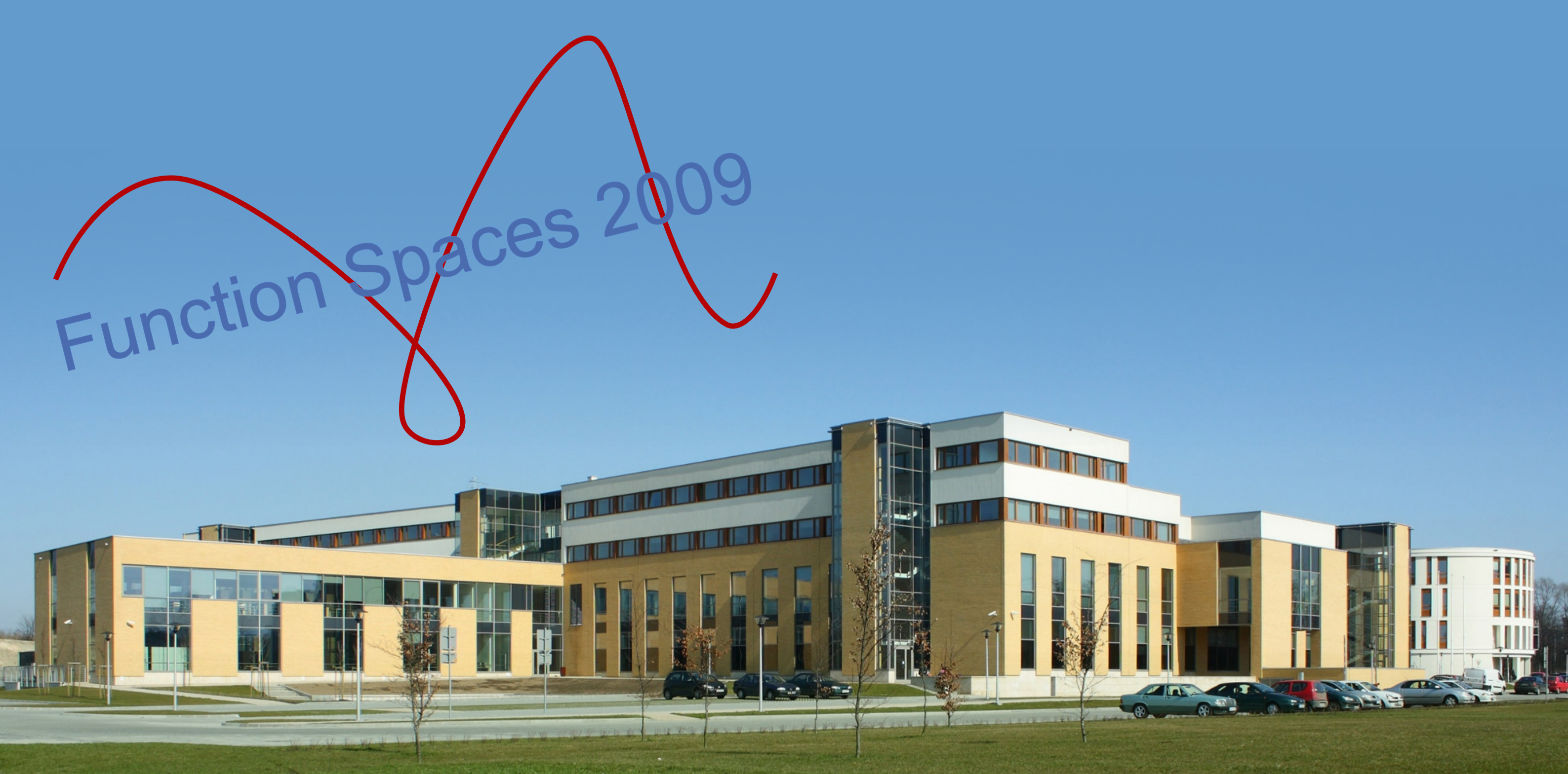

FUNCTION SPACES IX

Krakow, 6th July 2009 - 11th July 2009

Organizers:

Faculty of Mathematics and Computer Science
Jagiellonian University, Krakow, Poland

Faculty of Mathematics and Computer Science
Adam Mickiewicz University, Poznan, Poland

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JULY 6 – 10, 2009

KRAKÓW, POLAND

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Invited speakers

1. **Carlo Bardaro**, University of Perugia, Perugia, Italy.
2. **Fernando Bombal**, Universidad Complutense de Madrid, Madrid, Spain.
3. **Paul Butzer**, University of Aachen, Aachen, Germany.
4. **Charles Castaing**, University of Montpellier, Montpellier, France.
5. **Yunan Cui**, Harbin University of Science and Technology, Harbin, China.
6. **Antonio Granero**, Universidad Complutense de Madrid, Madrid, Spain.
7. **Dorothee Haroske**, Friedrich Schiller University of Jena, Jena, Germany.
8. **Nigel Kalton**, University of Missouri-Columbia, Columbia, USA.
9. **Anna Kamińska**, University of Memphis, Memphis, USA.
10. **Miroslaw Krbec**, Czech Academy of Science, Czech Republic.
11. **Lech Maligranda**, Lulea University of Technology, Lulea, Sweden.
12. **Józef Myjak**, University of L'Aquila and AGH University of Science and Technology, L'Aquilla and Krakow, Italy and Poland.
13. **Zuhair Nashed**, University of Central Florida, Orlando, Florida, USA.
14. **Pier Luigi Papini**, University of Bologna, Bologna, Italy.
15. **Aleksander Pełczyński**, Instytut Matematyczny PAN, Warsaw, Poland.
16. **Beata Randrianantoanina**, Miami University, Oxford, Ohio, USA.
17. **Brailey Sims**, University of Newcastle, Newcastle, Australia.
18. **Hans Triebel**, Friedrich Schiller University, Jena, Germany.
19. **Gianluca Vinti**, University of Perugia, Perugia, Italy.
20. **Baoxiang Wang**, Beijing University, Beijing, China.
21. **Yuwen Wang**, Harbin Normal University, Harbin, China.
22. **Przemyslaw Wojtaszczyk**, University of Warsaw, Warsaw, Poland.
23. **Congxin Wu**, Harbin Normal University, Harbin, China.
24. **Dachun Yang**, School of Mathematical Sciences, Beijing Normal Universiy, Beijing, China.

CONTENTS

PLENARY LECTURES

ABSTRACTS

Approximation by Mellin-type convolution operator

Carlo Bardaro

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Our objective is a sequence of integral operators of the form

$$(T_n f)(s) = \int_G K_n(st^{-1}) f(t) dt, \; s \in G$$

where G is a topological group and dt is the Haar measure over G. The above operators were studied in an extensive paper by P.L. Butzer and S. Jansche ([4]) in connection with the Mellin Trasform Theory. Here we study the pointwise convergence to f and some asymptotic formulae are established for twice differentiable functions. In particular the multiplicative groups $(\mathbb{R}_+, \cdot)$ and $(\mathbb{R}^2_+, \cdot)$ are considered. Applications to some classical operators are given. Moreover some extensions are obtained in case of certain special nonlinear Mellin-type operators. The theory of approximation by nonlinear operators was introduced by J. Musielak in 1986 (see [5]) and then was outlined in a systematic form in [3].

References

[1] C. Bardaro, I. Mantellini, *Voronovskaya-type estimate for Mellin convolution operators*, Results in Math, 50(1-2)(2007), 1-16.

[2] C. Bardaro, I. Mantellini, *Asymptotic formulae for bivariate Mellin convolution operators*, Analysis in Theory and Appl., 24(4)(2008),377- 394.

[3] C. Bardaro, J. Musielak, G. Vinti, *Nonlinear integral oper- ators and applications*, De Gruyter Series in Nonlinear Analysis and Appl., Vol.9, W. De Gruyter, Berlin, New York, 2003.

[4] P.L. Butzer, S. Jansche, *A direct approach to the Mellin trans- from theory*, J. Fourier Analysis and Appl. 3(4)(1997), 325-376.

[5] J. Musielak, *On some approximation problems in modular spaces*, in Constructive Function Theory 1981, (Proc. Int. Conf. Varna, June 1-5, 1981), 455-461, Publ. House Bulgarian Acad. Sci, Sofia, 1983.

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Factorization of multilinear and holomorphic functions on Banach spaces

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It is known that a holomorphic function of bounded type $f \in \mathcal{H}_b(E)$ admits a factorization of the form $f = g \circ S$, with S a (weakly) compact linear operator and g a holomorphic function of bounded type if and only if the derivative $df : E \to E^*$ takes bounded sets into relatively (weakly) compact sets. We extend these results to many other operator ideals by associating to them suitable families of bounded sets. This method allows us to obtain new and some known results with simpler proofs. Of course, The starting point is a factorization theorem for multilinear mappings and polynomials.

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Large The Sampling Theorem — A Central Theorem of Analysis

P.L. Butzer, P.J.S.G. Ferreira, J.R. Higgins, G. Schmeisser, R.L. Stens

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The classical sampling theorem of signal analysis, connected with the names of C. Shannon (1948/49), V. A. Kotelnikov (1933), E. T. Whittaker (1915), and many others, states that a function $f \in B_{\pi w}$ has the representation

$$f(z) = \sum_{k\in\mathbb{Z}} f\Big(\frac{k}{w}\Big)\text{sinc}w\Big(z - \frac{k}{w}\Big) \quad (z \in \mathbb{C}), \tag{1}$$

where the Berstein space B^p_σ for $\sigma > 0$, $1 \le p \le \infty$, is the space of all entire functions $f\colon \mathbb{C} \to \mathbb{C}$ that belong to $L^p(\mathbb{R})$ when restricted to the real axis as well as are of exponential type σ, so that they satisfy the inequality $f(z) = \mathcal{O}_f\big(\exp(\sigma|\mathfrak{Im} z|\big)$ for $|z| \to \infty$; the sinc-function is given by $\text{sinc}z := \sin(\pi z)/(\pi z)$ for $z \ne 0$, and $:= 1$ for $z = 0$.

The first aim of this lecture is to show that this formula is equivalent to several other striking formulae of mathematical analysis such as:

General Parseval formula for $f, g \in B^2_{\pi w}$:

$$\int_{\mathbb{R}} f(t)\overline{g}(t)\,dt = \frac{1}{w}\sum_{k\in\mathbb{Z}} f\Big(\frac{k}{w}\Big)\overline{g}\Big(\frac{k}{w}\Big), \tag{2}$$

Reproducing kernel formula for $f \in B^2_{\pi w}$:

$$f(z) = w\int_{\mathbb{R}} f(t)\,\text{sinc}w(z-t)\,dt \qquad (z \in \mathbb{C}), \tag{3}$$

such as **Poisson's summation formula (particular case)** for $f \in B^1_{\pi w}$:

$$\int_{\mathbb{R}} f(t)\,dt = \frac{2}{w}\sum_{k\in\mathbb{Z}} f\Big(\frac{2k}{w}\Big). \tag{4}$$

In the second part of the lecture we discuss the equivalence of these results in the more general instance of the spaces

$$F^p := \big\{f\colon \mathbb{R} \to \mathbb{C}\,;\, f \in L^p(\mathbb{R}) \cap C(\mathbb{R}),\, \widehat{f} \in L^1(\mathbb{R})\big\} \qquad (p = 1, 2),$$

where $\widehat{f}$ denotes the Fourier transform of f, and

$$S^p_w := \big\{f\colon \mathbb{R} \to \mathbb{C}\,;\, \{f(k/w)\}_{k\in\mathbb{Z}} \in \ell^p(\mathbb{Z})\big\} \qquad (w > 0).$$

In this frame all the formulae mentioned above hold only approximately in the sense that they have to be equipped with remainder (additional) terms. More precisely, the classical sampling theorem (1) is replaced by the

Approximate sampling theorem for $f \in F^2 \cap S^1_w$:

$$f(t) = \sum_{k\in\mathbb{Z}} f\Big(\frac{k}{w}\Big)\operatorname{sinc}(wt-k) + (R_w f)(t) \qquad (t \in \mathbb{R}), \tag{5}$$

where the error term is given by

$$(R_w f)(t) := \frac{1}{\sqrt{2\pi}} \sum_{n\in\mathbb{Z}} \Big(1 - e^{-it2\pi wn}\Big) \int_{(2n-1)\pi w}^{(2n+1)\pi w} f^\wedge(v)e^{itv}\,dv.$$

In case of the general Parseval formula (2) one has even to add two remainder terms, leading to

Generalized Parseval decomposition formula for $f \in F^2 \cap S^1_w$, and $g \in F^2$:

$$\begin{aligned}\int_{\mathbb{R}} f(u)\overline{g}(u)\,du &= \frac{1}{w}\sum_{k\in\mathbb{Z}} f\Big(\frac{k}{w}\Big)\overline{g}\Big(\frac{k}{w}\Big)\\ &\quad - \frac{1}{w}\sum_{k\in\mathbb{Z}} f\Big(\frac{k}{w}\Big)\frac{1}{\sqrt{2\pi}}\int_{|v|\ge\pi w} \widehat{\overline{g}}(v)e^{ikv/w}\,dv\\ &\quad + \int_{\mathbb{R}} (R_w f)(u)\overline{g}(u)\,du.\end{aligned} \tag{6}$$

Similarly, the reproducing kernel formula (3) has to be equipped with two additional terms:

Approximate reproducing kernel formula for $f \in F^2 \cap S^1_w$:

$$\begin{aligned}f(t) &= w\int_{\mathbb{R}} f(u)\operatorname{sinc}w(t-u)\,du\\ &\quad + (R_w f)(t) - w\int_{\mathbb{R}} (R_w f)(u)\operatorname{sinc}w(t-u)\,du.\end{aligned} \tag{7}$$

Finally, the particular case of Poisson's summation formula for $f \in B^1_{\pi w}$ (4) is generalized to the classical form:

Poisson's summation formula for $f \in F^1$ with $\widehat{f} \in S^1_w$:

$$\sqrt{2\pi}w\sum_{k\in\mathbb{Z}} f(x+2k\pi w) = \sum_{k\in\mathbb{Z}} \widehat{f}\Big(\frac{k}{w}\Big)e^{ikx/w} \quad (a.\,e.). \tag{8}$$

Clearly, if the functions involved belong to the (particular) Bernstein spaces $B^2_{\pi w}$, then, according to the Paley-Wiener theorem, the remainder terms in (5), (6) and (7) vanish, and one obtains the particular versions (1), (2) and (3). Similarly, for $f \in B^1_{\pi w}$ and $x = 0$, Poisson's summation formula (8) reduces to the particular case (4).

A note on James boundaries

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The aim of this talk is to study the behavior of $\overline{co}(B)$ with respect to $\overline{co}^{w^*}(K)$, when K is a w^*–compact subset of a dual Banach space X^* and B is a boundary of K. Recall that a subset $B \subset K$ is said to be a *(James) boundary* of K if every $x \in X$ attains on B its maximum on K. For instance, the set of extreme points $Ext(K)$ of K is the "most natural" boundary of K. If B is a boundary of K, then $\overline{co}^{w^*}(B) = \overline{co}^{w^*}(K)$ but, in general, $co(B) \neq \overline{co}^{w^*}(K)$. The investigation of the conditions under which $\overline{co}(B) = \overline{co}^{w^*}(K)$ and the properties transferred from the boundary B to the whole $\overline{co}^{w^*}(K)$ is a research field of increasing interest. We give two types of results:

(1)**Localization results.** In [1, 2] it is proved that K contains a structure, that we call a $w^* - \mathbb{N}$-family, and also a copy of the basis of $\ell_1(\mathfrak{c})$, whenever $\overline{co}(K) \neq \overline{co}^{w^*}(K)$. What happens if $\overline{co}(B) \neq \overline{co}^{w^*}(K)$? We show that in many cases, when $\overline{co}(B) \neq \overline{co}^{w^*}(K)$, there exist inside K — even inside B in some situations — a $w^* - \mathbb{N}$–family and a copy of the basis of $\ell_1(\mathfrak{c})$.

