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Direct theorems in the theory of approximation of vectors in a Banach space with exponential type entire vectors. (English summary)

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Let $\mathfrak{X}$ be a Banach space, $A\colon \mathfrak{X} \supset \mathfrak{D}(A) \to \mathfrak{X}$ a closed linear operator with domain dense in $\mathfrak{X}$ and $\mathfrak{E}(A)$ the space of all exponential-type entire vectors of A.

The main result of this paper (see Theorem 3.1) states that, if A is the generator of a C_0-group $\{U(t)\}_{t\in\mathbb{R}}$ satisfying the condition $\int_{-\infty}^{+\infty}(1+t^2)^{-1}\ln\|U(t)\|\,dt<\infty$, then there exists a positive constant $m_k=m_k(A)$ such that, for every $x\in\mathfrak{X}$,

(1) $$\mathcal{E}_r(x,A)\le m_k\cdot\widetilde{\omega}_k\left(r^{-1},x,A\right)\quad (r\ge 1),$$

where $\mathcal{E}_r(x,A)$ is the best approximation of x by exponential-type entire vectors of A of type less than or equal to r [see V. I. Gorbachuk and M. L. Gorbachuk, Algebra i Analiz **9** (1997), no. 6, 90–108; MR1610243 (99f:47020); M. L. Gorbachuk, Ya. Ī. Grushka and S. M. Torba, Ukraïn. Mat. Zh. **57** (2005), no. 5, 633–643; MR2209488 (2006m:41053)] and $\widetilde{\omega}_k(t,x,A)$ $(t>0)$ is a generalization of the k-modulus of continuity proposed in [N. P. Kupcov, Uspehi Mat. Nauk **23** (1968), no. 4 (142), 117–178; MR0234189 (38 #2507)].

Inequality (1) allows one not only to prove the analogue of the classic Jackson's inequality for m-times differentiable functions (see Corollary 3.1), but also to obtain Jackson's inequalities in the spaces $L_p(2\pi)$, $C(2\pi)$ and $L_p(\mathbb{R},\mu^p)$ (see Section 4).

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.