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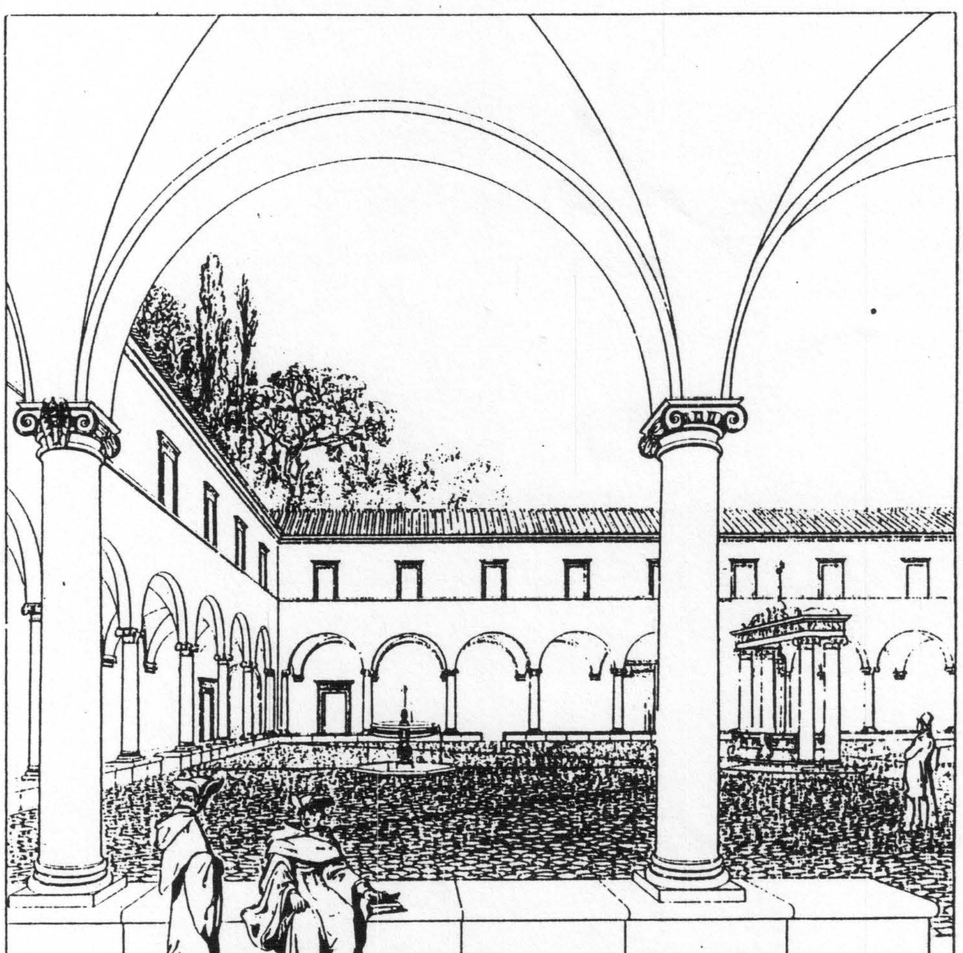

University of Rome "La Sapienza", Faculty of Engineering: The Cloister (Paul Letarouilly, 1820)

VOLUME II

CALCULATION OF TRANSIENT PULSE PROPAGATION THROUGH NON-LINEAR MAGNETIC SHEETS WITH AN ERROR-BASED APPROACH

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Abstract - The paper deals with a numerical formulation of Maxwell equations based on the edge elements and the minimization of the constitutive error. The formulation provides two dual solutions which can also be used to have an indication of the distribution of the numerical error in the solution domain. The method is applied to the calculation of transient pulse propagation through conducting nonlinear magnetic sheets. In this case, an iterative technique which is very efficient in matching the wave propagation in the vacuum to the magnetoquasistatic diffusion inside the conducting material is also proposed and applied.

I. INTRODUCTION

Sheets of ferromagnetic material can effectively be utilised for the shielding of interferences caused by transient phenomena in the low-frequency range. In fact, if the amplitude of the incident field is not so intense to lead the sheet in the saturation state, screens of ferromagnetic materials exhibit superior shielding effectiveness, for a given thickness, with respect to the common metallic (copper or aluminium) sheets because their skin depths are thinner. This assumption is not satisfied for the class of the safety problems connected to the lightning flashes or nuclear explosions, when the large external interferences create violent transients and the ferromagnetic material can saturate. The analysis of this phenomena can be particularly complicated because the magnetic material, even if hysteresis is neglected, shows a non linear behaviour and has a finite electric conductivity. The classical finite difference techniques [1] have extensively been used to solve the appropriate non-linear diffusion equation for the distribution field inside the ferromagnetic sheet with all the limitations associated with stability problems and reduction of the time-step size, evaluation of the error, refinement of the grid, treatment of the boundary conditions for 2D and 3D problems.