(2)**Quantitative results**. We show that for certain classes of convex subsets C of X^* there exist a constant M such that $dist(\overline{co}^{w^*}(K), C) \leq M dist(B, C)$ for every w^*–compact subset K of X^* and every boundary $B \subset K$.

References

[1] A. S. Granero and M. Sánchez, *Distances to convex sets*, Studia Math., 182 (4) (2007),165- 181.
[2] A. S. Granero and M. Sánchez, *Convex w^*-closures versus convex norm-closures*, J. Math. Anal. Appl., 350 (2009), 485-497.

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Muckenhoupt weighted function spaces of Besov and Triebel-Lizorkin type: embeddings, traces, and singularities

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We study weighted spaces of type $B^s_{p,q}(\mathbb{R}^n, w)$ and $F^s_{p,q}(\mathbb{R}^n, w)$, where $w \in \mathcal{A}_\infty$ belongs to some Muckenhoupt class. Our approach starts from typical examples of weight functions like

$$w_{\alpha,\beta}(x) = \begin{cases} |x|^\alpha, & |x| \le 1 \\ |x|^\beta, & |x| \ge 1 \end{cases}$$

and leads to the general setting $w \in \mathcal{A}_\infty$. We investigate such weighted function spaces with respect to embeddings of type

$$B^{s_1}_{p_1,q_1}(\mathbb{R}^n, w_1) \hookrightarrow B^{s_2}_{p_2,q_2}(\mathbb{R}^n, w_2)$$

to their traces on the hyperplane $x_n = 0$ in $\mathbb{R}^n$ (usually identified with $\mathbb{R}^{n-1}$),

$$\mathrm{tr}_{\mathbb{R}^{n-1}} B^s_{p,q}(\mathbb{R}^n, w_{\alpha,\beta})$$

and, finally, characterise singularity behaviour in terms of their growth envelopes,

$$\mathfrak{E}_G \left(B^s_{p,q}(\mathbb{R}^n, w) \right).$$

Here the prototype $w_{\alpha,\beta}$ is meanwhile well understood, whereas there remain some open questions in the general setting. In some cases, however, like $w \in \mathcal{A}_1$, we have complete answers now. We briefly discuss some phenomena and describe ideas how to control the interplay between typical features of the weight and the considered function spaces. We shall also indicate a few applications of our results.

Partly, this is based on joint work with L. Skrzypczak (Poznań) and H.-J. Schmeisser (Jena).

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Recent developments in nonlinear Banach space theory

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We will talk about some recent advances in the uniform and Lipschitz classification of Banach spaces, concentrating on developments since 2000 when the book of Benyamini and Lindenstrauss appeared.

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Isomorphic copies in the lattice E and its symmetrization $E^{(*)}$ with applications to Orlicz-Lorentz spaces

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The paper is devoted to the isomorphic structure of symmetrizations of quasi-Banach ideal function or sequence lattices. The symmetrization $E^{(*)}$ of a quasi-Banach ideal lattice E of measurable functions on $I = (0, a)$, $0 < a \leq \infty$, or $I = \mathbb{N}$, consists of all functions with decreasing rearrangement belonging to E. For an order continuous E we show that every subsymmetric basic sequence in $E^{(*)}$ which converges to zero in measure is equivalent to another one in the cone of positive decreasing elements in E, and conversely. Among several consequences we show that, provided E is order continuous with Fatou property, $E^{(*)}$ contains an order isomorphic copy of ℓ^p if and only if either E contains a normalized ℓ^p-basic sequence which converges to zero in measure, or $E^{(*)}$ contains the function $t^{-1/p}$.

We apply these results to the family of two-weighted Orlicz-Lorentz spaces $\Lambda_{\varphi,w,v}(I)$ defined on $I = \mathbb{N}$ or $I = (0, a)$, $0 < a \leq \infty$. This family contains usual Orlicz-Lorentz spaces $\Lambda_{\varphi,w}(I)$ when $v \equiv 1$ and Orlicz-Marcinkiewicz spaces $M_{\varphi,w}(I)$ when $v = 1/w$. We show that for a large class of weights w, v, it is equivalent for the space $\Lambda_{\varphi,w,v}(0, 1)$, and for the non-weighted Orlicz space $L_\varphi(0, 1)$ to contain a given sequential Orlicz space h_ψ isomorphically as a sublattice in their respective order continuous parts. We provide a complete characterization of order isomorphic copies of ℓ^p in these spaces over $(0, 1)$ or $\mathbb{N}$ exclusively in terms of the indices of φ. If $I = (0, \infty)$ we show that the set of exponents p for which ℓ^p lattice embeds in the order continuous part of $\Lambda_{\varphi,w,v}(I)$ is the union of three intervals determined respectively by the indices of φ and by the condition that the function $t^{-1/p}$ belongs to the space.

This is a joint work with Yves Raynaud from University VI in Paris, France.

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On dimension-free Sobolev inequalities

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Imbedding theorems for Sobolev spaces give an information about local improvement of integrability, which depends on the dimension of the underlying Euclidean space and this gives rise to a natural question what can be said about integrability properties of Sobolev functions independently of the dimension when the other parameters (smoothness and integrability of the weak derivatives) remain the same.

This talk will concern such residual phenomenon either in terms of imbeddings into Zygmund spaces, that is, establishing a logarithmic improvement of the integrability, or with use of weighted targets for the imbeddings. The topic is related to the Gross logarithmic inequality and its generalizations, best constants in imbedding theorems and extrapolation properties of Sobolev imbeddings.

This is a joint work with H.-J. Schmeisser.

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On Cesàro sequence and function spaces

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Cesàro sequence spaces ces_p for $1 < p < \infty$ are defined as the set of all real sequences $x = x_k$ such that $\|x\|_{c(p)} = \left[\sum_{n=1}^{\infty} \left(\frac{1}{n} \sum_{k=1}^{n} |x_k|\right)^p\right]^{1/p} < \infty$. They are separable, refexive, not symmetric and not B-convex Banach spaces (cf. [1, 2]) but they have the fixed point property (cf. [4, 5]). The corresponding *Cesàro function spaces* $Ces_p(I)$ on both $I = [0, 1]$ and $I = [0, \infty)$ are the classes of Lebesgue measurable real functions f on I such that

$$\|f\|_{C(p)} = \left[\int_I \left(\frac{1}{x} \int_0^x |f(t)| dt\right)^p dx\right]^{1/p} < \infty$$

Cesàro function spaces are separable, strictly convex, not symmetric but, in contrast to the sequence spaces, not reflexive and they do not have the fixed point property (cf. [1]). The structure of the Cesàro function spaces $Ces_p(I)$ is investigated. Their dual spaces, which equivalent norms have different description on $[0, 1]$ and $[0, \infty)$, are described. The spaces $Ces_p(I)$ for $1 < p < \infty$ are not isomorphic to any $L^q(I)$ space with $1 \le q \le \infty$. They have "near zero" complemented subspaces isomorphic to l^p and "in the middle" contain an asymptotically isometric copy of l^1 and also a copy of $L^1[0, 1]$. They do not have Dunford-Pettis property. Cesàro function spaces on $[0, 1]$ and $[0, \infty)$ are isomorphic for $1 < p < \infty$. Moreover, the Rademacher functions span in $Ces_p[0, 1]$ an uncomplemented space which is isomorphic to l^2.

References

[1] S. Astashkin and L. Maligranda, *Cesàro function spaces fail the fixed point property*, Proc. Amer. Math. Soc. **136**(2008), no. 12, 4289-4294.

[2] S. Astashkin and L. Maligranda, *Structure of Cesàro function spaces*, manuscript.

[3] G. Bennett, *Factorizing the Classical Inequalities*, Mem. Amer. Math. Soc. **120**, Providence 1996.

[4] Y. Cui and H. Hudzik, *Some geometric properties related to fixed point theory in Cesàro sequence spaces*, Collect. Math. **50**(1999), 277-288.

[5] Y. Cui, C. Meng and R. Płuciennik, *Banach-Saks property and property (fi) in Cesàro sequence spaces*, Southeast Asian Bull. Math. **24**(2000), 201-210.

[6] L. Maligranda, N. Petrot and S. Suantai, *On the James constant and B-convexity of Cesàro and Cesàro-Orlicz sequence spaces*, J. Math. Anal. Appl. **326**(2007), no. 1, 312-331.

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Inverse Problems, Moment Problems, and Signal Processing: Un Menage a Trois

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Inverse Problems deal with determining for a given input-output system an input that produces an observed output, or of determining an input that produces a desired output (or comes as close to it as possible), often in the presence of noise. Most inverse problems are ill-posed.

Signal Analysis/Processing deals with digital representations of signals and their analog reconstructions from digital representations. Sampling expansions, filters, reproducing kernel spaces, various function spaces, and techniques of functional analysis, computational and harmonic analysis play pivotal roles in this area.

Moment problems deal with recovery of a function or signal from its moments, and the construction of efficient stable algorithms for determining or approximating the function. Again this is an ill-posed problem. Interrelated applications of inverse problems, signal analysis and moment problems arise, in particular, in image analysis and recovery and many areas of science and technology.

Several decades ago the connections among these areas (inverse problems, signal processing, and moment problems) was rather tenuous. Researchers in one of these areas were often unfamiliar with the techniques and relevance of the other two areas. The situation has changed drastically in the last 20 years. The common thread among inverse problems, signal analysis, and moment problems is a canonical problem: recovering an object (function, signal, picture) from partial or indirect information about the object.

In this talk we will provide perspectives on some aspects of this interaction with emphasis on ill-posed problems for operator and integral equations, and ill-posed problems in signal processing. We will show that function spaces, in particular reproducing kernel spaces and certain subspaces of L^p, play a pivotal role in this interaction.

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Some parameters of Banach spaces

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Among the many contants that can be defined in an infinite dimensional Banach space X, Kottman's constant, defined and studied in the seventies, has been used also recently, to estimate extension constants concerning Lipschitz mappings in some spaces. Kottman's constant, $K(X)$, measures the largest possible separation that infinite sequences on the unit sphere (or the unit ball) can have. We give some estimates for $K(X)$, mainly based on the modulus of continuity of X. We also compare such constant with some other ones (like the so called thickness constant).

Moreover, we discuss some related constants, introduced and studied recently. Finally we indicate some problems concerning $K(X)$ which appear to be still open.

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On properties (k) and Dunford-Pettis

Aleksander Pełczyński

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The material is a part of the forthcoming paper: "Some approximation properties of Banach spaces and Banach lattices" joint with T. Figiel and W.B. Johnson.

Recall that (y_m) is a CCC sequence of (x_n) provided that there are a sequence (c_n) of non-negative numbers and a strictly increasing sequence of positive integers (n_m) such that

$$y_m = \sum_{n=n_m}^{n_{m+1}-1} c_n x_n; \quad \sum_{n=n_m}^{n_{m+1}-1} c_n = 1; \quad c_n \geq 0 \quad (n, m = 1, 2, \ldots).$$

Definition 1. *Banach space X has property (k) provided that for every $(x_n^*) \subset X$ with $x_n^* \overset{w^*}{\to} 0$ there exists a CCC sequence (y_m^*) of (x_n^*) such that for each linear operator $u : L_1[0,1] \to X$,*

$$\lim_m y_m^*(u f_m) = 0; \quad \left((f_m) \subset L_1[0,1],\ f_m \overset{weakly}{\to} 0,\ \sup_m \|f_m\|_\infty < \infty \right).$$

Property (k) appears implicitly in [JOHNSON 1997]; it is a modification of an invariant invented by KWAPIEŃ (cf.Kalton- Pełczyński 1997). There is some analogy between property (k) and Grothendiecks characterization of the Dunford-Pettis property. Weakly sequentially complete lattices with weak unity and separable preduals of von Neumann algebras has property (k). As application we give a criterion for kernels of quotient maps of Banach lattices to be uncomplemented in their second duals, an example which shows that the SCP (=separable complementation property) is not preserved when passing to subspaces. This answers a question posed by [PLICHKO -YOST 2001].

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The fixed point property via dual space properties

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We show that if a Banach space X fails to have the fixed point property for nonexpansive mappings, then, for every $\varepsilon > 0$, there exists a countably infinite set $A = \{y_1*, y_2*, \ldots\}$ in the unit sphere of $X*$ such that, for every $i, j \in \mathbb{N}$, $\|y_i * \pm y_j *\| \geq 2 - \varepsilon$. In particular, this generalizes a recent result of J. García-Falset, E. Llorens-Fuster and E.M. Mazcuñan-Navarro that uniformly nonsquare Banach spaces have the fixed point property (J. Functional Analysis 2008).

We also prove that a Banach space has the weak fixed point property if its dual space has a weak* sequentially compact unit ball and the dual space satisfies the weak* uniform Kadec-Klee property.

Joint work with P.N. Dowling and B. Turett.

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Bases in function spaces, numerical integration and discrepancy

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We survey some recent results about Haar bases and Faber bases in function spaces with dominating mixed smoothness and their use in connection with numerical integration and discrepany.

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Approximation Theory and Signal/Image Reconstruction. Applications to Endovascular Surgery

Gianluca Vinti

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In the last century, Whittaker, Kotelnikov and Shannon stated the celebrated WKS-sampling theorem, which can be formulate as follows ([4]):

Let $f \in L^2(\mathbb{R})$ be a function with the support of its Fourier transform $\widehat{f}$ contained in an interval $[-\pi w, \pi w]$, for $w > 0$, (i.e. f is a *band-limited* function); then f can be completely reconstructed on the whole real time-axis from its samples values by means of the interpolation series:

$$f(t) = \sum_{k=-\infty}^{+\infty} f\left(\frac{k}{w}\right) sinc[\pi(wt-k)],\ t \in \mathbb{R}$$

where $sinc(t) = \frac{\sin t}{t}$, $t \neq 0$ and $sinc(0) = 1$.

Although this theorem had a strong impact in communication theory as in image processing, the above interpolation formula has some disadvantages in terms of its applications; moreover the band-limitation to $[-\pi w, \pi w]$ is a rather restrictive condition. Several contributions have been given in order to weaken the band-limitation, but the most important contribution, based on an approximation theorys approach, has been given by P.L. Butzer and his school at Aachen ([3]) considering a family of discrete operators, called "generalized sampling series" of the form

$$(S_w^{\varphi} f)(t) := \sum_{k=-\infty}^{+\infty} f\left(\frac{k}{w}\right) \varphi(wt-k),\ t \in \mathbb{R},\ k \in \mathbb{Z},\ w > 0$$

where φ is a continuous function with compact support on $\mathbb{R}$.

For the above operators, behind pointwise and uniform convergence results for continuous signals together with the study of the rate of approximation, we will discuss the approach in L^p-setting ([2]), which allow to treat signals not necessarily continuous nor of finite energy, and some extension of the theory to the more general frame of Orlicz spaces ([1]). The above approach naturally brings to BV-functions, which seems to be the "good space" in order to reconstruct discontinuous signals which play an important rule in image analysis since discontinuities are usually located in the edge or contours of the images where jumps of grey levels occur. It is well-known the importance of images reconstruction in medical diagnostics. Recently the development of the endovascular surgery increased considerably the impact of the image processing in medical treatments. In the last part of this talk, some applications in this direction will be discussed.

