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On the iterates of positive linear operators. (English summary)
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In this paper the authors give a new method for the study of iterates of positive linear operators that also allows one to compute the iterate limit for many classical operators.

Namely, consider a bounded real interval, for instance $[0,1]$, and let $B_0[0,1]$ be the space of bounded real-valued functions on $[0,1]$ that are continuous at 0 and 1. Moreover, let X and Y be two linear subspaces such that

$$\{e_0, e_1, e_2\} \subset Y \subseteq X \subseteq B_0[0,1],$$

where $e_i(x) = x^i$ ($0 \le x \le 1$, $i = 0, 1, 2$).

Then the main result of this paper states that, if the linear and positive operator $U\colon X \to Y$ preserves linear functions, and if there exists $\varphi \in X$ such that $U\varphi - \varphi$ is continuous and has no roots in $(0,1)$, then one has

$$\lim_{k\to+\infty} U^k f = Lf := f(0)e_0 + (f(1) - f(0))e_1, \quad \text{for all } f \in X,$$

with a limit being uniform provided that $U^k f$ are continuous for each $k \in \mathbb{N}$.

Note that the condition $X \subseteq B_0[0,1]$ and that of the preservation of linear functions by U are essential.

Finally observe that, if in addition φ is such that e_0, e_1, φ form a Chebyshev system (e.g. $\varphi = e_2$) and $Y = C[0,1]$, then the following propositions are equivalent:

(1) $U^k f$ converge uniformly to Lf for all $f \in X$;
(2) $U\varphi - \varphi$ has no roots in $(0,1)$;
(3) U has no interior interpolation points.

This is the case of all classical positive operators that preserve linear functions.

Related results can be found in [I. Raşa, Jaen J. Approx. **1** (2009), no. 1, 27–36; MR2597905 (2011a:47088); Rend. Circ. Mat. Palermo (2) Suppl. No. 82 (2010), 123–142; per bibl.].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.