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Positive semigroups behave asymptotically as rotation groups. (English summary)

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Let $(T(t))_{t \geq 0}$ be a bounded positive irreducible C_0 -semigroup on a Banach lattice X with order continuous norm. In this paper the authors characterize the asymptotic behavior of such a semigroup by using the full range of the Perron-Frobenius-Schaefer spectral theory and some results from harmonic analysis concerning solenoidal rotation groups. In particular their main result states that, if A is the generator of the semigroup $(T(t))_{t \geq 0}$ and $H := P\sigma(A) \cap i\mathbb{R} \neq \emptyset$, then the semigroup converges almost weakly to the rotation group on a Banach function space on the character group \widehat{H} , which is solenoidal and compact. Furthermore if, in addition, the semigroup is relatively compact in the strong operator topology on X , then it converges strongly to a rotation group as above. A time-discrete version of these results is also obtained. The paper ends with some concrete and interesting examples in which different rotation groups occur as limit of a positive C_0 -semigroup.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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