References

[1] C. Bardaro, J.Musielak and G. Vinti, *Nonlinear Integral Operators and Applications*, De Gruyter Series in Nonlinear Analysis and Appl., W. de Gruyter, Berlin, New York, Vol. 9 (2003).
[2] C.Bardaro, P.L. Butzer, R.L. Stens and G. Vinti, *Prediction by samples from the past with error estimates covering discontinuous signals*, to appear in IEEE- Transaction on Information Theory.
[3] P.L. Butzer and R.J. Stens, *Linear prediction by samples from the past Shannon Sampling & Interpolation Theory II*, R.J. Marx II Ed, Springer-Verlag, New York (1993), 157-183.
[4] C.E. Shannon, *Communication in the presence of noise*, Proc. IRE 37 (1949), 10-21.

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Metric Generalized Inverse and Geometry of Banach Spaces

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In this paper, we summarize some new results on the continuity of single-valued metric generalized inverses for linear operators in Banach spaces, and on the single-valued homogenous selections and the single-valued continuous selections of set-valued metric generalized inverse for linear operators in Banach spaces. All these results were obtained by using the geometric properties of Banach spaces, such as the approximate compactness, middle-point locally uniform convexity, H-property and generalized orthogonal decomposition in Banach spaces, e.t.. Furthermore, we also give some new results on the metric generalized inverses of multi-valued linear operators in Banach spaces.

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Approximation of Functions of Few Variables in High Dimensions

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Let f be a continuous function defined on $\Omega := [0,1]^N$ which depends on only k coordinate variables, $f(x_1, \dots, x_N) = g(x_{i_1}, \dots, x_{i_k})$. We assume that we are given a budget m and are allowed to ask for the values of f at m points in Ω. If g is in Lip_1 and the coordinates $i_1, \dots, i_k$ are known to us than asking for the values of f at $m = L^k$ appropriately spaced points, we could recover f to the accuracy $\|g\|_{\mathrm{Lip}_1} L^{-1}$ in the norm of $C(\Omega)$. This paper studies whether we can obtain similar results when the coordinates $i_1, \dots, i_k$ are not known to us. A prototypical result of this paper is that by asking for $O(C(k)L^{k+1}(\log N)^4)$ point values of f (prescribed in advance) will provide the same approximation error as in the case when the coordinates are known. Results are also proven for other smoothness conditions on g and for the case when it is only known that f can be approximated by a function $g(x_{i_1}, \dots, x_{i_k})$ to some accuracy ϵ. In this case, the approximation will return an error of the form $C(k)(L^{-1}\|g\|_{\mathrm{Lip}_1} + \epsilon)$ in the case $g \in \mathrm{Lip}_1$ with similar results for more general smoothness conditions on g.

This is a joint work with R. DeVore and G. Petrova.

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COMMUNICATIONS

ABSTRACTS

On some new Korovkin-type theorems on L^p spaces

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The talk will be devoted to discuss several new Korovkin-type theorems, established in [3], in the setting of $Lp(X, \mu)$ spaces, $1 \leq p < +\infty$, where X is a locally compact Hausdorff space and μ is a regular positive Borel measure on X. Among other things these results furnishe simple tools to easily construct Korovkin subsets in $L^p(\mathbb{R}^N, \mu)$.

We show, indeed, that, if μ is a positive Borel measure of $\mathbb{R}^N$ and $1 \leq p < +\infty$, and if $f_0 \in C(\mathbb{R}^N) \cap L^p(\mathbb{R}^N, \mu)$ is a strictly positive function such that $\| \cdot \|^2 f_0 \in L^p(\mathbb{R}^N, \mu)$, then the subset formed by $\{f_0, f_0 pr_1, \ldots, f_0 pr_N, \| \cdot \|^2 f_0\}$ is a Korovkin subset in $Lp(\mathbb{R}^N, \mu)$ (here pr_i denotes the i-th coordinate function on $\mathbb{R}^N$, $1 \leq i \leq N$).

In particular, if μ is finite and $\| \cdot \|^2 \in L^p(\mathbb{R}^N, \mu)$, then $\{1, pr_1, \ldots, pr_N, \| \cdot \|^2\}$ is a Korovkin subset in $L^p(\mathbb{R}^N, \mu)$.

As an application we shall investigate the approximation properties on $L^p(\mathbb{R}^N, \mu)$ of a sequence of positive linear operators which generalize Gauss-Weierstrass operators and which are defined by

$$G_n(f)(x) := \left(\frac{n}{4\pi\alpha(x)}\right)^{\frac{N}{2}} \int_{\mathbb{R}^N} f(t) e^{-\frac{n}{4\alpha(x)} \left\| t - x - \frac{\beta(x)}{n} \right\|^2} dt,$$

for every real valued Borel measurable function f on $\mathbb{R}^N$ for which the integral to the right-hand side is absolutely convergent. Here $\alpha : \mathbb{R}^N \to \mathbb{R}$ and $\beta = (\beta_1, ..., \beta_N) : \mathbb{R}^N \to \mathbb{R}^N$ are given Borel-measurable functions with α strictly positive.

The one-dimensional version of these operators was recently introduced and studied in [1] and [2] in the setting of weighted spaces of continuous functions on the real line.

Among other things, we showed that these operators are a useful tool to approximate not only continuous functions but also the solutions of some classes of (possibly degenerate) diffusion equations on the real line.

The main motivation for extending our investigations to multidimensional settings was based on the aim to furnish appropriate positive approximation processes in the setting of large classes of weighted Lp spaces as well.

References

[1] F. Altomare and S. Milella, *Integral-type operators on continuous function spaces on the real line*, J. Approx. Theory, 152 (2008), 107-124.

[2] F. Altomare and S. Milella, *On the C_0-semigroups generated by second order differential operators on the real line*, to appear in Taiwanese J. of Math., 2009.
[3] F. Altomare and S. Milella, *On a sequence of integral operators on weighted L^p spaces*, Analysis Math. 34(2008), 237-259.

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Approximation with respect to Goffman-Serrin variation by means of non-convolution integral operators

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In [1] we study approximation properties for functions belonging to $BV_{\mathcal{F}}(\mathbb{R}^N_+)$, where $BV_{\mathcal{F}}(\mathbb{R}^N_+)$ denotes the space of L^1–functions with bounded $\mathcal{F}$–variation, namely the distributional variation introduced by Goffman and Serrin ([2]). Here $\mathcal{F} : \mathbb{R}^N \longrightarrow \mathbb{R}^+_0$ is a positive sublinear functional and $BV_{\mathcal{F}}(\mathbb{R}^N_+)$ is endowed with the norm

$$||f||_{BV_{\mathcal{F}}} = ||f||_1 + \mathcal{F}_{\mu f}(\mathbb{R}^N_+),$$

where μf is the derivative vector measure associated to f. In particular, we study the following family of non-convolution linear integral operators

$$(T_w f)(s) = \int_{\mathbb{R}^N_+} K_w(s,t) f(t) dt,\ w > 0,\ s \in \mathbb{R}^N_+,$$

where $f \in L^1_{loc}(\mathbb{R}^N_+)$ and $K_w : \mathbb{R}^N_+ \times \mathbb{R}^N_+ \longrightarrow \mathbb{R}^+$, $w > 0$, is a net of positive kernels satisfying a general η–homogeneity condition. For such operators, we obtain a convergence result which proves that

$$\lim_{w \to +\infty} ||T_w f - g||_{BV_{\mathcal{F}}} = 0,$$

where $g(s) := \langle s \rangle \eta(s) f(s)$, $s \in \mathbb{R}^N_+$, provided that $g \in W^{1,1}(\mathbb{R}^N_+)$. This result is quite natural since it can be proved that it is not possible to achieve, in general, convergence to f, even if f is smooth enough, except in the case of kernels homogeneous of degree -1. Preliminary to this result are some estimates and a convergence result for the $\mathcal{F}$–modulus of smoothness of g. The problem of the order of approximation for $||T_w f - g||_{BV_{\mathcal{F}}}$ is also investigated, introducing suitable Lipschitz classes which take into account of the functional of the space involved, i.e., the $BV_{\mathcal{F}}$–norm, and of the multidimensional frame.

References

[1] L. Angeloni and G. Vinti, *Approximation with respect to Goffman-Serrin variation by means of non-convolution integral operators*, preprint (2009).

[2] C. Goffman and J. Serrin, *Sublinear functions of measures and variational integrals*, Duke Math. J., **31** (1964), 159–178.

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Banach function spaces related to nonlinear partial differential equations

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We survey some Banach function spaces, such as Besov spaces, modulation spaces and Bourgain's spaces which have been widely applied in the study of nonlinear partial differential equations in the past twenty years. I will talk the essential connections between those function spaces and nonlinear PDE and show how to use space theory to PDE problems. It seems that many questions on those function spaces are not clear for us. Some open problems will be given as the concluding remarks.

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Tensor products of ordered Banach spaces

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We study a modification of the tensor Lapreste norms, within the framework of ordered Banach spaces. We characterize some families of maps between ordered Banach spaces.

If (x_i) is a sequence in a Banach space B, we set:

$$M_p((x_i)) = \sup\{(\sum |x'(x_i)|^p)^{1/p} : x' \in B',\ ||x'|| \leq 1\}$$

A convex cone Z in a Banach space B is said to be *generating* if $B = Z - Z$.

Definition: Let E and F be two Banach spaces and $X \subset E$, $Y \subset F$ two generating closed convex cones. If $1 \leq p, q, r \leq \infty$ satisfy $1/r + 1/p' + 1/q' = 1$ we equip $E \otimes F$ with the norm:

$$\alpha^+_{p,q}(z) = \inf\{||\lambda_i||_r M_{q'}((x_i)) M_{p'}((y_i)) : z = \sum \lambda_i x_i \otimes y_i\}$$

where $x_1, \ldots, x_n \in X$ and $y_1, \ldots, y_n \in Y$.

When $X = E$ and $Y = F$ we retrieve the classical Lapreste norms.

We compare these norms, using the following index: If Z is a convex cone in a Banach space B we set:

$$i(Z) = \inf\{p : p \geq 1,\ (\sum ||x_i||^p)^{1/P} \leq C_p M_1((x_i))\}$$

for every finite sequence (x_i) in Z.

Theorem: If $q_1, q_2 > i(X)$ and $p_1, p_2 > i(Y)$, then the two norms $\alpha^+_{p_1,q_1}$ and $\alpha^+_{p_2,q_2}$ are equivalent.

Within this framework we are lead, for each $1 < s < \infty$, to characterize the maps $\psi : U \to V$, where V is ordered by a *normal* cone, such that, for every positive map $T : V \to L^{s'}$, the map $T \circ \psi$ is *integral*.

References

[1] Richard Becker, *Convex cones in analysis*, Hermann, Paris.
[2] Richard Becker, *Ordered Banach spaces*, Hermann, Paris.

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Sobolev's type inequality and Markov's inequalities in the complex plane

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Let $\mathcal{A}^\infty(E)$ be the space of functions which are $\overline{\partial}$-flat on a fixed compact set $E \subset \mathbb{C}$. Some relationships between two versions of polynomial Markov's inequality and a Sobolev's type inequality in $\mathcal{A}^\infty(E)$ with quotient norms will be discussed.

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On some fixed point theorems of Krasnoselskii type

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The talk will deal with fixed point theorems of Krasnoselskii type for hyperconvex metric spaces. The classical Krasnoselskii theorem states that a sum of a contraction and a compact mapping on a convex subset of a Banach space has a fixed point. In 2000, Bugajewski proved an analogue of this theorem in hyperconvex setting. Recall that hyperconvex metric spaces, introduced by Aronszajn and Panitchpakdi in 1956 for the sake of HahnBanach type theorems, are known to have many fixed-point properties, often similar to convex sets. We will show some theorems of similar kind for both single-valued and multi-valued mappings on hyperconvex spaces.

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On superposition operators in spaces of functions of bounded variations with applications to nonlinear integral equations

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In this talk we are going to present some suffcient conditions under which the classical superposition operator maps the space of functions of bounded variation into itself. We will consider the variation in the sense of Jordan, Young and also so-called generalized φ-variation. As applications of those results we are going to present some theorems on the existence and uniqueness of solutions to the nonlinear Hammerstein as well as Volterra-Hammerstein integral equations. We will consider solutions to these equations (local as well as global) belonging to the space of functions of bounded variation in the above mentioned meanings.

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Degenerate elliptic operators, Feller semigroups and modified Bernstein-Schnabl operators

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The main object of interest in this talk is a class of second-order elliptic differential operators on finite dimensional convex compact sets, whose principal part degenerates on a subset of the boundary of the domain. More precisely, if K is a convex compact subset of $\mathbb{R}^p$, $p \geq 2$, and $T : C(K) \to C(K)$ a positive projection satisfying suitable assumptions, we consider the operator defined by setting for every $u \in C^2(K)$ and $x = (x_1, \ldots, x_p) \in K$

$$V(u)(x) := \frac{1}{2} \sum_{i,j=1}^{p} (T(pr_i pr_j)(x) - x_i x_j) \frac{\partial^2 u}{\partial x_i \partial x_j}(x) + \sum_{i=1}^{p} \beta_i(x) \frac{\partial u}{\partial x_i}(x) + \gamma(x) u(x),$$

where $\beta_1, \ldots, \beta_p, \gamma \in C(K)$.

That kind of operators is of concern in many differential problems arising from genetics, physics, financial mathematics and other fields.

We show that $(V, C^2(K))$ is closable and its closure generates a Feller semigroup $(T(t))_{t\geq 0}$, that may be approximated by means of powers of suitable positive linear operators, which we refer to as modified Bernstein-Schnabl operators. In the talk we also present the main approximation and shape preserving properties of modified Bernstein-Schnabl operators and, as a consequence, we investigate some regularity properties preserved by the semigroup $(T(t))_{t\geq 0}$.

Finally, we present a generalization of those results to infinite-dimensional settings.

REFERENCES

[1] F. Altomare and M. Campiti, *Korovkin-Type Approximation Theory and its Applications,* De Gruyter Studies in Mathematics **17**, Walter de Gruyter, Berlin-New York, 1994.
[2] M. Cappelletti Montano and S. Diomede, *Modified Bernstein-Schnabl operators on compact convex subsets of locally convex spaces and their limit semigroups*, Comm. Appl. Anal., to appear.
[3] F. Altomare, M. Cappelletti Montano and S. Diomede, *Degenerate elliptic operators, Feller semigroups and modified Bernstein-Schnabl operators*, preprint (2008).

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Variational convergence for conditionalexpectation and martingales

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We state some variational convergence results for conditional expectations and martingales in Banach spaces and their duals. The main tool is to construct the conditional expectations for Pettis-integrable functions and convex weakly compact valued Pettis-integrable multifunctions in separable Banach spaces and also the conditional expectations for Gelfand-integrable functions and convex weakly star valued Gelfand-integrable multifunctions in the dual space of a separable Banach space.

