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Gupta, Vijay [Gupta, Vijay[1]]
A note on modified Phillips operators. (English summary)
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Let P_n be the n-th Phillips operator defined as follows:

$$(1)\quad P_n(f,x) = n\sum_{k=1}^{+\infty} e^{-nx}\frac{(nx)^k}{k!}\int_0^{+\infty} e^{-nt}\frac{(nt)^{k-1}}{(k-1)!}f(t)\,dt + e^{-nx}f(0),\quad x\geq 0.$$

In this paper the author shows how to modify such operators in order to improve the rate of convergence on the space $C_{\rm b}([0,+\infty))$ of all bounded and continuous functions on $[0,+\infty)$.

The modification presented is obtained by replacing in (1) the variable x with a sequence $(r_n(x))_{n\geq 1}$ of real-valued continuous functions defined on $[0,+\infty)$ such that $0\leq r_n(x) < +\infty$. By setting

$$r_n(x) := r_n^*(x) = \frac{-1+\sqrt{1+n^2x^2}}{n},$$

it can be seen that the relevant operators $P_n^*(f,x)$ map $C_{\rm b}([0,+\infty))$ into itself and, for every $f\in C_{\rm b}([0,+\infty))$ and $x\geq 0$, one has

$$|P_n^*(f,x)-f(x)|\leq 2\omega(f,\delta_{n,x}),$$

where $\delta_{n,x}=\sqrt{2x(x-r_n^*(x))}$ and $\omega(f,\delta)$ is the usual modulus of smoothness of first order. This error estimate is better than

$$|P_n(f,x)-f(x)|\leq 2\omega\left(f,\sqrt{\frac{2x}{n}}\right),$$

which is known to hold for each $f\in C_{\rm b}([0,+\infty))$ in the whole $[0,+\infty)$.

It is also proved that the rate of convergence of $P_n^*(f,x)$ is better than that of $P_n(f,x)$ also for f from a Lipschitz class.

Reviewed by *Vita Leonessa*