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Asymptotic expansion of L^2-norms of solutions to the heat and dissipative wave equations. (English summary)

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Let $v(t,x)$ and $u(t,x)$ be solutions of the following differential equations:

$$\begin{cases} v_t - \Delta v = 0, & t \geq 0,\ x \in \mathbb{R}^N, \\ v(0,\cdot) = v_0, & v_0 \in L^2(\mathbb{R}^N) \end{cases}$$

and

$$\begin{cases} u_{tt} + u_t - \Delta u = 0, & t \geq 0, x \in \mathbb{R}^N, \\ u(0,\cdot) = u_0, & u_0 \in L^2(\mathbb{R}^N), \\ u_t(0,\cdot) = u_1, & u_1 \in L^2(\mathbb{R}^N). \end{cases}$$

In this paper the author furnishes, under suitable hypotheses on the related initial values, asymptotic expansions of $\|v(t,\cdot)\|_{L^2}^2$, $\|u(t,\cdot)\|_{L^2}^2$ and $\|u(t,\cdot)-v(t,\cdot)\|_{L^2}^2$, as well as of their partial derivatives, as $t \to \infty$.

As far as $\|u(t,\cdot)-v(t,\cdot)\|_{L^2}^2$ is concerned, H. Yang and A. J. Milani [Bull. Sci. Math. **124** (2000), no. 5, 415–433; MR1781556 (2001f:35271)] showed the following decay estimate:

$$\|u(t,\cdot)-v(t,\cdot)\|_{L^2}^2 = O(t^{-N/2-2}) \quad \text{as } t \to \infty, \tag{1}$$

under the assumption

$$v_0 = u_0 + u_1. \tag{2}$$

In the last section of the paper under review, the author gives some conditions on v_0, u_0, u_1 under which the decay estimate (1) is improved. Moreover, he states some results where condition (2) is necessary to get (1).

The method used hinges on Fourier analysis.

Reviewed by *Vita Leonessa*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.