in this work by considering the three masonry fatigue models previously described: Ronca et al. (2004), Roberts et al. (2006), and Casas (2009).

It is worth to note that, in the case of knowledge level 3 (KL3) we obtain for the two arches S_{max}/S_u ratios on the sections investigated always less than the ratio 0.5, that is the value often indicated as the fatigue endurance limit. Therefore, it would result that in these cases the residual life under cyclic loading is infinite.

On the contrary, the fatigue assessment would be necessary if one would attain the Knowledge Level 1 (S_{max}/S_u values greater than 0.5). In this case the fatigue model proposed by Ronca et al. cannot be used for both the arches investigated because of the found ratios S_{max}/S_u fall beyond the factored stress-life curve (Figure 13). Hence, no evaluation of the residual life for the two arches may be performed with this model. In Figure 14 and Figure 15 are reported the fatigue curves obtained with the models proposed by Roberts et al. (2006) and Casas (2009) for the main and the secondary arches, respectively. In particular, in Figure 14 are reported the fatigue curves for the two different values of the ratio S_{min}/S_{max} found, respectively, for the Section 1, and for the Sections 2 and 3.

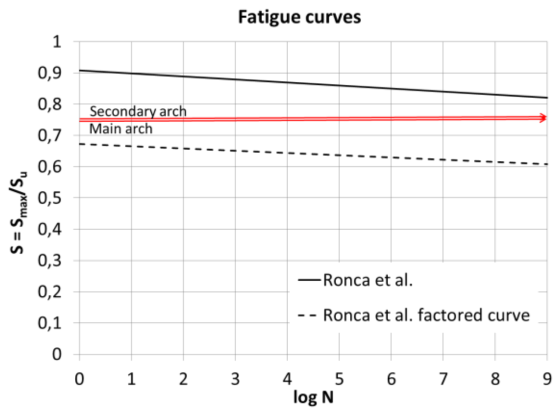


Figure 13- Main and secondary arch: stress-life curve proposed by Ronca et al. (2004).

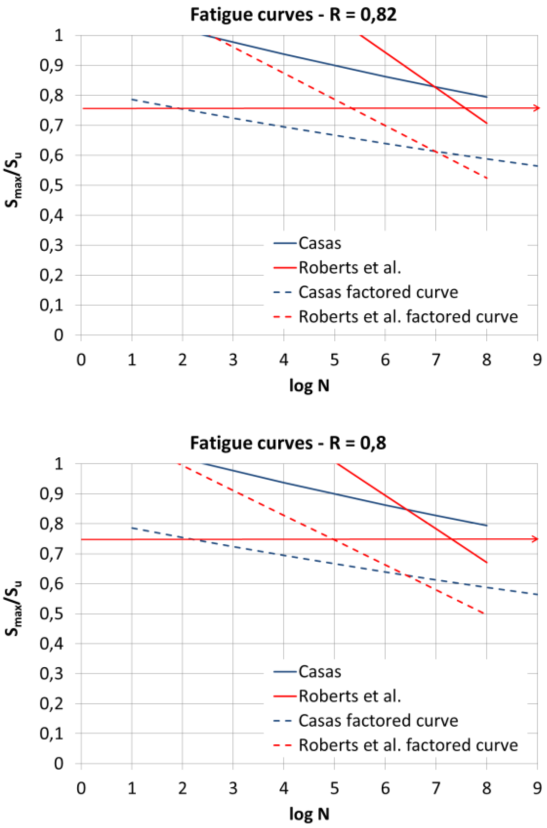


Figure 14. Main arch: stress-life curves of Casas (2009) with survival probability of 95% and Roberts et al. (2006). Fatigue curves of section 1 (above), and of sections 2 and 3 (below).