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Lebesgue's Differentiation Theorems in the Lorentz spaces $\Gamma_{p,w}$ for $0 < p < \infty$

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Let L^0 be a space of all Lebesgue measurable extended real valued functions defined over $[0,\alpha)$, where $0 < \alpha \leq \infty$. The *distribution function* d_f of a function $f \in L^0$ is given by $d_f(\lambda) = \mu(\{x \in [0,\alpha) : |f(x)| > \lambda\})$, for all $\lambda \geq 0$. For any $f \in L^0$ its *decreasing rearrangement* is defined as $f^*(t) = \inf\{s > 0 : d_f(s) \leq t\}$, $t > 0$. The *maximal function* of f^* is $f^{**}(t) = \frac{1}{t}\int_0^t f^*(s)ds$. Let $1 \leq p < \infty$ and w be a measurable positive weight function. The *Lorentz space* $\Gamma_{p,w}$ is a subspace of L^0 equipped with the norm

$$\|f\|_{\Gamma_{p,w}} := \left(\int_0^\alpha f^{**p} w\right)^{1/p}.$$

Let $B(x,\epsilon)$ be a ball with a center $x \in (0,\alpha)$ and a radius $\epsilon > 0$ and $\chi_{B(x,\epsilon)}$ be a characteristic function of $B(x,\epsilon)$. The fundamental function of the Lorentz space $\Gamma_{p,w}$ is given by $\phi(x) = \|\chi_{(0,x)}\|_{\Gamma_{p,w}}$.

We investigate two types of Lebesgue's Differentiation (LDT) Theorem, which are generalizations of the classical LDT, in the Lorentz spaces $\Gamma_{p,w}$ for $0 < p < \infty$ and non-negative weight function w. In the first theorem we prove that certain integral averages of a function $f \in \Gamma_{p,w}$ converges to f a.e. Theorem 1 is showed as a consequence of weak inequalities for the maximal function Mf, given by

$$Mf(x) = \sup\left\{\frac{\|f\chi_{B(x,\epsilon)}\|_{\Gamma_{p,w}}}{\|\chi_{B(x,\epsilon)}\|_{\Gamma_{p,w}}} : \epsilon > 0 \text{ and } B(x,\epsilon) \subset (0,\alpha)\right\}.$$

Theorem 1. *Let $0 < p < \infty$ and $f \in \Gamma_{p,w}$. If the Lorentz space $\Gamma_{p,w}$ satisfies lower ϕ-estimate then for a.a. x,*

$$\lim_{\epsilon \to 0} \frac{\|(f - f(x))\chi_{B(x,\epsilon)}\|_{\Gamma_{p,w}}}{\|\chi_{B(x,\epsilon)}\|_{\Gamma_{p,w}}} = 0.$$

The second strain of generalization of LDT is written in the spirit of the best constant approximation of a given function $f\chi_{B(x,\epsilon)}$ from a linear subspace of constant functions defined on the ball $B(x,\epsilon)$. We expand the definition of a best constant approximant operator $T^{(p)}_{B(x,\epsilon)}$ from the Lorentz space $\Gamma_{p,w}$ to $\Gamma_{p-1,w}$, for $p > 1$ and from $\Gamma_{1,w}$ to L^0, for $p = 1$. $T^{(p)}_{B(x,\epsilon)}$ is a set-valued operator defined for each $f \in L^0$ if $p = 1$ and $f \in \Gamma_{p-1,w}$ if $p > 1$, which assumes the values as a set of constant numbers $c \in \mathbb{R}$, for which the right-hand Gâteaux derivatives of the norm $\|\cdot\|_{\Gamma_{p,w}}$ at the points $(f-c)\chi_{B(x,\epsilon)}$ and $(c-f)\chi_{B(x,\epsilon)}$ in the direction $\chi_{B(x,\epsilon)}$ are non-negative. In fact, to prove that the best constant approximants converges a.e. to f, is to show weak inequalities for maximal function associated with the best constant approximants.

Theorem 2. *Let $f \in L^0$ and $f_\epsilon(x) \in T^{(1)}_{B(x,\epsilon)}(f)$, for $x \in (0,\alpha)$ and $\epsilon > 0$. Then for a.a. x,*

$$f_\epsilon(x) \to f(x) \qquad \text{as} \qquad \epsilon \to 0. \tag{9}$$

In case when $1 \leq p < \infty$ and $\Gamma_{p,w}$ satisfies a lower ϕ-estimate then for any $f \in \Gamma_{p,w}$ and $f_\epsilon(x) \in T^{(p)}_{B(x,\epsilon)}(f)$ we have the condition (9).

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Fixed point properties for C^*-algebras

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This paper derives fixed point properties of nonabelian C^*-algebras. We show that the following conditions on a C^*-algebra are equivalent: (i) it has the fixed point property for nonexpansive mappings, (ii) the spectrum of every self adjoint element is finite, (iii) it is finite dimensional. We prove that (i) implies (ii) using constructions given by Goebel, that (ii) implies (iii) using projection operator properties derived from the spectral and Gelfand-Naimark-Segal theorems, and observe that (iii) implies (i) by Brouwer's fixed point theorem. As a consequence we obtain: If G is a locally compact group, and $C^*(G)$, the group C^*-algebra of G has the fpp, then G must be a finite group.

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A fixed point theorem in the strictly convex space

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We present a fixed point theorem for a continuous self mapping on a compact and connected subset of a strictly convex space.

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Integral representation of γ-linear operators on Orlicz-Bochner spaces

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Let (Ω, Σ, μ) be a finite measure space and let $\mathcal{L}(X, Y)$ stand for the space of all bounded linear operators between real Banach spaces X and Y. An operator measure $m : \Sigma \to \mathcal{L}(X, Y)$ is said to be variationally μ-absolutely continuous if $\widetilde{m}_{\varphi^*}(A_n) \to 0$ whenever $\mu(A_n) \to 0$, $(A_n) \subset \Sigma$ (here $\widetilde{m}_{\varphi^*}(A)$ stand for the φ^*-semivariation of m on $A \in \Sigma$). A bounded linear operator T from the Orlicz-Bochner space $L^\varphi(\mu; X)$ is said to be γ-linear if $||T(f_n)||_Y \to 0$ whenever (f_n) converges to 0 in measure μ and $\sup_n ||f_n||_\varphi < \infty$. It is shown that a bounded linear operator $T : L^\varphi(\mu; X) \to Y$ is γ-linear if and only if its representing operator measure m is variationally μ-absolutely continuous.

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Some fundamental geometric and topological properties of generalized Orlicz-Lorentz function spaces

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Generalized Orlicz-Lorentz function spaces Λ_φ generated by Musielak-Orlicz functions φ satisfying some growth and regularity conditions (cf. [4] and [5]) are investigated. A regularity condition Δ_2^Λ for φ is defined in such a way that it guarantees many positive topological and geometric properties of Λ_φ. The problems of the Fatou property, order continuity (separability) and the Kadec-Klee property with respect to the local convergence in measure of Λ_φ are considered. Moreover, some embeddings between Λ_φ and their two subspaces are established and strict monotonicity as well as lower and upper local uniform monotonicities are characterized. Finally, necessary and sufficient conditions for rotundity of Λ_φ are presented. Presented results are generalizations of the results from [3]. Analogous results in the sequence case were presented in [1] and [2], but the techniques in the function case are different from those for the sequence case.

References

[1] P. Foralewski, H. Hudzik and L. Szymaszkiewicz, *On some geometric and topological properties of generalized Orlicz-Lorentz sequence spaces*, Math. Nachr. 281(2) (2008), 181-198.
[2] P. Foralewski, H. Hudzik and L. Szymaszkiewicz, *Local rotundity structure of generalized Orlicz-Lorentz sequence spaces*, Nonlinear Analysis 68(9) (2008), 2709-2718.
[3] A. Kamińska, *Some remarks on Orlicz-Lorentz spaces*, Math. Nachr. 147 (1990), 29-38.
[4] G. G. Lorentz, *An inequality for rearrangements*, Amer. Math. Monthly 60 (1953), 176-179.
[5] M. Nawrocki, *The Mackey topology of some F-spaces*, Ph.D. thesis, Adam Mickiewicz University (1984) (in Polish).

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Vector convex differences controlled by their scalar counterparts

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A sandwich type theorem of K. Baron, J. Matkowski and K. Nikodem (Mathematica Pannonica 5 (1994), 139-144) will be applied and developed to deal with the functional inequality

$$\|F(\lambda x+(1-\lambda)y)-\lambda G(x)-(1-\lambda)G(y)\| \leq$$
$$\leq \lambda g(x)+(1-\lambda)g(y)-f(\lambda x+(1-\lambda)y).$$

Under some mild regularity assumptions upon the functions in question we shall also show the equivalence of the latter inequality with its Jensen type analogue

$$\left\|F\left(\frac{x+y}{2}\right)-\frac{G(x)+G(y)}{2}\right\| \leq \frac{g(x)+g(y)}{2}-f\left(\frac{x+y}{2}\right).$$

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A new hybrid algorithm for mixed equilibrium problems, variational inequality problems and optimization problems in Hilbert spaces

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In this talk, we present a new hybrid iterative method for finding a common element of the set of solutions of a mixed equilibrium problem, the set of fixed points of an infinite family of nonexpansive mappings and the set of solutions of variational inequalities for a relaxed (u, v)-cocoercive and ξ-Lipschitz continuous mappings in Hilbert spaces. Furthermore, we show that the strong convergence theorem of the iterative sequence generated by the proposed iterative algorithm under some suitable conditions, which solves some optimization problems.

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Function spaces on the Koch curve

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We consider two types of Besov spaces on the Koch curve, defined by traces and with the help of the snowflaked transform. We compare these spaces and give their characterization in terms of Daubechies wavelets.

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Monotonicity characteristic of Köthe-Bochner spaces

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Estimates of the characteristic of monotonicity in Köthe-Bochner function spaces $E(X)$, which are the best possible in general, and some consequences are given. Geometric interpretation of the characteristic of monotonicity of any finite dimensional Banach lattice is presented.

Based on the joint paper: H. Hudzik and R. Kaczmarek, *Monotonicity characteristic of Köthe-Bochner spaces*, J. Math. Anal. Appl. 349 (2009) 459-468.

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On the Diamond Operator and Some Related Equations

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In this talk, we introduce and study fundamental properties of the Diamond operator. We also give elementary solutions of the equations related to the operator. Finally, we apply the kernel of the Diamond operator to study some heat and wave equations.

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Singular integral operators on variable Lebesgue spaces

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We describe essential spectra of one-dimensional singular integral operators with piecewise continuous coefficients acting on variable Lebesgue spaces (sometimes also called Nakano spaces) with radial oscillating weights over Carleson curves. These spectra consist of the ranges of the coefficients complemented by some connected sets joining the values of the coefficients at jumps. The complementary sets are circular arcs if weights are power weights and underlying curves are sufficiently smooth. However, if we admit oscillations of weights or curves, then the complementary sets become sets with nonzero plane measure whose boundaries consist of double logarithmic spirals. The precise shape of these sets is determined by the indices of some submultiplicative functions associated with weights and curves, as well as with values of the variable exponent.

The existence of continuous operator between two Musielak-Orlicz spaces

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Let(Ω, Σ, μ) be a σ-finite and atomless measure space. Denote by $L^0(\Omega)$ the set of equivalence classes of real μ-measurable functions defined on Ω. We find a necessary and sufficient condition for an existence a nonzero linear and continuous operator from a Musielak-Orlicz space L^Φ into an Orlicz space L^ψ in case when the function Φ satifies the Δ_2-condition and ψ is concave. We give also some necessary and sufficient conditions for an existance such operators between two Musielak-Orlicz spaces.

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Remarks on the spaces of differentiable multifunctions

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In this talk we give some remarks on some spaces of differentiable multifunctions and we give some theorems. In particular we define the generalized Orlicz-Sobolev spaces of multifunctions and we study completennes of them. Now we introduce a simple example of this space.

Let $W^k_\varphi(T)$ denote gerneralized Orlicz-Sobolev space. Let $X = \{F : T \to 2^{\overline{R}}\}$, $\underline{f}(F)(t) = \inf_{t\in T} F(t)$, $\overline{f}(F)(t) = \sup_{t\in T} F(t)$, $X_m = \{F \in X : F \text{ is measurable }\}$. If $F \in X_m$, then we define $convF$ by $(convF)(t) = conv(F(t))$ for every $t \in T$.

Denote

$$X_{1,\varphi,k} = \{F \in X_m : F(t) \text{ is convex for every } t \in T \text{ and}$$
$$\underline{f}(F), \overline{f}(F) \in W^k_\varphi(T)\},$$
$$\tilde{X}_{\varphi,k} = \{F \in X_m : convF \in X_{1,\varphi,k}\}.$$

It is easy to see that $X_{1,\varphi,k}$, $\tilde{X}_{\varphi,k}$ are linear subsets of X and we will be call them the generalized Orlicz-Sobolev spaces of multifunctions.

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On the topological structure of solution sets to certain fractional differential equations

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Although the investigation of the fractional differential equations (or differential equations of generalized order) was started not so long ago, it has found many applications in different branches of science and engineering such as: electrical networks, electromagnetic theory,fluid flow and probability. One can also find its applications in statistic and in viscoelasticity. In this talk we are going to discuss so-called Aronszajn type theorems describing the topological structure of solution sets to some initial value problems; for example to the following problem

$$(D^{\alpha_n} - a_{n-1}D^{\alpha_{n-1}} - \ldots - a_1 D^{\alpha_1})(x) = f(t,x), \quad x(0) = 0,$$

for $t \in [0,T]$, where T is a positive number, $f : [0,T] \times \mathbb{R} \to \mathbb{R}$ is a given continuous function, $0 < \alpha_1 < \ldots < \alpha_n < 1$ and $a_j > 0$ for $j = 1, \ldots, n-1$ (D^{α_j} denotes the standard Riemann-Liouville fractional derivative of order α_j). We are also going to present some new results on the existence and uniqueness of positive solutions to the problems in question. The talk is based on the common paper *On the existence, uniqueness and topological structure of solution sets to a certain fractional differential equations* with Daria Bugajewska.

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Probability distribution solutions of a general linear equation of infinite order

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Let $(\Omega, \mathcal{A}, P)$ be a probability space and let $\tau : \mathbb{R} \times \Omega \to \mathbb{R}$ be a mapping strictly increasing and continuous with respect to the first variable, $\mathcal{A}$-measurable with respect to the second one. We obtain a partial characterization and a uniqueness-type result concerning solutions of the general linear equation

$$F(x) = \int_{\Omega} F(\tau(x, \omega)) P(d\omega)$$

in the class of probability distribution functions.

joint work with Janusz Morawiec

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Local $\Delta_2^E(x)$ condition as a crucial tool for local structure of Calderón-Lozanovskiĭ spaces

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We will study the structure of Calderón-Lozanovskiĭ space E_φ. Let (T, Σ, μ) be a $\sigma-$finite and complete measure space. By $L^0 = L^0(T)$ we mean the set of all μ-equivalence classes of real valued measurable functions defined on T. A Banach space $E = (E, \|\cdot\|_E)$ is said to be a *Köthe space* if E is a linear subspace of L^0 and:

(i) if $x \in E, y \in L^0$ and $|y| \leq |x|$ μ-a.e., then $y \in E$ and $\|y\|_E \leq \|x\|_E$;
(ii) there exists a function x in E that is positive on the whole T.

