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Quantitative convergence theorems for a class of Bernstein-Durrmeyer operators preserving linear functions. (English summary)

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In this paper the authors continue the study of a class of one-parameter operators U_n^ρ ($\rho > 0, n \geq 1$) of Bernstein-Durrmeyer type started in 2007 by R. Păltănea and extended in [H. H. Gonska and R. Păltănea, Czechoslovak Math. J. **60(135)** (2010), no. 3, 783–799; MR2672415 (2011h:41025)], where several properties of the U_n^ρ's can be found. In particular one has that for $\rho = 1$ they turn into the so-called "genuine Bernstein-Durrmeyer operators" U_n; moreover, for every $f \in C([0,1])$,

$$\lim_{\rho\to\infty} U_n^\rho(f) = B_n(f) \quad \text{uniformly on } [0,1],$$

$B_n(f)$ being the n-th Bernstein polynomial.

In the present paper the authors first discuss the case $\rho \to 0$, proving, among other things, that, for every $f \in C([0,1])$,

$$\lim_{\rho\to 0} U_n^\rho(f) = B_1(f) = f(0)(e_0 - e_1) + f(1)e_1 \quad \text{uniformly on } [0,1],$$

where $e_i(t) = t^i$ for every $t \in [0,1]$ and $i = 0, 1$.

Moreover they prove that, for a fixed $0 < \rho < \infty$, for every $f \in C([0,1])$,

$$\lim_{n\to\infty} U_n^\rho f = f \quad \text{uniformly on } [0,1],$$

where the rate of convergence becomes slower and slower as ρ approaches 0. They remark also that the overiterated operator images $(U_n^\rho f)^m$ converge uniformly to $B_1(f)$ as $m \to \infty$, with ρ and $n \geq 1$ being fixed.

Finally, they prove two quantitative Voronovskaya theorems by using first- and second-order moduli of continuity, respectively, generalizing and improving, in some cases, estimates from previous works.

Reviewed by *Vita Leonessa*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.