This paper deals with a new formulation and solution of the problem based on the edge elements and on the minimization of the constitutive error. The conventional methods enforce the constitutive relations and only one of the field equation (Faraday's law or Ampere's law) explicitly, whereas the other one is satisfied approximately. The approach used here is based on the minimization of the constitutive error [2, 3] and makes use of a global formulation in terms of two unknown (electric and magnetic) vector potentials with which both field equations are automatically and exactly verified. Thus the solution can be found by imposing the minimum of a certain functional, associated with the error in the constitutive equations.

The method provides an immediate information about the distribution on the local and global errors of the numerical solution, information that can be used to refine spatial and temporal discretizations.

The formulation allows for a unified treatment of eddy current and wave propagation problems. Nevertheless, the coexistence of vacuum and conducting materials may lead to an ill conditioned numerical formulation, due to the presence of regions having properties and characteristic parameters several orders of magnitude different. To tackle this difficulty an iterative technique is proposed and applied to the simple nonlinear conducting slab; its key aspect is based on an asymptotic expansion about the limit case of infinite electrical conductivity.

II. FORMULATION

A detailed description of the error based approach to the solution of full Maxwell equations is given in [3]. Here, we will summarize the main aspects. In particular, we refer to the following form of Maxwell equations:

$$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t \qquad (1)$$

$$\nabla\times\mathbf{H} = \partial\mathbf{D}_T/\partial t \qquad (2)$$

where

$$\mathbf{D}_T(\mathbf{x}, t) = \mathbf{D}(\mathbf{x}, t) + \int_0^t \mathbf{J}(\mathbf{x}, t)\, dt \qquad (3)$$

with the proper boundary and initial conditions and the monotonic constitutive properties

$$\mathbf{B} = \mathbf{f}_B(\mathbf{H}, \mathbf{x}, t) \qquad (4)$$

$$\mathbf{D} = \mathbf{f}_D(\mathbf{E}, \mathbf{x}, t) \qquad (5)$$

$$J = f_J(E, x, t) \tag{6}$$

Introducing the vector potentials **A** e **F** :

$$A = -\int_0^t E\,dt \tag{7}$$

$$F = \int_0^t H\,dt \tag{8}$$

the following fields:

$$E=-\partial A/\partial t, \qquad H=\partial F/\partial t,$$

$$B=B_o+\nabla\times A, \qquad D_T=D_o+\nabla\times F \tag{9}$$

automatically satisfy Eqs. (1) and (2) and the initial conditions

$$B(x,0)=B_o(x),\ D(x,0)=D_o(x) \tag{10}$$

From the numerical point of view [3] the vector potentials **A**(x,t) and **F**(x,t) are approximated in space using edge-element based functions [4] which possess full continuity inside elements and tangential continuity across adjacent elements. Piecewise linear approximation is assumed in time.

With time stepping the problem reduces to the solution of a number of subproblems defined in the time steps in which the only unknowns are the vector potentials at final time. Each subproblem is solved by minimizing a global error functional related to the constitutive relationship:

$$\Lambda = \int_{t_k}^{t_{k+1}} \int_V \lambda(x,t)\,dv dt \tag{11}$$

where V is the domain of integration, (t_k, t_{k+1}) is the k-th time step and:

$$\lambda(x,t) =$$

$$\alpha_H\left(\int_{H^*}^{H} f_B(h)\cdot dh + \int_{B^*}^{B} f_H(b)\cdot db + H^*\cdot B^* - H\cdot B\right) +$$

$$\alpha_E\left(\int_{D_T^*}^{D_T} f_E(d_T)\cdot dd_T + \int_{E^*}^{E} f_{D_T}(e)\cdot de + E^*\cdot D_T^* - E\cdot D_T\right) \tag{12}$$

$f_{D_T}(E)$ is the non linear mapping from **E** to D_T; f_H and f_E are the inverse mappings of f_B and f_{D_T}; (H^*, B^*) and (E^*, D_T^*) are two pairs which satisfy Eqs. (3), (4), taking also account of Eqs. (5) and (6); α_H and α_E are weighting factors. The minimization of Λ is here carried out using the Newton-Raphson algorithm.