In the whole talk φ denotes an *Orlicz function*, i.e. $\varphi : \mathbb{R} \to [0, \infty]$, it is convex, even, vanishing and continuous at zero, left continuous on $(0, \infty)$ and not identically equal to zero. Define on L^0 a convex modular I_φ by

$$I_\varphi(x) = \begin{cases} \|\varphi \circ x\|_E & \text{if } \varphi \circ x \in E \\ \infty & \text{otherwise} \end{cases},$$

where $(\varphi \circ x)(t) = \varphi(x(t))$, $t \in T$. By the Calderón-Lozanovskiĭ space E_φ we mean

$$E_\varphi = \{x \in L^0 : I_\varphi(cx) < \infty \ \text{ for some } \ c > 0\}$$

equipped with so called *Luxemburg norm* defined by

$$\|x\|_\varphi = \inf\{\lambda > 0 : I_\varphi(x/\lambda) \leq 1\}.$$

The order continuity is a fundamental tool in the theory of Banach lattices. It is natural to study not only the global property P but also the behaviour of the separated point x in the Banach space (Banach lattice) from this property P point of view (see [2] for the study of local rotundity structure). This leads to the notion of point of order continuity.

A point $x \in E$ is said to have *order continuous norm* if for any sequence (x_m) in E such that $0 \leq x_m \leq |x|$ and $x_m \to 0$ μ-a.e. we have $\|x_m\|_E \to 0$. A Köthe space E is called *order continuous* ($E \in (OC)$) if every element of E has an order continuous norm.

Recall that, under some additional assumptions, the Δ_2^E condition is necessary and sufficient for order continuity of E_φ ([1]). Given a point $x \in E_\varphi$ we introduce a new condition (called local $\Delta_2^E(x)$) which appears to be necessary and sufficient for x to be a point of order continuity of E_φ. The $\Delta_2^E(x)$ condition is naturally weaker than the global one Δ_2^E.

It is known that monotonicity properties (strict and uniform monotonicity) play analogous role in the best dominated approximation problems in Banach lattices as do the respective rotundity properties (strict and uniform rotundity) in the best approximation problems in Banach spaces. Clearly, the points of lower (upper) monotonicity of a Banach lattice E play an analogous role as the extreme points in a Banach space X. Similarly, the role of points of upper (lower) local uniform monotonicity in Banach lattices is analogous to that of points of local uniform rotundity in Banach spaces.

A point $x \in E_+ \setminus \{0\}$ is a *point of lower monotonicity* (*upper monotonicity*) if for any $y \in E_+$ such that $y \leq x$ and $y \neq x$ ($x \leq y$ and $y \neq x$), we have $\|y\|_E < \|x\|_E$ ($\|x\|_E < \|y\|_E$). A point $x \in E_+$ is called a *point of lower local uniform monotonicity* (*upper local uniform monotonicity*) if for any sequence $x_n \in E$ such that $0 \leq x_n \leq x$ and $\|x_n\|_E \to \|x\|_E$ ($x \leq x_n$ and $\|x_n\|_E \to \|x\|_E$) there holds $\|x_n - x\|_E \to 0$. We will write shortly that x is a *LM-point, UM-point, LLUM-point* and *ULUM-point,* respectively.

The local monotonicity structure of Calderón-Lozanovskiĭ spaces has been considered in [3]. However, the precise full criteria have been presented only for points of lower and upper monotonicity. Considering $LLUM$ points, the authors of [3] gave only some sufficient and some necessary conditions, often using too strong assumptions. We shall give a full criterion for $LLUM$ points of Calderón-Lozanovskiĭ spaces. It appears that the structure of $LLUM$ and $ULUM$ points is quite different in E_φ. Namely, considering the $LLUM$ point x, the local $\Delta_2^E(x)$ condition is crucial, while in the case of $ULUM$ point x the respective global Δ_2^E is essential (see [4]) which of course is really stronger than the local $\Delta_2^E(x)$ in general.

All results presented in this talk are obtained in the paper "Local $\Delta_2^E(x)$ condition as a crucial tool for local structure of Calderón-Lozanovskiĭ spaces" jointed with Ryszard Płuciennik to appear in Journal of Mathematical Analysis and Applications.

References

[1] H. Hudzik, A. Kamińska and M. Mastyło, *Geometric properties of some Calderón-Lozanovskiĭ spaces and Orlicz-Lorentz spaces*, Houston J. Math. **22** (1996), 639-663.
[2] H. Hudzik, P. Kolwicz and A. Narloch, *Local rotundity structure of Calderón-Lozanovskiĭ spaces*, Indag. Math. N. S. **17(3)**, (2006), 373-395.
[3] H. Hudzik, A. Narloch, *Local monotonicity structure of Calderón-Lozanovskiĭ spaces*, Indag. Math. N. S. **15(1)** (2004), 1-12.
[4] P. Kolwicz, R. Płuciennik, *Points of upper local uniform monotonicity of Calderón-Lozanovskiĭ spaces,* Journal of Convex analysis, to appear.

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Stability and non-stability of convexity

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Let $\mathbb{K}$ be a fixed number field contained is the space of all reals $\mathbb{R}$ and let X be a linear space over $\mathbb{K}$. A subset $D \subset X$ is said to be $\mathbb{K}$-convex iff for all $x, y \in D$ and each $\lambda \in \mathbb{K} \cap (0,1)$ we have $\lambda x + (1-\lambda)y \in D$. Assume that $\varepsilon \geq 0$ is a fixed real number and D is a non-empty $\mathbb{K}$-convex subset of X. A function $f : D \to \mathbb{R}$ satisfying the inequality

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + \varepsilon,$$

for all $x, y \in D$, $t \in \mathbb{K} \cap (0,1)$ is called $\varepsilon - \mathbb{K}$-convex. In 1940 D.H. Hyers and S. Ulam (also J.W. Green) proved that if D is an open and convex subset of $\mathbb{R}^n$ and $f : D \to \mathbb{R}$ is an $\varepsilon - \mathbb{R}$-convex function, then there exist a convex function $g : D \to \mathbb{R}$ and a constant M (depending on the dimension n) such that

$$(1) |f(x) - g(x)| \leq M, \quad x \in D.$$

It is not longer true if D is a subset of an infinite-dimensional real linear space. It was first observed by E. Casini and P.L. Papini and this statement was proved for specially constructed domains. We show that if D is an infinite-dimensional $\mathbb{K}$-convex subset of an arbitrary (infinite-dimensional) real linear space then there exists an $\varepsilon - \mathbb{K}$-convex function $f : D \to \mathbb{R}$ such that for every $\mathbb{K}$-convex function $g : D \to \mathbb{R}$ we have

$$\sup |f(x) - g(x)|;\ x \in D = \infty.$$

As a consequence we obtain a nonstability result for an inequality defining real Jensen-convex functions defined on an arbitrary real interval.

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Strong unicity of best approximation in a space of compact operators

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Let $\mathcal{K}(c_0, c_0)$ denote the space of all compact operators from c_0 to c_0 equipped with the operator norm. We are interested in Chebyshev subspaces of $\mathcal{K}(c_0, c_0)$. In particular, we show that for every $k < \infty$ there exists a k-dimensional non-interpolating Chebyshev subspace of $\mathcal{K}(c_0, c_0)$.
Note that in the space $\mathcal{L}(l_1^n, c_0)$ any finite-dimensional Chebyshev subspace is an interpolating subspace.

Additionally, we consider the strong unicity of best approximation in some (not necessarily Chebyshev) subspaces of $\mathcal{K}(c_0, c_0)$.

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Arithmetic separation and Banach-Saks property

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We introduce the arithmetic separation of a sequence in a Banach space. This geometric characteristic describes the Banach-Saks property of spaces and bounded linear operators. Next we define an operator seminorm vanishing for operators with the Banach-Saks property. We show logarithmically convex-type estimates of the seminorm for operators interpolated by the real method of Lions and Peetre.

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A note on Cesàro-Orlicz sequence spaces

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The (non-identically equal zero) function $\varphi : [0,\infty) \to [0,\infty]$ is called an Orlicz function if it is convex, continuous at 0, left continuous on $(0,\infty)$ and $\varphi(0) = 0$. We say that the Orlicz function φ satisfies the δ_2 condition (we will write $\varphi \in \delta_2$) if there are $K > 0$ and $u_0 > 0$ such that $\varphi(u_0) > 0$ and $\varphi(2u) \leq K\varphi(u)$ for all $u \in [0, u_0]$. This condition plays the crucial role in the theory of Orlicz spaces.

Given an Orlicz function φ, the modular

$$I_{ces_\varphi}(x) = \sum_{n=1}^{\infty} \varphi\left(\frac{1}{n}\sum_{i=1}^{n}|x(i)|\right)$$

is convex and defines so called Cesàro-Orlicz sequence space

$$ces_\varphi = \{x \in \ell^0 : \ I_{ces_\varphi}(\lambda x) < \infty \quad \text{forsome } \lambda > 0\}$$

with the Luxemburg norm given by

$$||x||_{ces_\varphi} = \inf\{\varepsilon > 0 : I_{ces_\varphi}(\frac{x}{\varepsilon}) \leq 1\}.$$

The space ces_φ^h is the subspaces of all order continuous elements of ces_φ. It is known that the condition $\varphi \in \delta_2$ implies the order continuity of ces_φ. Using the notion of Matuszewska-Orlicz indices (α_φ and β_φ) we obtained the converse which was not known for some time.

Theorem 1. *If $ces_\varphi^h = ces_\varphi$ and $\alpha_\varphi > 1$ then $\varphi \in \delta_2$. In particular, if $\alpha_\varphi > 1$ then $ces_\varphi^h = ces_\varphi$ if and only if $\varphi \in \delta_2$.*

We also show the theorem which allows to compare Cesàro-Orlicz spaces.

Theorem 2. *Let φ_1 and φ_2 be Orlicz functions such that $\varphi_1(u) > 0$, $\varphi_2(u) > 0$ for all $u > 0$, and $\alpha_\varphi > 1$. The following conditions are equivalent*

i: $ces_{\varphi_1} \subset ces_{\varphi_2}$

ii: *There exist $b, t_0 > 0$ such that $\varphi_2(t) \leq \varphi_1(bt)$ for all $t \in [0, t_0]$.*

iii: *There exist $C > 0$ such that $||x||_{ces_{\varphi_2}} \leq ||x||_{ces_{\varphi_1}}$ for all $x \in ces_{\varphi_1}$.*

Some important examples will be presented.

References

[1] D. Kubiak, *A note on Cesàro-Orlicz sequence spaces.* J. Math. Anal. Appl. 349 (2009), 1 , 291–296.

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Generalized f-contraction for multivalued maps without T-weakly commuting condition and invariant approximations

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In this paper, we establish the existence of common fixed point results for multivalued generalized f-contraction and generalized f-nonexpansive maps. In our results we may not assume the T-weakly commuting. Several invariant approximation results are obtained as applications. Our results improve and extend the recent results of M.A. Al-Thagafi and N. Shahzad [1], N. Shashzed and H. Hussain [2] and many authors.

References

[1] M.A. Al-Thagafi, Naseer Shahzad, *Coincidence points, generalized Inonexpansive multimaps, and applications*, Nonlinear Analysis 67 (2007) 2180-2188.

[2] N. Shahzad and N. Hussain, *Deterministic and random coincidence point results for f-nonexpansive maps*, J. Math. Anal. Appl. 323 (2006) 1038-1046.

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Energy functionals for elastic thin films in the setting of OrliczSobolev spaces

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Let ω be an open, bounded subset of $\mathbb{R}^2$. Define $I := (-\frac{1}{2}, \frac{1}{2})$, $\Omega := \omega \times I$. Let $L^M(\Omega, \mathbb{R}^3)$ be the Orlicz space of measurable functions from Ω to$\mathbb{R}^3$ generated by some non-power OrliczYoung function M. Let $W^{1,M}(\Omega; \mathbb{R}^3)$ be the OrliczSobolev space modeled by $L^M(\Omega, \mathbb{R}^3)$. Let $W : M_3 \to [0, \infty)$ be a continuous function satisfying $\frac{1}{C}(M(|F|) - 1) \leq W(F) \leq C(1 + M(|F|))$ for all $F \in M_3$ and for some $C \in (0, \infty)$ where M_3 is the vector spaces of real 3×3 matrices. Let the bulk energy functionals $J^1_\varepsilon : W^{1,M}(\Omega; \mathbb{R}^3) \times L^M(\Omega, \mathbb{R}^3) \to \mathbb{R}$ and $J^2_\varepsilon : W^{1,M}(\Omega; \mathbb{R}^3) \times L^M(\omega, \mathbb{R}^3) \to \mathbb{R}$ be defined by

$$J^1_\varepsilon(u, \bar{b}) := \begin{cases} \int_\Omega W(\frac{\partial u}{\partial x_1} | \frac{\partial u}{\partial x_2} | \frac{1}{\varepsilon} \frac{\partial u}{\partial x_3}) dx & \text{if} \frac{1}{\varepsilon} \int_I \frac{\partial u}{\partial x_3} dx_3 = \bar{b} \\ +\infty & \text{otherwise;} \end{cases}$$

$$J^2_\varepsilon(u, b) := \begin{cases} \int_\Omega W(\frac{\partial u}{\partial x_1} | \frac{\partial u}{\partial x_2} | \frac{1}{\varepsilon} \frac{\partial u}{\partial x_3}) dx & \text{if} \frac{1}{\varepsilon} \frac{\partial u}{\partial x_3} = b \\ +\infty & \text{otherwise.} \end{cases}$$

We present the 3D-2D dimension reduction via Γ-convergence of J^1_ε and J^2_ε as $\varepsilon \to 0$ for finding the effective energy functionals of elastic thin films (membranes) defined respectively on $\mathcal{V} \times L^M(\omega; \mathbb{R}^3)$ and $\mathcal{V} \times L^M(\Omega; \mathbb{R}^3)$ where $\mathcal{V} := \{u \in W^{1,M}(\Omega; \mathbb{R}^3) : \frac{\partial u}{\partial x_3} = 0\}$. Our main theorems generalize theorems due to G. Bouchitte/ I. Fonseca/ M. L. Mascarenhas in [1], [2] proved therein in the setting of Sobolev $W^{1,p}$-spaces for the power case $M(t) = t^p$. We also obtain the $W^{1,M}$-generalization of $W^{1,p}$-theorems for thin films reduced from the bulk energy functionals involving higher order perturbations due to I. Fonseca/ G. Francfort/ G. Leoni in [3]. The proofs of our results are based also on recent relaxation theorems for multiple integral functionals in the setting of Orlicz-Sobolev spaces due to prof. Hong Thai Nguyen.

References

[1] G. Bouchitté, I. Fonseca, and M. L. Mascarenhas, *Bending moment in membrane theory*, J. Elasticity **73** (2004), 7599.
[2] ———, *The Cosserat Vector In Membrane Theory: A Variational Approach*, J. Convex Anal. **16** (2009).
[3] I. Fonseca, G. Francfort, and G. Leoni, *Thin elastic films: The impact of higher order perturbations*, Quart. Appl. Math. **65** (2007), 6998.

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Selected properties of Calderón-Lozanovskiĭ spaces

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The theory of Orlicz spaces L_φ is well known. The Banach lattices called E_φ generated by the Köthe space E and the Orlicz function φ, are generalization of Orlicz spaces and Orlicz-Lorentz spaces. The structure of spaces E_φ have been intesively developed during the last 20 years (see for example [1] or [2]). Although E_φ are often called Calderón- Lozanovskiĭspaces they are only a particular case of Calderón-Lozanovskiĭconstruction $\rho(X,Y)$ for $X = L^\infty$. We shall study the structure of general Calderón-Lozanovskiĭ construction $\rho(X,Y)$ as well as E_φ spaces.

Let (T,Σ,μ) be a σ-finite and complete measure space. By $L^0 = L^0(T)$ we denote the set of all μ-equivalence classes of real valued measurable functions defined on T.

