MR2785700 (2012d:41026) 41A36 (41A10)
Pop, Ovidiu T.; **Fărcaş, Mircea D.**; **Bărbosu, Dan** (R-NBM-CS)
On a general class of linear and positive operators. (English summary)
An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat. **19** (2011), *no. 1,* 237–254.1844-0835

Starting with the given positive approximation process $(L_m)_{m\geq 1}$ acting on the weighted space $E_w(I)$ (I being a real interval and $w\colon I\to(0,\infty)$ such that $M\leq w(x)$ for some $M>0$ and for any $x\in I$), the authors construct a new positive approximation process $(K_m)_{m\geq 1}$ for which an asymptotic formula as well as an estimate of the rate of convergence hold.

Namely, for every $m\geq 1$, let $L_m\colon E_w(I)\to F(J)$ be the positive linear operator given by

$$L_m(f)(x):=\sum_{k=0}^{p_m}\varphi_{m,k}(x)f(x_{m,k})\quad(x\in J,\ f\in E_w(I)),$$

where J is an interval of $\mathbf{R}$ such that $I\cap J\neq\varnothing$, $x_{m,k}\in I$, $\varphi_{m,k}\geq 0$ on J and p_m can be equal to m or to ∞. Moreover, suppose that

$$\lim_{m\to\infty} L_m(f)(x)=f(x)$$

uniformly on every compact $K\subset I\cap J$. Then the mth operator $K_m\colon E_w(I)\to F(J)$ is defined as

$$K_m(f)(x):=\sum_{k=0}^{p_m}\varphi_{m,k}(x)f(y_{m,k})\quad(x\in J,\ f\in E_w(I)),$$

where the new nodes are chosen in the following way: for $m\geq 1$ and $k\in\{0,1,\dots,p_m\}\cap\mathbf{N}_0$, the nodes $y_{m,k}\in I$ are such that

$$\beta_m:=\sup_{k\in\{0,1,\dots,p_m\}\cap\mathbf{N}_0}|x_{m,k}-y_{m,k}|<\infty$$

for every $m\geq 1$, and

$$\lim_{m\to\infty} m^{2-\alpha_2}\beta_m=0,$$

where $0<\alpha_2<2$ is a suitable constant that depends on L_m.

The class of operators K_m contains many well-known linear operators such as Bernstein operators, Stancu operators, Mirak′yan-Favard-Szász operators, Bleimann-Butzer-Hahn operators, Meyer-König-Zeller operators and Ismail-May operators. The authors end the paper with an application of their construction to all these operators, obtaining new positive approximation processes, the relevant Voronovskaja-type formulas and estimates of the rate of convergence by means of the first-order modulus of smoothness.

Reviewed by *Vita Leonessa*