Numerical Formulations for Accurate Magnetic Field Flow Tracing in Fusion Tokamaks

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Abstract

We consider the problem of accurate tracing of long magnetic field lines in tokamaks for the determination of scrape off layers and plasma boundaries. The problem of the accuracy of the numerical integration is tackled. Criteria for the specific problem are introduced and discussed. Standard ODE (Ordinary Differential Equation) integrators are considered and compared with a volume preserving algorithm, with reference to prescribed accuracy requirements.

1 Introduction

Basic equilibria in Tokamaks are mostly 2-D axisymmetric. Perturbations of such axisymmetric equilibria are then considered in different context, mainly related to plasma stability and control. When realistic estimations of the 3-D plasma (shape, scrape off layer, connection length) have to be evaluated, an accurate tracing of magnetic field lines is required. Once the field is known, either in an analytic representation or as numerical values on a grid, the problem is easily recognized as the tracing of a vector field flow, which turns in the solution of a given ODE system.

Since the length of the field lines in tokamaks can be considerably long, it is not trivial to pursue accurate tracing at an affordable computational cost. The problem of long-term behaviour in ODE systems is faced in several science areas, as for example in classical nonlinear dynamics for the determination of bifurcation diagrams and chaotic attractors. Some research has been carried out to improve properties of algorithms, mainly in the area of the so-called geometric integrators [1]. Differently from that case, where the main interest is in the preservation of some average properties of the solutions, in plasma shape determination in Tokamaks the accuracy of any single line tracing is crucial for the reliable estimation of the quantities under investigation. We here discuss the performance and results of some ODE integrators for the 3-D determination of connection length [2] and the plasma boundary in tokamaks.

2 Field Lines Evaluation

The problem of field lines tracing with assigned field can be viewed as the solution of an ODE:

$$d\mathbf{x}/d\tau = \mathbf{B}(\mathbf{x}) \tag{1}$$

from a specified starting point $\mathbf{x}_0$ at $\tau = 0$, where $\mathbf{B}(\mathbf{x})$ is the magnetic field as function of position $\mathbf{x}$ (we consider stationary magnetic configurations). Therefore, once the required spatial resolution in the mapping of the plasma contour is assigned, we need an accurate solution of (1) also for very long integration lengths, at an affordable computational burden.

The problem can be successfully faced with standard ODE integrators, but strict control and verification of integration error is needed in order to ensure reliable results in the plasma shape evaluation. Moreover, it is desirable that intrinsic invariant properties are preserved in the numerical solution. In particular, due to the solenoidal property of the magnetic field ($\nabla \cdot \mathbf{B} = 0$) the correct integration of (1) is "volume preserving", like Lagrangian trajectories in incompressible fluids, which coincide with the velocity field lines in stationary conditions. Such basic property can be used a posteriori as a figure of merit of the integrator.

It is possible to implement volume preserving integration schemes [1,3] that a priory guarantees such invariance. In particular, we implemented a vector potential based splitting method, belonging to the class of "Generation Functions" algorithms [1]. It is based on the idea of splitting the 3-D $\mathbf{B}$ field as a sum of 2-D divergence free components, as properly obtained from a vector potential $\mathbf{B} = \nabla \times \mathbf{A}$, integrating them via any simplectic method, so preserving volume. By expressing both the flux density field and the vector potential in Cartesian components, it is possible to consider the splitting $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_2$ with:

$$\mathbf{B}_1 = \nabla A_x \times \mathbf{i}_x;\ \mathbf{B}_2 = \nabla A_y \times \mathbf{i}_y;\ \mathbf{B}_3 = \nabla A_z \times \mathbf{i}_z \tag{2}$$

Each $\mathbf{B}_i$ component is 2-D and divergence free. For such problem the Midpoint Rule (MR) method:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta \cdot \mathbf{B}_i\left(\frac{\mathbf{x}_{k+1} + \mathbf{x}_k}{2}\right) \tag{3}$$

is demonstrated to be symplectic (area preserving) [3].

On that basis an algorithm for the tracing and representation of the field lines in Tokamaks has been implemented in Matlab. It allows (i) to specify the field configuration both directly from numerical data, as well as from a proper analytical reconstruction of the field; (ii) to perform accurate 3D simulation of the field lines allowing the choice of different ODE integrators; (iii) to evaluate accuracy in terms of magnetic flux and volume preservation; (iv) to calculate the connection length (the length of the field line up to the point where it intersects the solid wall), Poincarè intersections on poloidal sections, visualizing results in different view modes.

