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Derivatives of Bernstein operators and smoothness with Jacobi weights. (English summary)

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For the function $\varphi(x) = \sqrt{x(1-x)}$, $x \in [0,1]$, the authors consider the modulus of smoothness with Jacobi weights, $\omega^2_{\varphi^\lambda}(f,t)_\omega$ $(0 \le \lambda \le 1)$, in the space $C_0([0,1])$ of all functions $f \in C([0,1])$ such that $f(0) = f(1) = 0$. Moreover, consider a sequence $(B_n)_{n\ge1}$ of the Bernstein operators on $C([0,1])$.

After showing that, for every $f \in C_0([0,1])$,

$$|\omega(x)\varphi^{2\lambda}B_n''(f)(x)| \le \frac{C}{\delta_n(x)}\omega^2_{\varphi^\lambda}(f, \sqrt{\delta_n(x)})_\omega,$$

where $\delta_n(x) = \varphi^{2(1-\lambda)}(x)\max\{n^{-1}\varphi^{-2}(x), 1\}/n$, the authors obtain an equivalence theorem between the derivatives of Bernstein operators and $\omega^2_{\varphi^\lambda}(f,t)_\omega$. Precisely, they show that, for every $f \in C_0([0,1])$ and for $0 < \alpha < 2$,

$$|\omega(x)\varphi^{2\lambda}B_n''(f)(x)| = O(\delta_n^{(\alpha-2)/2})$$

if, and only if,

$$\omega^2_{\varphi^\lambda}(f,t)_\omega = O(t^\alpha).$$

Thus, the corresponding results without weights are generalized.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.