As an example, in the linear, isotropic, time-invariant case ($B=\mu H$, $J=\sigma E$, $D=\varepsilon E$), with zero initial conditions, we have $f_H(B)=B/\mu$, $f_B(H)=\mu H$ with $H^* = 0$ and $B^* = 0$, $f_{D_T}(E, t) = [\sigma(t-t_k) + \varepsilon]E - \sigma A(t_k)$, $f_E(D_T, t) = (D_T + \sigma A(t_k))/[\sigma(t-t_k) + \varepsilon]$ and the local error (12) becomes

$$\lambda = \alpha_H\{\mu H^2/2 + B^2/2\mu - B\cdot H\} + \alpha_E[D_T - f_{D_T}(E, t)]\cdot[E - f_E(D_T, t)]/2, \tag{13}$$

The main advantages of this formulation are that the numerical solution is obtained via minimization of a global error functional leading to a symmetric and positive definite matrix and that an error estimation is readily available, being the numerical error concentrated in the constitutive equations. Moreover it allows for the unified treatment of quasistationary and wave propagation problems.

III. THE PROBLEM

The method is applied to the analysis of a transient pulse propagation through a nonlinear conducting magnetic sheet. Although the formulation of Section II is intrinsically three-dimensional, here we limit the study to a one dimensional sample case, because, anyway, most of the basic features of this kind of problems are present in these test cases as well. In particular we refer to an infinite sheet of ferromagnetic material (Fig. 1) whose thickness d, conductivity σ, nonlinear permeability $\mu(H)$ and incident electric field E_i are those of [5] and are summarized in Table I. A plane wave with the electric field polarized along the x axis and travelling along z is incident from region 1 on the sheet. It is required to compute the field transmitted through the sheet in region 3.

In this particular 1D case it is possible to restrict the analysis to region 2 by imposing the following well known boundary conditions at z=0 and z=d:

$$E_x(0, t) + \zeta_o H_y(0, t) = 2\,E_i(t) \tag{14}$$

$$E_x(d, t) - \zeta_o H_y(d, t) = 0 \tag{15}$$

where ζ_o is the impedance of the vacuum.

Table I – Nonlinear slab: relevant data.

d	1.26×10^{-4}m
σ	10^7 S/m
μ	$\mu_m + B_s \exp(-\lvert H\rvert/H_c)/H_c$
μ_m	1.67×10^{-4}H/m
B_s	1.53 T
H_c	120 A/m
E_i	$E_i(t)\,i_x$
$E_i(t)=$	$E_{i\,max}\sin 2\pi f t$ for $0 \le t \le 1/2f$; 0 otherwise
E_{imax}	10^4 V/m
f	1000 Hz

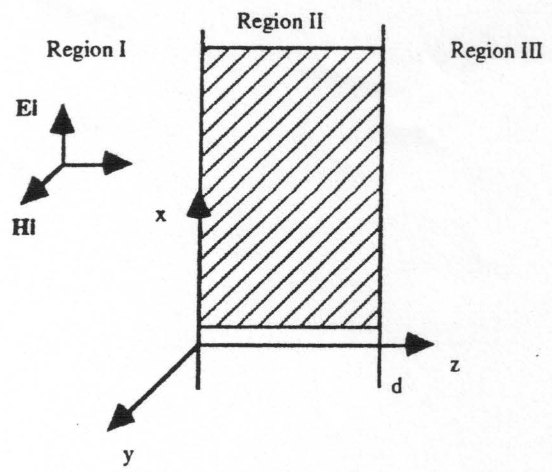

Fig. 1 – The nonlinear slab.

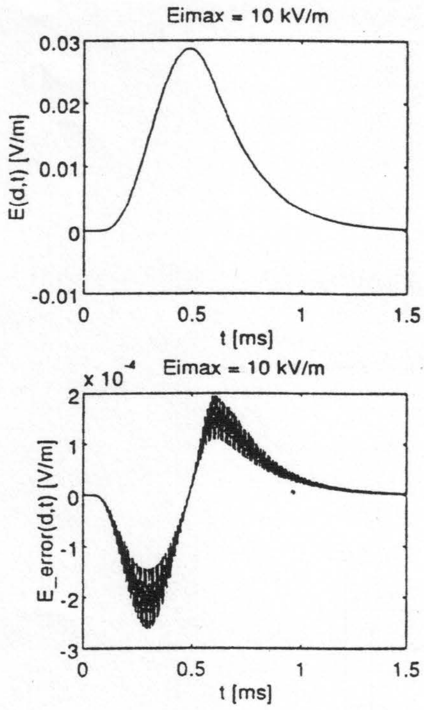

Fig. 2 – Nonlinear slab. The two estimates $-\partial A/\partial t$ (solid) and $\sigma^{-1}\nabla\times\partial F/\partial t$ (dashed) of the transmitted electric field and their difference.