A Banach space $E = (E, ||\cdot||_E)$ is said to be a *Köthe space* if E is a linear subspace of L^0 and :

i if $x \in E$, $y \in L^0$ and $|y| \le |x|$ μ-a.e., then $y \in E$ and $||y||_E \le ||x||_E$;
ii for all $A \in \Sigma$ with $\mu(A) < \infty$ we have $\chi_A \in E$.

We say that φ is an *Orlicz function* whenever $\varphi : [0,\infty) \to [0,\infty]$, it is convex, vanishing and continuous at zero, left continuous on $(0,\infty)$ and not identically equal to zero. Additionally, if φ vanishes only at zero, takes only finite values and

$$\lim_{u\to\infty}\frac{\varphi(u)}{u} = \infty \text{ and } \lim_{u\to 0}\frac{\varphi(u)}{u} = 0$$

then φ is called an $\mathcal{N}$-function. Denote the class of $\mathcal{N}$ functions by $\mathcal{O}$. For each $\varphi \in \mathcal{O}$ one can associate a function $\rho_{\varphi,R} : [0,\infty)\times[0,\infty) \to [0,\infty)$ by

$$\rho_{\varphi,R}(u,v) = \begin{cases} u\varphi^{-1}\left(\frac{u}{v}\right) & \text{if} u > 0 \\ 0 \quad \text{for} u = 0 \end{cases}$$

Denote the family of all such functions $\rho_{\varphi,R}$ by U.

Definition 1.*S*uppose that $\rho \in U$ and X, Y are Köthe spaces over the same measure space (we will also say that a couple (X,Y) is compatible). Let $1 \le p \le \infty$. By the Calderón-Lozanovskiĭspace $\rho(X,Y)$ we mean

$$\rho(X,Y) = \{z \in L^0 : |z| \le \rho(x,y) for some x \in X_+, y \in Y_+\}$$

equipped with the norm

$$_p||z||_{\rho(X,Y)} = \inf\left\{||(x,y)||_p : x \in X_+, y \in Y_+ with |z| \le \rho(x,y)\right\}.$$

We should investigate such properties like *order continuity*, *strict monotonicity or property (β) of Rolewicz* in Calderón-Lozanovskiĭspaces.

A point $x \in E$ is said to have *order continuous* norm if for any sequence (x_m) in E such that $0 \leq x_m \leq |x|$ and $x_m \to 0$ μ-a.e. we have $||x_m||_E \to 0$. A Köthe space E is called *order continuous* ($E \in (OC)$) if every element of E has an order continuous norm. As usual E_α stands for the subspace of order continuous elements of E. It is known that $x \in E_\alpha$ iff $||x\chi_{A_i}||_E \downarrow 0$ for any sequence $\{A_n\}$ satisfying $A_n \searrow 0$ (that is $A_n \supset A_{n+1}$ and $\chi_{A_n} \to 0$ μ-a.e.).

E is said to be *strictly monotone* ($E \in (SM)$) if for each $0 \leq y \leq x$ with $y \neq x$ we have $||y||_E < ||x||_E$.

We will say that E has the *Fatou property* if $0 \leq x_n \uparrow x \in L^0$ with $(x_n)_{n=1}^\infty$ in E and $\sup_n ||x_n||_E < \infty$, then $x \in E$ and $||x||_E = \lim_n ||x_n||_E$.

A Banach space X has *property* (β) if and only if for every $\varepsilon > 0$ there exist $\delta > 0$ such that for each element $x \in B(X)$ and each sequence (x_n) in $B(X)$ with $\operatorname{sep}(x_n) \geq \varepsilon$ there is an index k for which

$$||x + x_k||_X \leq 2(1 - \delta).$$

References

[1] H. Hudzik, A. Kamińska and M. Mastyło, *Geometric properties of some Calderón- Lozanovskiĭspaces and Orlicz-Lorentz spaces*, Houston J. Math. 22, (1996), 639-663.
[2] P. Kolwicz, *Rotundity properties in Calderón-Lozanovskiĭ spaces*, Houston J. Math. **31.3** (2005), 883-912.
[3] L. Maligranda, *Orlicz spaces and interpolation*, SemMath. 5 Univ. of Campinas, Camp- inas SP, Brazil, 1989.
[4] G. Ya. Lozanowskiĭ, *On some Banach lattices*, Sibirsk. Math. J. 12 (1971), 562-567.
[5] S. Reisner, *On two theorems of Lozanovskiĭ concerning intermediate Banach lattices*, Geometric aspects of functional analysis (1986/87), 67-83, Lecture Notes in Math. 1317, Springer Berlin 1988.

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Pointwise convergence for discrete operators

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In [2] we consider a sequence of general bivariate discrete operators of type

$$(S_n f)(x,y) = \sum \sum_{(k,j)\in\mathcal{I}^2} K_n(x,y,\nu_{n,k},\mu_{n,j}) f(\nu_{n,k},\mu_{n,j}),$$

where $(x,y) \in I \subset \mathbb{R}^2$, I an interval, and $\mathcal{I}$ is an infinite subset of $\mathbb{Z}$. Here $(\nu_{n,k})_k, (\mu_{n,j})j$ are two sequences of real numbers satisfying suitable assumptions. Moreover (K_n) is a kernel function (not necessarily positive). A one-dimensional version was considered in [1]. We study some pointwise convergence theorems and we obtain a Voronovskaja formula when the function f is twice differentiable at a point (x,y). We apply the general asymptotic formula to the generalized sampling series of bivariate functions, generated by various kinds of kernel functions: box-type kernels, radial kernels and bivariate spline functions. We use some basic results proved by P.L. Butzer, A. Fischer and R.L. Stens about the multivariate generalized sampling series (see [3]).

References

[1] C. Bardaro, I. Mantellini, *A Voronovskaya-type theorem for a general class of discrete operators*, Rocky Mountain J. Math., (2009), to appear.

[2] C. Bardaro, I. Mantellini, A*symptotic formulae for bivariate discrete operators. Applications to generalized sampling series and Szasz- Mirak'jan type operators*, submitted, 2009.

[3] P.L. Butzer, A. Fischer and R.L. Stens, *Generalized sampling approximation of multivariate signals; general theory*, Atti Sem. Mat. Fis. Univ. Modena, **41**, (1993), 17-37.

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On the boundedness of singular integral operators on some Herz spaces

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[joint work with Yasuo Komori (Tokai U.,Japan)]

A. Beurling (1964) introduced the Beurling algebra and its dual space $B^p(\mathbb{R}^n)$, which is a particular case of Herz spaces $K^{\alpha}_{p,r}(\mathbb{R}^n)$, i.e. $B^p(\mathbb{R}^n) = K^{-n/p}_{p,\infty}(\mathbb{R}^n)$.

Concerning the boundedness of operators, X. Li and D. Yang (1996) proved that the singular integral operators are bounded on $K^{\alpha}_{p,r}(\mathbb{R}^n)$, where $1 < p < \infty$, $0 < r \leq \infty$ and $-n/p < \alpha < n(1 - 1/p)$.

On the other hand, when we consider the boundedness of singular intregral operators on $B^p(\mathbb{R}^n)$, we can't use the general theory of Herz spaces. Therefore, Y. Chen and K. Lau (1989) and J. García-Cuerva (1989) introduced the spaces $CMO^p(\mathbb{R}^n)$, which are the dual spaces of Beurling type Hardy spaces, and showed that the singular integral operators are bounded from $B^p(\mathbb{R}^n)$ to $CMO^p(\mathbb{R}^n)$, where $1 < p < \infty$.

Furthermore, for a close study about the Hardy spaces, J. García-Cuerva and M.-J. L. Herrero (1994) generalized the spaces $B^p(\mathbb{R}^n)$ and $CMO^p(\mathbb{R}^n)$, and introduced the spaces $B^p_q(\mathbb{R}^n)$ and $CMO^p_q(\mathbb{R}^n)$.

In this talk, we consider the boundedness of singular integral operators on $B^p_q(\mathbb{R}^n)$, and also investigate the weighted estimates.

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Minimal multi-convex projections onto subspaces of incomplete algebraic polynomials

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Let $X = C^N[0,1]$, where $N \geq 3$ and let V be a linear subspace of Π_N, where Π_N denotes the space of algebraic polynomials of degree less than or equal to N.

Denote by

$$\mathcal{P}(X,V) = \{P : X \to V \ : \ P - \text{linear and bounded} P_{|V} = \mathrm{id}_V\}.$$

Let S denote a cone of multi-convex functions and let

$$\mathcal{P}_S = \{P \in \mathcal{P}(X,V) \ : \ PS \subset S\}.$$

In [1, 2] the multi-convex projections are defined and it is shown explicite formula for projection with minimal norm in $\mathcal{P}_S$ for $V = \Pi_N$.

In my talk I will present a generalization of these results in the case of V being subspaces of incomplete polynomials.

References

[1] G. Lewicki, M. Prophet *Minimal Shape-Preserving Projections Onto Π_n: Generalizations and Extensions*, Numerical Functional Analysis and Optimization, 27 (7-8) **(2006)**, 847–873.
[2] G. Lewicki, M. Prophet *Minimal multi-convex projections*, Studia Mathematica 178 (2) **(2007)**, 99–124.
[3] J. Meissner, *Minimal multi-convex projections onto subspaces of incomplete algebraic polynomials*, in preparation.

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Equality of two strongly unique minimal projection constants

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Let $\mathcal{P}(X,Y)$ denote the set of all linear, continuous projections from a Banach space X onto its linear subspace Y. Let $\hat{f} = (f_1,\ldots,f_k) \in \mathbb{R}^k$ be such that $0 < f_1 \le f_2 \le \ldots \le f_k$, $\sum_{i=1}^k f_i = 1$. Define $f^{(0)} = (f_1, f_2, \ldots, f_k, 0, \ldots, 0)$, $f^{(j)} = (0,\ldots,0,1_{j+k},0,\ldots,0)$ for $j = 1,2,\ldots,n-k$. Let $\hat{H} = \ker \hat{f}$ and $H = \bigcap_{j=0}^{n-k} \ker f^{(j)}$. We prove that the strongly unique minimal projection constant (SUP-constant) of the space $\mathcal{P}(l_\infty^{(k)}, \hat{H})$ is equal to the SUP-constant of the space $\mathcal{P}(l_\infty^{(n)}, H)$. This solves a conjecture stated in [1]. The main tool applied in our proof is Kolmogorov's type theorem for strongly unique best approximation.

References

[1] W. Odyniec, M.P.Prophet, *A lower bound of the strongly unique minimal projection constant of $l_\infty^n, n \ge 3$*, Journal of Appproximation Theory 145 (2007), 111–121.

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Bernstein-Schnabl operators and degenerate parabolic problems on weighted spaces of continuous functions

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We present some new results on Bernstein-Schnabl operators

$$B_n(f)(x) := \int_J \cdots \int_J f\left(\frac{x_1 + \ldots + x_n}{n}\right) d\mu_x(x_1) \cdots d\mu_x(x_n),$$

associated with a continuous selection of probability Borel measures on a noncompact real interval, acting on the weighted space of continuous functions $C_0^w(J) := \{f \in C(J) \mid wf \text{ vanishes at the boundary of } J\}$, w being a strictly positive continuous weight on J.

Our approach allows to include in a unique unifying setting several classical sequences of positive linear operators currently studied in Approximation Theory as Szàsz-Mirakjan operators, Post-Widder operators, Gauss-Weierstrass operators and many others.

We investigate shape-preserving, regularity and approximation properties of the operators B_n.

In particular, we establish an asymptotic formula which relates them to a degenerate differential operator of the form

$$Au := \alpha u''$$

defined on a suitable subspace $D_w(A)$ of $C_0^w(J)$, where α is a strictly positive continuous function satisfying some growth assumptions at the boundary of J. Accordingly, we show that the positive C_0-semigroup generated by $(A, D_w(A))$, and then the solution ot the parabolic problem associated with A, can be approximated by means of iterates of the B_n's.

References

[1] F. Altomare, V. Leonessa and S. Milella, *Bernstein-Schnabl operators on noncompact real intervals,* preprint 2008, submitted.

[2] F. Altomare, V. Leonessa and S. Milella, *Cores for second-order differential operators on real intervals*, to appear in Communications in Applied Analysis, 2009.

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Generators of positive semigroups in weighted continuous function spaces

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In this talk we present some generation results of strongly continuous positive semigroups for degenerate second-order differential operators in weighted continuous function spaces.

Our approach is essentially based on semigroup theory and on some general generation results we established in [1].

Consider a weight function w on a real interval J, i.e. $w \in C(J)$ and $w(x) > 0$ for every $x \in J$, and set $r_1 := \inf J \in \mathbb{R} \cup \{-\infty\}$ and $r_2 := \sup J \in \mathbb{R} \cup \{+\infty\}$; our purpose is to find under which conditions a differential operator of the form

$$Lu = \alpha u'' + \beta u' + \gamma u, \tag{10}$$

with continuous coefficients and with maximal type boundary conditions, i.e.

$$\lim_{x \to r_i} w(x)(\alpha u''(x) + \beta u'(x)) \in \mathbb{R}, \quad (i = 1, 2), \tag{11}$$

or Wentzel type boundary conditions, i.e.

$$\lim_{x \to r_i} w(x)(\alpha u''(x) + \beta u'(x)) = 0, \quad (i = 1, 2), \tag{12}$$

or generalized reflecting barriers boundary conditions, i.e.

$$\lim_{x \to r_i} \frac{(u(x)w(x))'}{w^2(x)W(x)} = 0, \quad (i = 1, 2), \tag{13}$$

where W is the Wronskian function, is the generator of a strongly continuous positive semigroup on the weighted space

$$E^w(J) := \left\{ f \in C(J) : wf \in C(\widetilde{J}) \right\}$$

where $\widetilde{J}$ indicates the two-point compactification of J.

We show some applications of generators of semigroups of the form 10 on a domain with boundary conditions respectively as 11, 12 and 13.

References

[1] F. Altomare and G. Musceo, *Markov processes and positive semigroups on some classes of weighted continuous function spaces*, Rend. Circ. Mat. Palermo, vol **57**, n. 1, April 2008, 61-82.

[2] F. Altomare and G. Musceo, *Positive semigroups generated by degenerate second-order differential operators*, Funkcialaj Ekvacioj, **51** (2008), 431-458.

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Relaxed multiple functionals and gradient Young measures generated by sequences in OrliczSobolev spaces

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We present two relaxation theorems for non-convex multiple integral functionals with non-power M-growth-type densities in the setting of Orlicz-Sobolev $W^{1,M}$-spaces (modeled by Orlicz L^M-spaces),which are generalizations of the well-known Dacorogna and Pedregal-Sychev relaxation theorems for functionals with p-power-growth-type densities in Sobolev $W^{1,p}$-spaces. In the connection with the formulation and the proofs of these theorems we present also two auxiliary results: the so-called decomposition lemma for sequences of functions in OrliczSobolev $W^{1,M}$-spaces and the characterization theorem for so-called $W^{1,M}$-gradient Young measures, i.e. measurable-dependent families of probabilistic Radon measures generated by sequences of gradients of OrliczSobolev functions. These theorems are $W^{1,M}$-generalizations of the well-known Fonseca-Müller-Pedregal and Kinderlehrer-Pedregal $W^{1,p}$-theorems for Sobolev functions.

