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On some convergence criteria for nets of positive operators on continuous function spaces. **(English summary)**

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In this paper the author states a new criterion for convergence of nets of positive linear operators on $C(X)$, X compact, towards a positive linear operator T. The main assumptions refer to the subset $\partial_T X$ of interpolation points of T, i.e.

$$\partial_T X := \{x \in X \,|\, T(f)(x) = f(x) \text{ for every } f \in C(X)\},$$

and the possibility to represent it in the following way:

$$\partial_T X = \{x \in X \,|\, \Psi(x) = 0\},$$

where $\Psi \in C(X)$, $\Psi \geq 0$.

This result generalizes a previous one that holds when T is also a projection on $C(X)$ [see F. Altomare, Rend. Mat. (6) **13** (1980), no. 3, 409–429 (1981); MR0609483 (82h:41029)], furnishing a new simple proof for it. Moreover, it can be applied for studying the asymptotic behavior of the iterates of Markov operators.

The paper ends with a discussion of some applications concerning, in particular, the asymptotic behavior of iterates of Bernstein-Schnabl operators that are associated with a positive linear operator (not necessarily a projection) and the iterates of a generalization of the so-called Cesàro operator on $C([0,1])$.

Reviewed by *Vita Leonessa*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.