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Growth order and stability of semigroups and cosine operator functions. (English summary)

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This paper treats the characterization for the growth orders $O(t^\alpha)$ and $o(t^\alpha)$ ($\alpha \geq 0$) of C_0 -semigroups $(T(t))_{t \geq 0}$ of $\mathcal{L}(X)$, X being a Banach space, by means of conditions involving Cesàro and Abel means of $\|T(\cdot)x\|^p$ and $\|T^*(\cdot)x^*\|^q$ for every $x \in X$ and $x^* \in X^*$, where $p \geq 1$ and $q = (p - 1)/p$.

Note that, when $\alpha = 0$, these results give equivalent conditions about uniform boundedness and uniform stability of $(T(t))_{t \geq 0}$.

It is also shown that, under some Tauberian condition, $(T(t))_{t \geq 0}$ is uniformly bounded (resp. strongly convergent) if and only if its Abel mean is.

Similar results for cosine operator functions are also obtained.

Reviewed by *Vita Leonessa*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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