Influence of hydraulic geometry ratios on the entropy parameter in open channel flow

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The knowledge of flow velocity distribution is an essential requirement in dealing with stage–discharge relationships, sediments transport processes and prediction of morphological behaviour in alluvial streams, design of stable channels, flood control works and mathematical and physical modelling of flows. Due to the limitations of classical hydraulic methods, Chiu (1987) derived the velocity distribution law basing on the concept of informational entropy introduced by Shannon (1948) and Tsallis (1988). Such velocity profile has been widely employed in many different flow cases and improved by relevant and meaningful both theoretical and applied contributions derived from robust experimental knowledge. Main aspect of such model is related to the need of one parameter M said as entropy parameter. Such parameter is depending on the ratio between the mean cross section velocity over maximum velocity, Φ(M). Thus, Chiu's velocity distribution results as:

$$u = \frac{U_{max}}{M} ln\left[1 + \left(e^M - 1\right)F(u)\right] = \frac{U_{max}}{M} ln\left[1 + \left(e^M - 1\right)\frac{\xi - \xi_0}{\xi_{max} - \xi_0}\right] \qquad (1)$$

where M is the dimensionless entropy parameter introduced in the entropy-based derivation (Chiu, 1988; Chiu and Said, 1995; Chiu and Tung 2002; Luo and Singh, 2011; Cui and Singh, 2013). Hence, M can be used as a measure of uniformity of probability and velocity distributions. The value of M can be determined by the mean, Um, and maximum velocity values derived from the following equation:

$$\Phi(M) = \frac{U_m}{U_{max}} = \left(\frac{e^M}{e^M - 1} - \frac{1}{M}\right) \qquad (2)$$

This parameter has proved to be useful for characterizing and comparing various patterns of velocity distributions and the status of open-channel flow system, which can be expressed by the location of mean and maximum velocity and their relationships. The mean velocity value, the location of the mean velocity (Chiu and Said, 1995; Chiu and Tung, 2002) and the energy coefficient (Chiu, 1992) can be obtained from M. The use of the entropy parameter predetermined for a channel section can greatly ease discharge estimation, especially in unsteady flow (Chiu, 1992).

The mean velocity, in fact, is another main characteristic of channel flow. With the known mean velocity value, the flow discharge, sediment transport and pollutant transport can be obtained. A linear relation between mean and maximum velocities was discovered by collecting the velocity data in some cross-sections of the Mississippi River (Xia, 1997). Eq. (2), indeed, represents the fundamental relationship, from applied point of view, of the entropy velocity distribution and the assessment of the entropy parameter pass through the knowledge of the ratio between mean and maximum velocities, Φ(M).

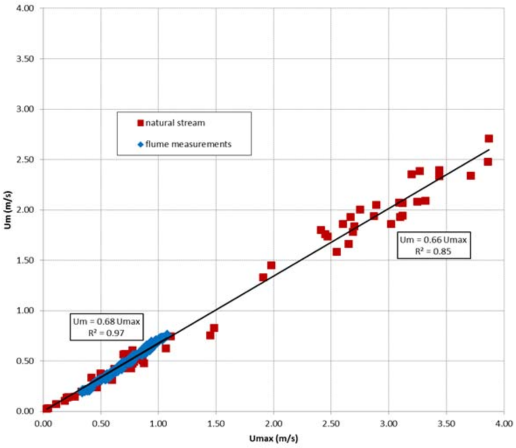

Figure 1 – Mean and maximum cross section velocities.

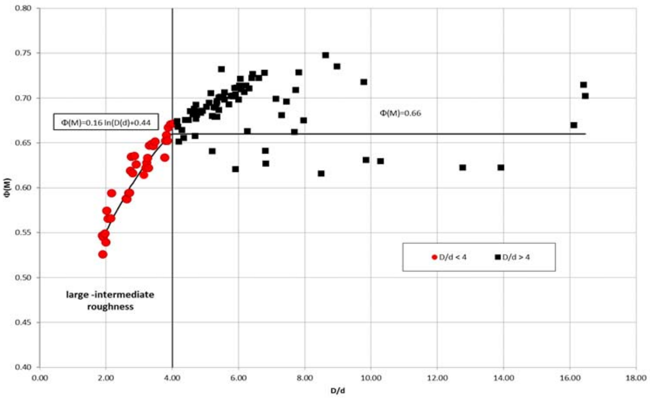

Figure 2 – Relation between velocities ratio Φ(M) and relative submergence, D/d.