The direct enforcement of these boundary conditions is not convenient in our formulation since the coupling between the electric and magnetic field at the boundary leads to a nonsymmetric matrix. In order to keep the symmetry, at every Newton-Raphson iteration inside a given time step $(t_{k+1}-t_k)$, the following three subproblems are solved:

I Solution: $E_x{}^{I}(0, t_{k+1}) = 1, E_x{}^{I}(d, t_{k+1}) = 0$
II Solution: $E_x{}^{II}(0, t_{k+1}) = 0, E_x{}^{II}(d, t_{k+1}) = 1$
III Solution: $E_x^{III}(0, t_{k+1}) = 0, E_x^{III}(d, t_{k+1}) = 0$

Then the three solutions are linearly combined in order to match boundary conditions (14)-(15).

However, this approach is of limited utility being restricted of course to 1D cases. In general one has to face a much more difficult problem due to the different behaviour of the fields in the vacuum and in the highly conducting material. In fact, in correspondence of an element size Δx, the orders of magnitude of the time steps should be $\sigma\mu\Delta x^2$ and $\Delta x/c$ in the conducting region (where the displacement current can be neglected since $f\varepsilon_0 << \sigma$) and in the vacuum, respectively. This means that the time steps required in the vacuum region are several orders of magnitude smaller than in the material [1] unless very different element sizes are used. Thus, the solution of more realistic 2D and 3D problems might become rather inefficient.

Even if the problem is one-dimensional, the case has been treated as a three-dimensional one, using a mesh consisting of a stack of hexahedral elements along z (the direction of propagation) and imposing suitable symmetry conditions ($\mathbf{E}\times\mathbf{n}=0$ hence $\mathbf{A}\times\mathbf{n}=0$ at the two x=constant symmetry planes; $\mathbf{H}\times\mathbf{n}=0$ hence $\mathbf{F}\times\mathbf{n}=0$ at the two y=constant symmetry planes) and zero initial conditions. We have chosen element size $\Delta x=6.3\mu m$ and time step $\Delta t=1\mu s$.

In Fig. 2, we show the electric field at z=d as a function of the time. The figure also shows the difference between the two estimates ($-\partial A/\partial t$ and $\sigma^{-1}\nabla\times\partial F/\partial t$) of the electric field. This quantity is a useful estimate of the local error. In fact, the difference between the results presented here and those of [1] are less than 1 mV/m.

Fig. 3 shows the two estimates of $E_x(0,t)$ and their difference as functions of the time.

Fig. 4 shows the estimates of E_x and B_y as functions of z at five different time instants. It is worth noticing how the estimates $\mathbf{E}=-\partial\mathbf{A}/\partial t$ and $\mathbf{H}=\partial\mathbf{F}/\partial t$ hence $\mathbf{B}=f_B(\partial\mathbf{F}/\partial t)$ are discontinuous with respect to the time but continuous along z. On the other hand, the two estimates $\mathbf{E}=\sigma^{-1}\nabla\times\partial\mathbf{F}/\partial t$ and $\mathbf{B}=\nabla\times\mathbf{A}$ are piecewise constant along z.

Fig. 5 shows the transmitted field in the case of a larger amplitude (E_{imax}=100kV/m) of the incident field. It can be noticed that the transmitted field now exceeds $3\cdot10^{-6}\,E_{imax}$ even if the electric field at z=0 is now less than $1\cdot10^{-5}\,E_{imax}$ (see Fig. 6). This deterioration of the shielding action, already pointed out in [1], is clearly due to the saturation of the magnetic material that results in an enlargement of the skin depth. The saturation of the magnetic material is evident in Fig. 7 where the magnetic field at z=0 is plotted as a function of the time for the two different amplitudes of the incident field.

