MR2493939 (2009m:41017) 41A25 (41A36)
Rempulska, Lucyna (PL-POZNT); **Tomczak, Karolina** (PL-POZNT)
On approximation of functions by certain operators preserving x^2. (English summary)
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For every $k = 0, 1, 2$ define the function $e_k(x) = x^k$ ($x \in X$, $X \subseteq \mathbb{R}$) and consider a sequence $(L_n)_{n\geq 1}$ of operators preserving e_0 and e_1 and satisfying the following condition:

$$L_n(e_2)(x) = x^2 + \frac{ax^2 + bx}{\lambda_n} \quad (n \geq 1,\ x \in X) \tag{1}$$

where $a, b \geq 0$, $a^2 + b^2 > 0$ and $(\lambda_n)_{n\geq 1}$ is a fixed increasing and unbounded sequence such that $\lambda_1 \geq 1$. Among the operators satisfying the above conditions are, for example, the Szász-Mirakyan operators, the Baskakov operators, the Post-Widder operators and the Stancu operators.

Extending an idea due to Duman and King [see M. A. Özarslan and O. Duman, Canad. Math. Bull. **50** (2007), no. 3, 434–439; MR2344178 (2008g:41008); Appl. Math. Lett. **20** (2007), no. 12, 1184–1188; MR2384243 (2008k:41034); J. P. King, Acta Math. Hungar. **99** (2003), no. 3, 203–208; MR1973095 (2004e:41027)], L. Rempulska and M. Skorupka ["On approximation by Post-Widder and Stancu operators preserving x^2", Kyungpook Math. J., to appear] introduced certain modified Post-Widder and Stancu operators, defined on polynomial weighted spaces, which preserve the functions e_0 and e_2 and have better approximation properties than classical operators.

Now, for every $n \geq 1$, let $L_{n,r}$ be the linear operator defined by means of an operator L_n preserving e_0 and e_1 as follows:

$$L_{n,r}(f)(x) := L_n(F_r(t, x))(x)$$

with

$$F_r(t, x) := \sum_{j=0}^{r} \frac{f^{(j)}(t)}{j!}(x - t)^j$$

for r-times differentiable functions f. The approximation properties of such operators were studied by G. Khr. Kirov [Math. Balkanica (N.S.) **6** (1992), no. 2, 147–153; MR1182946 (94a:41015)] and other authors [e.g., G. Khr. Kirov and L. Popova, Math. Balkanica (N.S.) **7** (1993), no. 2, 149–162; MR1270375 (95i:41048); L. Rempulska and Z. Walczak, Math. Balkanica (N.S.) **18** (2004), no. 1-2, 53–63; MR2076077 (2005e:41064)].

The aim of the paper under review is to extend the Duman-King idea to operators L_n and $L_{n,r}$ satisfying condition (1) and defined on certain polynomial weighted spaces. In fact, using a modification of certain operators L_n preserving e_0 and e_1, the authors introduce the operators L_n^* which preserve x^2 and next they define the operators $L_{n,r}^*$ for r-times differentiable functions. They show that L_n^* and $L_{n,r}^*$ have better approximation properties than L_n and $L_{n,r}$.

Reviewed by *Vita Leonessa*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.