The way to evaluate the entropy parameter M through the ratio Φ(M), still represents a relevant issue nourishing a reach discussion among researchers mainly addressed to the reasonable ground about the invariance of Φ(M) for sections along the same river only at high flow while for low stage the ratio Φ(M) can be affected by the influence of roughness through the relative submergence (the ratio between water depth and roughness height: D/d) as well as by the aspect ratio related to the cross section geometry (the ratio between flow width and flow depth: B/D). Using laboratory and field data, the classical hydraulic relationships on entropy velocity profile, the uniform flow and regime theory, a predictor for entropy parameter is proposed for open channel flow both natural and artificial ones. The mentioned dependence between the ratio Φ(M) and the relative submergence, D/d, has been studied referring to a wide volume of data, collected both in laboratory and in field. Such database covers a relevant interval of relative submergence values, ranging from 1.9 up to 17, water discharge, from few litres up to some cubic metres per second, and slope, that is:

- ✓ *field data* referred to different rivers in Southern Italy: slope 0.2-1%, water discharge 0.017-9 mc/sec, mean sediment diameter, d_{50}, 3-8.6 cm and relative submergence 5-17;
- ✓ *laboratory data* sampled on a rectangular rough flume with a regular bed roughness (wooden sphere, *d*= 3.5 cm), slope 0.05-1%, water discharge 7-72 lts/sec and relative submergence 2-7 (Mirauda et al., 2011).

The field (U_m; U_{max}) is shown in Fig. 1, where linear regressions differentiated among the two data set are reported, showing quite similar value of the ratio Φ(M) close to the value 0.66. Further, focusing on low values of velocities, both set of data, field and laboratory, exhibit a lowest value of the Φ(M) which suggest to investigate whether or not there should exist a possible influence of the bed roughness on such ratio.

In fact, as reported in figure 2, Φ(M) is strongly depending on the ratio depth/roughness for values of D/d less than 4 when large and intermediate roughness scale occurs (Bathurst et al., 1981; Bathurst, 1985), while it might be assumed almost constant, equal to 0.66, for small roughness scale (D/d>4), according to literature for high flow stage (Moramarco and Singh, 2010). The implementation of the proposed relationships [Φ(M) - D/d] to assess the water discharge, gives up a good response as proposed in figure 3 which reports the distribution of percentage error between the observed discharge and the computed ones and the relative submergence. The variance of about 10% represents an acceptable operative uncertainty.

Further, a possible influence on the ratio Φ(M) can be induced by the aspect ratio (B/D), which plays a relevant role on the velocity profile in terms of velocity dip, that is on maximum velocity location (y_{max}). Basing on the available data, the relationship between Φ(M) and B/D seems to be depending on whether or not the flow is confined, like artificial channel instead of natural cross section, as outlined in figure 4. The comparison between the two set of data, laboratory versus field, enlighten the effect of the aspect ratio which is strongly related to Φ(M) for flume velocity data while it results not depending on Φ(M) for river measurements. Finally, even this last issue enforce the difference between the Φ(M) ratio behaviours for high roughness flow and low roughness one, remarking the value of D/d=4 as operative threshold.

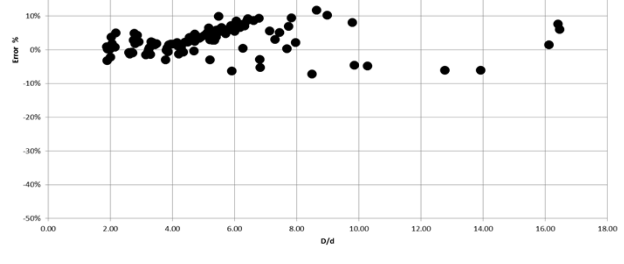

Figure 3 – Percentage error on the water discharge assessment.

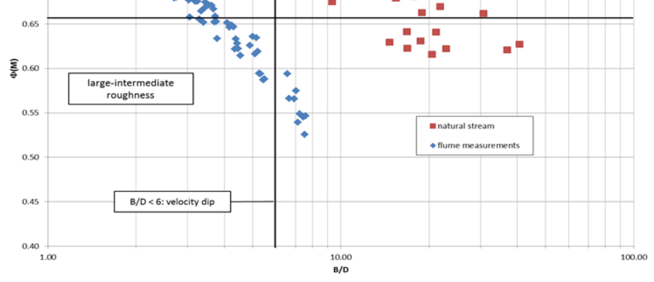

Figure 4 – Relation between velocities ratio Φ(M) and aspect ratio, B/D.

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