All these results have been recently and successfully applied by us such as first technical results: for establishing, in the setting of "interpolation" Orlicz-Sobolev spaces, (W^{1,M_1}-W^{1,M_2})-relaxation theorems for multiple integral functionals with non-standard non-power (M_1, M_2)-growth-type densities (which generalize the known (W^{1,p_1}-W^{1,p_2})-relaxation theorems for functionals with non-standard (p_1, p_2)-power-growth-type densities due to Fonseca-Malý, Bouchitté-Fonseca-Malý, Acerbi-Bouchitté-Fonseca) as well as characterization theorems for so-called $W^{1,M}$-gradient Young-DiPerna and $W^{1,M}$-A-Young-DiPerna measures (which generalize recent $W^{1,p}$-theorems due to Fonseca-Müller, Braides-Fonseca-Leoni, Fonseca-Müller-Leoni, and Kalamajska-Kruzik, Fonseca-Kruzik).

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The Euler–Lagrange differential inclusion in Orlicz–Sobolev spaces and its applications

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We present the Euler–Lagrange differential inclusion in Orlicz–Sobolev spaces established by the results [1, 2, 4] on the integral representation of Clarke's generalized gradient of locally Lipschitz integral functionals defined on Orlicz spaces. We give applications of this result in the C^1-smooth case of integral functionals (resp., together with [3]) in the proof of the existence of global weak solutions on $[0, T]$ (resp., Young measure solutions) in the Orlicz–Sobolev space setting for the nonlinear hyperbolic problems in nonlinear elasticity $u''_{tt} = \operatorname{div}(\sigma(Du)) + \mu(\triangle u)'_t$ (resp., for $u''_{tt} = \operatorname{div}(\sigma(Du))$) subject to the classical Cauchy initial condition and the Dirichlet boundary condition, where $u\colon \Omega\times[0,T] \to \mathbb{R}^d$ is unknown, $\Omega \subset \mathbb{R}^n$ is a bounded Lipschitz domain, $T, \mu > 0$, $W\colon \mathbb{M}^{n\times d} \to \mathbb{R}$ is the stored-energy function, and $\sigma = \partial W/\partial F$ is the Piola–Kirchhoff stress function.

References

[1] H.T. Nguyêñ and D. ączka, *Existence theorems for the Dirichlet elliptic inclusion involving exponential-growth-type multivalued right-hand side*, Bull. Polish Acad. Sci. Math. **53** (2005), 361–375.

[2] H.T. Nguyêñ and D. Pączka, *Generalized gradients for locally Lipschitz integral functionals on non-L^p-type spaces of measurable functions*, in: Function Spaces VIII (H. Hudzik, J. Musielak, M. Nowak, and L. Skrzypczak, eds.), Banach Center Publ., vol. 79, 2008, pp. 135–156.

[3] H.T. Nguyêñ and D. Pączka, *The Young measure representation for weak cluster points of sequences in M-spaces of measurable functions*, Bull. Polish Acad. Sci. Math., **56** (2008), 109–120.

[4] R. Płuciennik, S. Tian, and Y. Wang, *Non-convex integral functionals on Musielak–Orlicz spaces*, Comment. Math. Prace Mat. **30** (1990), 113–123.

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Minimality properties of mixed Tsirelson spaces

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Mixed Tsirelson spaces form now an important class of spaces in the theory of Banach spaces extensively studied with respect to their distortability, local structure as well as minimality properties. In the talk various types of minimality properties of mixed Tsirelson spaces $T[(\mathcal{A}_{n_j}, \theta_j)_{j\in\mathbb{N}}]$ and $T[(\mathcal{S}_{n_j}, \theta_j)_{j\in\mathbb{N}}]$ and their dual spaces will be discussed, basing on the recent work with A. Manoussakis.

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Rotundity and monotonicity properties in a certain Banach function spaces

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It is well known that rotundity properties of Banach spaces have applications in various branches of mathematics. Moreover, if the focus of the study is Banach lattices, then there are strong relationships between rotundity properties and monotonicity properties. In this work, we study the local rotundity and local monotonicity structures of a certain Banach lattice, namely generalized Calderón-Lozanovskiǐ spaces. The result will be a generalization of two excellent papers ([H. Hudzik, P. Kolwicz, A. Narloch, Local rotundity structure of Calderón-Lozanovskiǐ spaces, Indag. Math. (NS) 17 (3) (2006) 373-395.], [H. Hudzik, A. Narloch, Local monotonicity structure of Calderón-Lozanovskiǐ spaces, Indag. Math. (NS) 15 (1) (2004) 1-12.]) by considering Orlicz function with parameter, called Musielak-Orlicz function, instead of Orlicz function.

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Points of upper local uniform monotonicity in Calderón-Lozanovskiĭ spaces

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Let $(E, \|\cdot\|_E)$ be a Köthe space with the Fatou property and φ be an Orlicz function. The *Calderón-Lozanovskiĭ space* E_φ generated by the couple (E, φ) is defined as the set of these measurable real functions (or these sequences, when E is a sequence space) that $I_\varphi(\lambda x) = \|\varphi \circ \lambda x\|_E < \infty$ for some $\lambda > 0$. The norm in E_φ is defined by

$$\|x\|_\varphi = \inf\{\lambda > 0 : I_\varphi(x/\lambda) \leq 1\}$$

The space E_φ is a special case of a general Calderón-Lozanovskiĭ construction of an intermediate space $\Psi(E_0, E_1)$ between Banach function lattices E_0 and E_1 (cf. [1], [4]). The class of Calderón-Lozanovskiĭ spaces contains among others Orlicz spaces and Orlicz-Lorentz spaces.

It is known that the points of lower (upper) monotonicity of a Banach lattice E play an analogous role as the extreme points in a Banach space X. Similarly, the role of points of upper (lower) local uniform monotonicity in Banach lattices is analogous to that of points of local uniform rotundity in Banach spaces.

The local monotonicity structure of Calderón-Lozanovskiĭ spaces has been considered in [2]. However, the precise full criteria have been presented only for points of lower and upper monotonicity ($LLUM$ and $ULUM$-points, respectively). Considering $LLUM$ and $ULUM$-points, the authors of [2] gave only some sufficient and some necessary conditions, basing often on too strong assumptions. The $LLUM$-points of Calderón-Lozanovskiĭspaces have been characterized in [3]. We will present full criteria for $ULUM$-points of Calderón-Lozanovskiĭ spaces. It is worth to mention that the structure of $LLUM$ and $ULUM$-points is quite different in E_φ. Namely, considering the $LLUM$-point x, the local $\Delta_2^E(x)$ condition is crucial, while in the case of an $ULUM$-point x the respective global condition Δ_2^E is essential which is indeed really stronger than the local $\Delta_2^E(x)$ in general.

All results that will be presented are obtained in collaboration with Paweł Kolwicz and they will be published in our joint paper "Points of upper local uniform monotonicity of Calderón-Lozanovskiĭ spaces" that will appear in Journal of Convex Analysis.

References

[1] A. P. Calderón, *Intermediate spaces and interpolation, the complex method*, Studia Math. (1964), 113-190.
[2] H. Hudzik, A. Narloch, *Local monotonicity structure of Calderón-Lozanovskiĭ spaces*, Indag. Math. N. S. 15.1 (2004), 1-12.

[3] P. Kolwicz and R. Płuciennik, *Local* $\Delta_2^E(x)$ *condition as a crucial tool for local struc- ture of Calderón-Lozanovskiĭ sequence spaces*, J. Math. Anal. Appl., to appear.
[4] G. Ya. Lozanovskiĭ, *On some Banach lattices*, Sibirsk. Math. J. 12 (1971), 562-567.

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On the numerical index of the real $L_p(\mu)$-spaces

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The numerical index of a Banach space is a constant introduced by G. Lumer in 1968 (see [3]). Given a Banach space X, we will write X^* for its dual and $\mathcal{L}(X)$ for the Banach algebra of all (bounded linear) operators on X. For an operator $T \in \mathcal{L}(X)$, its numerical radius is defined as

$$v(T) := \sup\{|x^*(Tx)| : x^* \in X^*,\ x \in X,\ \|x^*\| = \|x\| = x^*(x) = 1\}.$$

Obviously, v is a seminorm on $\mathcal{L}(X)$ smaller than the operator norm. The *numerical index* of $X$is the constant given by

$$n(X) := \inf\{v(T) : T \in \mathcal{L}(X),\ \|T\| = 1\}.$$

Classical references here are the aforementioned paper [3] and the monographs by F. Bonsall and J. Duncan [1, 2] from the seventies. The reader will find the state-of-the-art on the subject in the recent survey paper [7] and references therein.

Evidently, $0 \leq n(X) \leq 1$ for every Banach space X, and $n(X) > 0$ means that the numerical radius and the operator norm are equivalent on $\mathcal{L}(X)$. In the real case, all values in $[0, 1]$ are possible for the numerical index. In the complex case one has $1/e \leq n(X) \leq 1$ and all of these values are possible. Remark that $n(X) \leq n(X^*)$, and $n(X) = n(X^*)$ if X is reflexive. There are some classical Banach spaces for which the numerical index has been calculated. For instance, the numerical index of $L_1(/mu)$ is 1, and this property is shared by any of its isometric preduals. In particular, $n\left(C(K)\right) = 1$ for every compact K and $n(Y) = 1$ for every finite-codimensional subspace Y of $C[0, 1]$. If H is a Hilbert space with $\dim X > 1$ then $n(H) = 0$ in the real case and $n(H) = 1/2$ in the complex case.

The problem of computing the numerical index of the L_p-spaces was posed for the first time in the seminal paper [3, p. 488]. There it is proved that $\{n(\ell_p^2) : 1 < p < \infty\} = [0, 1]$ in the real case, though the exact computation of $n(\ell_p^2)$ is not achieved for $p \neq 2$ (even now!). Recently, some results have been obtained on the numerical index of the L_p-spaces [4, 5, 6, 8, 9].

(a) The sequence $\left(n(\ell_p^m)\right)_{m\in\mathbb{N}}$ is decreasing in the non strict sense.
(b) $n\left(L_p(\mu)\right) = \inf_m n(\ell_p^m)$ for every measure μ such that $\dim(L_p(\mu)) = \infty$.
(c) In the real case, $\max\left\{\frac{1}{2^{1/p}}, \frac{1}{2^{1/q}}\right\} M_p \leq n(\ell_p^2) \leq M_p$ where

$$M_p = \max_{t\in[0,1]} \frac{|t^{p-1} - t|}{1 + t^p} = \max_{t\geq 1} \frac{|t^{p-1} - t|}{1 + t^p},$$

which is the numerical radius of the operator $T(x,y) = (-y,x)$ defined on the real space ℓ_p^2, see [8, Lemma 2] for instance.

(d) In the real case, $n(\ell_p^m) > 0$ for $p \neq 2$ and $m \in \mathbb{N}$.

Our main result is the following theorem.

Theorem 1. *The numerical index for any real infinite dimensional space $L_p(\mu)$ satisfies*

$$n(L_p(\mu)) \geq \frac{M_p}{8e} \tag{14}$$

Since $M_p > 0$ for $p \neq 2$, this extends item (d) for infinite-dimensional real $L_p(\mu)$-spaces, meaning that the numerical radius and the operator norm are equivalent on $\mathcal{L}(L_p(\mu))$ for every $p \neq 2$ and every positive measure μ. This answers in the positive a question raised by C. Finet and D. Li (see [5, 6]) also posed in [7, Problem 1].

The key idea to get this result is to define a new seminorm on $\mathcal{L}(L_p(\mu))$ which is in between the numerical radius and the operator norm, and to get constants of equivalence between these three seminorms.

For any $x \in L_p(\mu)$, we denote $x^\# = |x|^{p-1}\mathrm{sign}(x)$ in the real case, which is the unique element in $L_q(\mu)$ such that $\int_\Omega x^\# x d\mu = \|x\|_p \|x^\#\|_q = \|x\|_p^p$. With this notation, for $T \in \mathcal{L}(L_p(\mu))$ one has

$$v(T) = \sup\left\{ \left| \int_\Omega x^\# T x d\mu \right| : x \in L_p(\mu),\ \|x\|_p = 1 \right\}.$$

Given an operator $T \in \mathcal{L}(L_p(\mu))$, the *absolute numerical radius* of T is given by

$$|v|(T) = \sup\left\{ \int_\Omega |x^\# T x| d\mu : x \in L_p(\mu),\ \|x\|_p = 1 \right\}.$$

Obviously, $v(T) \leq |v|(T) \leq \|T\|$ for any $T \in \mathcal{L}(L_p(\mu))$. Given an operator T on the *real* space $L_p(\mu)$, we will show that

$$v(T) \geq \frac{M_p}{4}|v|(T) \quad \text{and} \quad |v|(T) \geq \frac{n\left(L_p^{\mathbb{C}}(\mu)\right)}{2}\|T\|,$$

where $n\left(L_p^{\mathbb{C}}(\mu)\right)$ is the numerical index of the *complex* space $L_p(\mu)$. Since

$$n\left(L_p^{\mathbb{C}}(\mu)\right) \geq \frac{1}{e}$$

(as for any complex space, see [1, Theorem 4.1]), the above two inequalities together give, in particular, the inequality 14.

References

[1] F. F. Bonsall and J. Duncan, *Numerical Ranges of operators on normed spaces and of elements of normed algebras*, London Math. Soc. Lecture Note Ser. 2, Cambridge, 1971.

[2] F. F. Bonsall and J. Duncan, *Numerical Ranges II*, London Math. Soc. Lecture Note Ser. 10, Cambridge, 1973.

[3] J. Duncan, C. McGregor, J. Pryce, A. White, *The numerical index of a normed space*, J. London Math. Soc. **2** (1970), 481-488.

[4] E. Ed-dari, *On the numerical index of Banach spaces*, Linear Algebra Appl. **403** (2005), 86-96.

[5] E. Ed-dari and M. Khamsi, *The numerical index of the L_p space*, Proc. Amer. Math. Soc. **134** (2006), 2019-2025.

[6] E. Ed-dari, M. Khamsi, and A. Aksoy, *On the numerical index of vector-valued function spaces*, Linear Mult. Algebra **55** (2007), 507-513.

[7] V. Kadets, M. Martłn, and R. Payá, *Recent progress and open questions on the numerical index of Banach spaces*, Rev. R. Acad. Cien. Serie A. Mat., **100** (2006), 155-182.

[8] M. Martín and J. Merí, *A note on the numerical index of the Lp space of dimension two*, Linear Mult. Algebra **57** (2009), 201-204.

[9] M. Martín, and R. Payá, *Numerical index of vector-valued function spaces*, Studia Math. **142** (2000), 269-280.

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Characterization of Radon Integrals as Linear Functionals

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We consider the following *problem of characterization of Radon integrals* on an arbitrary Hausdorff space T: to characterize the family of all integrals with respect to Radon measures on T and its important subfamilies among all linear functionals on an appropriate linear space $A(T)$ of integrable functions.

In this article we give a solution of the problem in the form of general parametric theorems. These theorems imply as partial cases the Riesz theorem on the Riemann-Stieltjes integral on a segment, the Radon theorem on the Lebesgue integral with respect to Radon measure on a compact in $\mathbb{R}^n$, the Kakutani, Halmos-Hewitt-Edwards, and Bourbaki-Prokhorov theorems on characterizations of Radon integrals for a compact, locally compact, and Tykhonoff spaces, respectively.