3 Examples, results, accuracy and CPU time

We discuss about the performance of a standard ODE algorithm available under the MATLAB platform (ode45 based on Runge-Kutta method) [4], as well as the implemented Volume Preserving MR formulation (3). A general example of the field lines tracing is shown in Figure 1. In order to achieve satisfactory accuracy, the maximum step is selected so as to get a maximum step of 2 deg in toroidal angle φ.

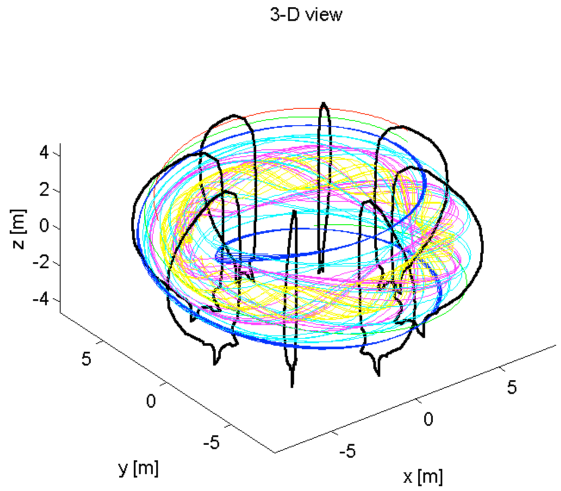

Figure 1: 3-D plot of typical field lines in a Tokamak

The error assessment was performed by evaluating the conservation of the Jacobian of the mapping (1), as well as the poloidal magnetic flux Ψ, which is invariant in axisymmetric cases. A comparison of the ODE integrators is given In Table I. The volume $V_h(\tau)$ is approximated by considering time evolution of the vertices of a finite tetrahedron (*h* is the edge size) around the starting point. The quantity $V(\tau)/V(0)$ is estimated via second order extrapolation as:

$$V(\tau)/V(0) = 2V_{h/2}(\tau)/V_{h/2}(0) - V_h(\tau)/V_h(0) \qquad (4)$$

As for the MR scheme implemented, comparable results are obtained. The volume preservation is paid in terms of higher computational time for a given accuracy, as expected [5].

Figure 2 shows how to use the field tracing to calculate the plasma-wall gap at a given position: the connection length is finite only outside the plasma.

Algorithm	ε[m]	V/V0-1	ΔΨ	CPU [s]
ODE 45	1e-7	-5.7936e-4	0.7e-3	153
MR	2e-4	-	0.91	3590

Table I. Comparison of algorithms on a typical configuration for 1 km long field line evaluation (ε is the absolute accuracy) and the edge size of the tetrahedron *h*=1e-6 m.

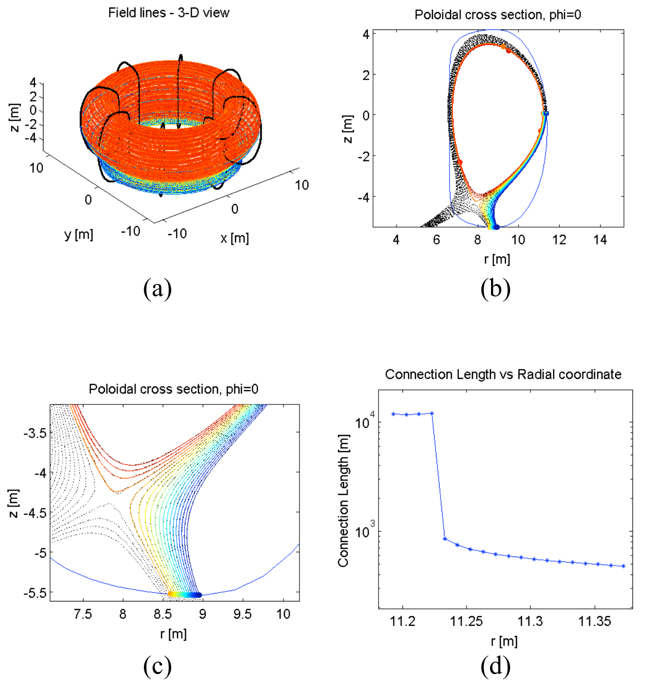

Figure 2: Plasma boundary: (a) 3-D plot; (b) the corresponding poloidal plane projection; (c) a detail of cross section close to the X–point; (d) connection lengths in a layer of 20 mm, identifying the plasma boundary (the integration is truncated after about 20 Km).

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