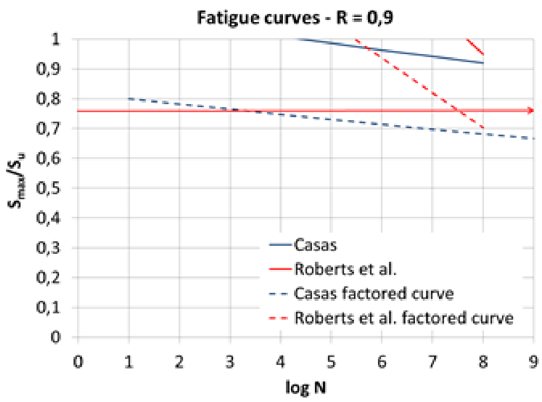


Figure 15. Secondary arch: stress-life curves of Casas (2009) with survival probability of 95% and Roberts et al. (2006). Fatigue curves of sections 1, 2 and 3.

The residual fatigue life evaluation for the arches, reported from Table 4 to 6 , is performed by assuming that the number *Nobs* of heavy vehicles passing on the bridge is 0.5 x 10^6, value indicated by NTC-08 and EC1 for roads and motorways with medium flow rates of lorries, considered compatible with the case studied.

The performed fatigue assessments show a high difference among the results. In particular with the model proposed by Casas (2009) for both the analyzed arches the residual service life is null. Instead, different results are obtained with the Roberts et al. (2006) model: the evaluated residual service life is very low (less than one year) in the case of the main arches. Whereas, for secondary arches the residual service life results greater than 60 years and, consequently, they would be verified for fatigue loads if a service life of 50 years (typical value for ordinary bridges) is assumed.

Table 4. Main arch: residual service life evaluation in correspondence of the section 1 for the KL1.

	logN	N	S_{max}/S_u	Residual service life (years)
Casas (2009)	2,01	102,7	0,754	0,00021
Roberts et al. (2006)	5,36	230585,5	0,754	0,46

Table 5. Main arch: residual service life evaluation in correspondence of the sections 2 and 3 for the KL1.

	logN	N	S_{max}/S_u	Residual service life (years)
Casas (2009)	1,80	62,8	0,754	0,0001
Roberts et al. (2006)	4,90	79198,9	0,754	0,16

Table 6. Secondary arch: residual service life evaluation in correspondence of the sections 1, 2 and 3 for the KL1.

	logN	N	S_{max}/S_u	Residual service life (years)
Casas (2009)	3,25	1783,16	0,76	0,004
Roberts et al. (2006)	7,51	32434427,8	0,76	64,87

It is evident that for the case analyzed the fatigue assessment with the stress-life curves to date available in literature provide conservative results. This is due most likely to the fact that these curves have been derived starting from elements with a masonry compressive strength very different with the respect to the ones of the two arches. From this stems the necessity of defining more appropriate stress-life curves covering the range of the compressive strengths usual for the old masonry existing structures.

5 CONCLUSIONS

In this paper modeling criteria and fatigue assessment procedures of masonry arch bridges have been discussed. An old existing multi-span Italian masonry bridge named “Cavone” bridge, with backfill above the arches has been studied for assessing its actual fatigue capacity and for evaluating the residual service life. The fatigue assessment has been focused on the two types of brick masonry arches by using different stress-life curves available to date in literature.

It must be pointed out that, differently from the steel structures, in the case of masonry elements no stress-life curve is indicated in the codes considered (NTC-08, EC1 and EC3).

In this work the fatigue assessment of the masonry brick arches of the Cavone bridge has been performed by the means of some stress-life curves published in literature. In the case analyzed, the considered fatigue models provide results very conservative as in the case of the main arches, where a residual service life less than one year has been estimated. Most likely this is due to the fact that the stress-life curves considered (Roca et al, Roberts, Casas et al.) have been established with few experimental data, furthermore obtained with masonry specimens with a significant different masonry compressive strength.

Finally, it has been found in the published literature a lack of stress-life curves referred to different types of masonry (such as brick masonry, stone masonry, etc.) with low compressive strength. Therefore, more realistic fatigue models in the case of old existing structures should be proposed.

6 ACKNOWLEDGEMENTS

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