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Lebesgue type points in strong $(\mathbf{C},\alpha)$ approximation of Fourier series

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We present an estimation of the $H^{q,\alpha}_{k_0,k_r}f$ and $H^{\phi,\alpha}_{\lambda,u}f$ means of the Fourier series of a function $f \in L^p$ $(1 < p < \infty)$ or $f \in C$ as approximation versions of the Totik type (see [2, 2]) generalization of the result of G. H. Hardy and J. E. Littlewood (see [1]), where the partial sums $S_k f$ are replaced by the (C,α)-means $\sigma^\alpha_k f$ of its Fourier series, i.e.:

$$H^{q,\alpha}_{k_0,k_r} f(x) := \left\{ \frac{1}{r+1} \sum_{\nu=0}^{r} \left| \sigma^{\alpha}_{k_\nu} f(x) - f(x) \right|^q \right\}^{1/q}, \qquad (q > 0, \quad \alpha > -1).$$

where $0 < k_0 < k_1 < k_2 < ... < k_r$, and

$$H^{\phi,\alpha}_{\lambda,u} f(x) := \sum_{\nu=0}^{\infty} \lambda_\nu(u)\, \phi\left(\left| \sigma^{\alpha}_{\nu} f(x) - f(x) \right| \right), \qquad (\alpha > -1)$$

where (λ_ν) is a sequence of positive functions defined on the set having at least one limit point and $\phi : [0,\infty) \to \mathbf{R}$.

As a measure of approximation by the above means we use the pointwise characteristic

$$w_x f(\delta)_p := \left\{ \frac{1}{\delta} \int_0^\delta |\varphi_x(t)|^p \, dt \right\}^{1/p},$$

$$\text{where} \quad \varphi_x(t) := f(x+t) + f(x-t) - 2f(x) \quad ,$$

constructed on the base of definition of Lebesgue points ($L^p - points$) (cf [4]).

We also give some corollaries on norm approximation.

References

[1] G.H. Hardy, J.E. Littlewood, *Sur la série de Fourier d'une function a caré sommable*, Comptes Rendus, Vol. 28, (1913), 1307-1309.

[2] V. Totik, *On Generalization of Fejér summation theorem*, Coll. Math. Soc. J. Bolyai 35 Functions series, operators, Budapest (Hungary) (1980), 1195-1199.

[3] V. Totik, *Notes on Fourier series: Strong approximation*, J. Approx. Th. 43 (1985), 105-111.

[4] W. Łenski, *On the rate of pointwise strong (C,α) summability of Fourier series*, Colloquia Math. Soc. János Bolyai, 58 Approx. Theory, Kecskemét (Hungary), (1990), 453-486.

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A characterization of bisymmetrical functionals

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Let $\Omega_i, i = 1, 2$ be compact sets. Consider spaces $B(\Omega_i, \mathbb{R})$ of bounded functions defined on Ω_i, and let F and G be functionals defined in $B(\Omega_1, \mathbb{R})$ and $B(\Omega_2, \mathbb{R})$, respectively. We characterize F and G such that the equation

$$G\left(F_t(x(s,t))\right) = F\left(G_s(x(s,t))\right)$$

holds for every $x \in B(\Omega_1 \times \Omega_2, \mathbb{R})$, under some additional regularity assumptions. It turns out that F and G are conjugated to an integral with respect to some Radon measure in B_i. The main tool in the proof is a result of Gy. Maksa from [1].

References

[1] Gy. Maksa, *Solution of generalized bisymmetry type equations without surjectivity assumptions*, Aequationes Math. 57(1999), 50-74.

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Some space structures and partially differential systems

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We have a survey about researching some space structures of Bochner-Orlicz spaces, some of Banach spaces. Finally an application is given in a class of elliptic partial differential systems.

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Characterization of the strict and uniform convexity of the Besicovitch-Musielak-Orlicz spaces of almost periodic functions

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The class of uniformly almost periodic functions (*u.a.p.*) of H. Bohr is subject to various generalizations. Following a topological characterization of almost periodicity, A. S. Besicovitch obtains a first extension in the context of L^p spaces. Precisely, he defined the $S^q_{a.p.}$, $W^q_{a.p.}$ and $B^q_{a.p.}$ spaces (resp. Stepanoff, Weyl and Besicovitch space of almost periodic functions). Later on, T.R. Hilmann considered such extensions in the context of Orlicz spaces using a similar approach. Most of Hilmann's works are concerned with topological and structural properties of the new spaces.

On the other hand, M. Morsli and F. Bedouhene considered the fundamental geometric properties of the Besicovitch-Orlicz spaces of almost periodic functions.

In this talk, we consider the natural extension of almost periodicity in the context of Musielak-Orlicz spaces, that is, the case when the φ function generating the spaces depends on a parameter. We characterize the strict and uniform convexity of this class.

References

[1] M. Morsli, M. Smaali, *Characterization of the strict convexity of the Besicovitch-Musielak- Orlicz space of almost periodic functions*, Comment. Math. Univ. Carolin.; 48 (2007), no. 3, 443-458.

[2] M. Morsli, M. Smaali, *Characterization of the uniform convexity of the Besicovitch-Musielak- Orlicz spaces of almost periodic functions*, Comment. Math. Prace Mat.; 46 (2006), no. 2, 215-231.

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Hybrid iterative scheme for generalized equilibrium problems and fixed point problems of finite family of nonexpansive mappings

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In this paper, we introduce a new mapping and a Hybrid iterative scheme for finding a common element of the set of solutions of a generalized equilibrium problem and the set of common fixed points of a finite family of nonexpansive mappings in a Hilbert space. Then, we prove the strong convergence of the proposed iterative algorithm to a common fixed point of a finite family of nonexpansive mappings which is a solution of the generalized equilibrium problem. The results obtained in this paper extend the recent ones of Takahashi and Takahashi [S. Takahashi, W. Takahashi, *Strong convergence theorem for a generalized equilibrium problem and a nonexpansive mapping in a Hilbert space*, Nonlinear Anal. 69 (2008) 1025-1033].

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On uniform convergence of sine series

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It is well know that there are a great number of interesting results in Fourier analysis established by assuming monotonicity of coefficients. The following classical convergence result can be found in many monographs (see [1] and [3], for example).

Theorem. *Suppose that $b_n \geq b_{n+1}$ and $b_n \to 0$. Then a necessary and sufficient condition for the uniform convergence of the series*

$$\sum_{n=1}^{\infty} b_n \sin nk$$

is $nb_n \to 0$.

This result have been generalized by weakening the monotone condition of the coefficient sequences. Generally speaking, it has become an important topic how to generalize monotonicity.

We introduced a new class of sequences called $GM\left(\beta, r\right)$, which is the generalization of a class considered by Tikhonov in [2]. Moreover, we obtain sufficient and necessary conditions for uniform convergence of sine series with (β, r) −general monotone coefficients.

References

[1] T. W. Chaundy and A. E. Jolliffe, *The uniform convergence of a certain class of trigonometrical series*, Proc. London Math. Soc., 15(1916), 214-216.

[2] S. Tikhonov, *Best approximation and moduli of smoothness: Computation and equivalence theorems*, J. Approx. Theory, 153 (2008), 19-39.

[3] A. Zygmund, *Trigonometric series*, Vol. I, University Press (Cambridge, 1959).

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On some special case of Beckenbach convexity

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The inequality

$$f\left(\frac{x+y}{2}\right) \le \gamma(a)[f(x)+f(y)], \quad x \le ay, \tag{15}$$

where $\gamma(a) \in \left(0, \frac{1}{2}\right)$, is used in Orlicz spaces theory to obtain some nice properties of spaces connected to this function. In order to obtain a geometrical description of this inequality we consider a special case of Beckenbach convexity. In this kind of convexity, called $\omega-$convexity, straight lines (used in the classical case) are replaced by suitable pieces of the graph of some function. We describe the general form of a function which may be used to this purpose and then we prove that if a function f is $\omega-$convex with respect to a function $\omega(x) = |x|^p, p > 1$, then f satisfies (15).

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An iterative approximation method for solving a general system of variational inequality problems and mixed equilibrium problems

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In this paper, we introduce an iterative algorithm for finding a common element of the set of solutions of a mixed equilibrium problem, the set of fixed points of an infinitely family of nonexpansive mappings and the set of solutions of general system of variational inequality for cocoercive mapping in a real Hilbert space. Furthermore, we prove that the proposed iterative algorithm converges strongly to a common element of the above three sets. Our results extend and improve the corresponding results of Ceng, Wang, and Yao [L.C. Ceng, C.Y. Wang, and J.C. Yao, Strong convergence theorems by a relaxed extragradient method for a general system of variational inequalities, Math. Meth.Oper. Res., 67, 375-390 (2008)], Ceng and Yao [L.C. Ceng and J.C. Yao, *A hybrid iterative scheme for mixed equilibrium problems and fixed point problems*, J. Comput. Appl. Math.,doi:10.1016/j.cam.2007.02.022], Takahashi and Takahashi [S. Takahashi and W. Takahashi, *Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces*, J. Math. Anal. Appl., 331(2007), 506-515] and many others.

Nonexpansive mapping and Equilibrium problem and Fixed point and General system of variational inequality.

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Convergence theorems of modified Ishikawa iterations for two hemi-relatively nonexpansive mappings in a Banach space

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In this paper, we prove strong convergence theorems of modified Ishikawa and Halpern iterations for countable family of two hemi-relatively nonexpansive mappings in Banach spaces. Moreover, we discuss the problem of strong convergence of relatively nonexpansive mappings and we also apply our results to generalizes extend and improve these announced by Qin and Su's result [*Strong convergence theorems for relatively nonexpansive mapping in a Banach space*, Nonlinear Anal. 67 (2007) 19581965.], Nilsrakoo and Saejung's result [*Strong convergence to common fixed points of countable relatively quasi- nonexpansive mappings*, Fixed Point Theory and Appl. (2008), doi:10.1155/ 2008/312454.] and Su et al.'s result [*Strong convergence of monotone hybrid algorithm for hemi-relatively nonexpansive mappings*, Fixed Point Theory and Appl. (2008), doi:10.1155/2008/284613.].

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New Besov-type Spaces and Triebel-Lizorkin-type Spaces

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Let $s, \tau \in \mathbb{R}$ and $q \in (0,\infty]$. We introduce Besov-type spaces $\dot{B}^{s,\tau}_{p,q}(\mathbb{R}^n)$ for $p \in (0,\infty]$ and Triebel-Lizorkin-type spaces $\dot{F}^{s,\tau}_{p,q}(\mathbb{R}^n)$ for $p \in (0,\infty)$, which unify and generalize the Besov spaces, Triebel-Lizorkin spaces and Q spaces. We then establish the φ-transform characterization of these new spaces in the sense of Frazier and Jawerth. Using the φ-transform characterization of $\dot{B}^{s,\tau}_{p,q}(\mathbb{R}^n)$ and $\dot{F}^{s,\tau}_{p,q}(\mathbb{R}^n)$, we obtain their embedding and lifting properties; moreover, for appropriate τ, we also establish the smooth atomic and molecular decomposition characterizations of $\dot{B}^{s,\tau}_{p,q}(\mathbb{R}^n)$ and $\dot{F}^{s,\tau}_{p,q}(\mathbb{R}^n)$. For $s \in \mathbb{R}$, $p \in (1,\infty)$, $q \in [1,\infty)$ and $\tau \in [0, \frac{1}{(\max\{p,q\})'}]$, via the Hausdorff capacity, we introduce certain Besov-Hausdorff spaces $B\dot{H}^{s,\tau}_{p,q}(\mathbb{R}^n)$ and Triebel-Lizorkin-Hausdorff $F\dot{H}^{s,\tau}_{p,q}(\mathbb{R}^n)$ and prove that the dual spaces of $B\dot{H}^{s,\tau}_{p,q}(\mathbb{R}^n)$ and $F\dot{H}^{s,\tau}_{p,q}(\mathbb{R}^n)$ are just, respectively, $\dot{B}^{-s,\tau}_{p',q'}(\mathbb{R}^n)$ and $\dot{F}^{-s,\tau}_{p',q'}(\mathbb{R}^n)$, where t' denotes the conjugate index of $t \in (1,\infty)$. Applications to trace theorems and boundedness of the pseudo-differential operators with homogeneous symbols are obtained. The inhomogeneous versions of these results are also given.

References

[1] D. Yang and W. Yuan, *A note on dyadic Hausdorff capacities*, Bull. Sci. Math. 132(2008), 500-509.
[2] D. Yang and W. Yuan, *A new class of function spaces connecting Triebel-Lizorkin spaces and Q spaces*, J. Funct. Anal. 255(2008), 2760-2809.
[3] D. Yang and W. Yuan, *New Besov-type spaces and Triebel-Lizorkin-type spaces including Q spaces*, Math. Z. (to appear).
[4] Y. Sawano, D. Yang and W. Yuan, *New applications of Besov-type and Triebel-Lizorkin-type spaces*, Submitted.
[5] W. Yuan, Y. Sawano and D. Yang, *Decompositions of Besov-Hausdorff and Triebel-Lizorkin-Hausdorff spaces and their applications*, Submitted.
[6] W. Yuan, W. Sickel and D. Yang, *New inhomogeneous Besov-type and Triebel-Lizorkin-type spaces*, in preparation.

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Some properties for duals of Musielak-Orlicz spaces

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It is known that any functional $f \in (L_\Phi)^*$, where L_Φ is a Musielak-Orlicz space, is of the form $f = v + \varphi, (v \in L_{\Phi^*}, \varphi \in F)$ where $\varphi \in F$ means that $\langle x, \varphi \rangle = 0$ for any $x \in E_\Phi = \{x \in L^0 : I_\Phi(\lambda x) < +\infty \quad \text{for any} \quad \lambda > 0\}$ and Φ^* is the Musielak-Orlicz function conjugate to Φ in the sense of Young, as well as that if L_Φ is equipped with the Luxemburg norm, then

$$\|f\|^o = \|v\|^o_{\Phi^*} + \|\varphi\|^o \, ,$$

and if L_Φ is equipped with the Orlicz norm, then

$$\|f\| = \inf \left\{ \lambda > 0 : \rho^*(\frac{f}{\lambda}) \leq 1 \right\},$$

where $\rho^*(f) = I_{\Phi^*}(v) + \|\varphi\|$ for any $f \in (L_\Phi)^*$. We will present relationships between the modular ρ^* and the norm $\|\|$ in the dual spaces $(L_\Phi)^*$ in the case when L_Φ is equipped with the Orlicz norm. Moreover, criteria for extreme points of the unit sphere of the dual space $(L^o_\Phi)^*$ will be presented.

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On complemented subspaces of non-archimedean power series spaces

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The non-archimedean power series spaces, $A_1(a)$ and $A_\infty(b)$, are the most known and important examples of non-archimedean nuclear Fréchet spaces. We prove that the range of every continuous linear map from $A_p(a)$ to $A_q(b)$ has a Schauder basis, if either $p = 1$ or $p = \infty$ and the set $M_{b,a}$ of all bounded limit points of the double sequence $(b_i/a_j)_{i,j\in\mathbb{N}}$ is bounded. It follows that every complemented subspace of a power series space $A_p(a)$ has a Schauder basis, if either $p = 1$ or $p = \infty$ and the set $M_{a,a}$ is bounded.

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Applications of coincidence equations to boundary value problems

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In [1] O'Regan and Zima obtained results on the existence of solutions in cones in Banach spaces for the equation $Lx = Nx$, where L is a Fredholm operator of index zero and N is a nonlinear mapping. We will discuss an application of these results to the non-local boundary value problems at resonance.

References

[1] D. O'Regan, M. Zima, *Leggett-Williams norm-type theorems for coincidences*, Arch. Math. 87 (2006), 233-244.

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