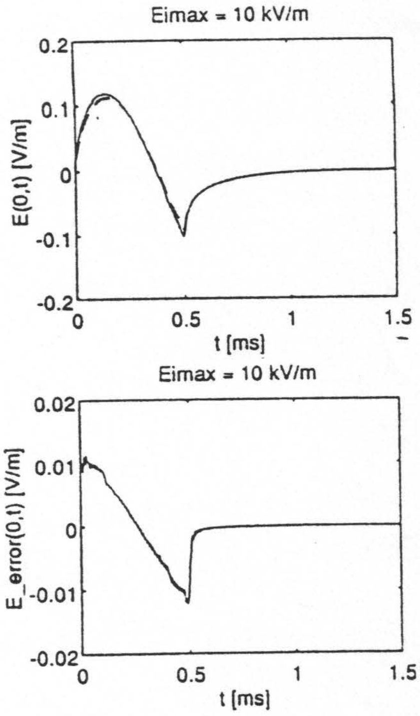

Fig. 3 – Nonlinear slab. The two estimates $-\partial \mathbf{A}/\partial t$ (solid) and $\sigma^{-1}\nabla\times\partial \mathbf{F}/\partial t$ (dashed) of the electric field and their difference, evaluated at z=0.

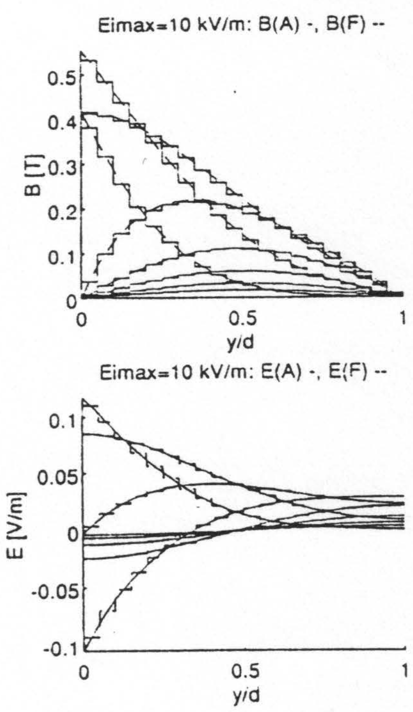

Fig. 4 – Nonlinear slab. Estimates of electric field and magnetic fields at five different time instants.

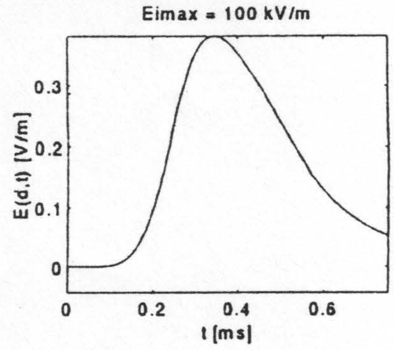

Fig. 5 – Nonlinear slab. Transmitted field, evaluated for $E_{imax} =$ 100 kV/m.

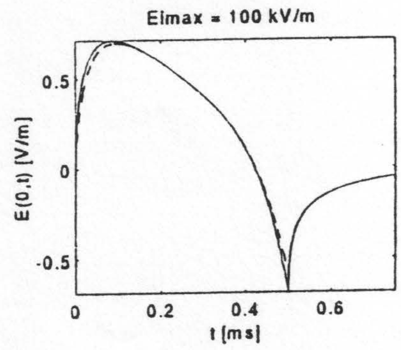

Fig. 6 – Nonlinear slab. Electric field at z=0, evaluated for $E_{imax} =$ 100 kV/m.

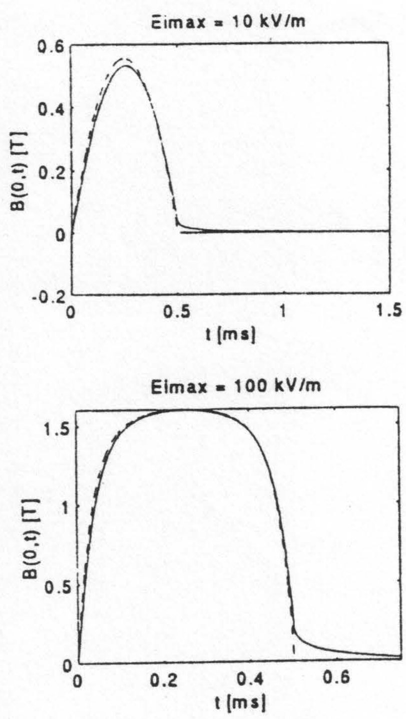

Fig. 7 – Nonlinear slab. Magnetic field at z=0, evaluated for two different amplitudes of the incident field.

IV. AN ITERATIVE TECHNIQUE

The problem can efficiently be solved by using an iterative technique, as described hereafter, valid in the limit $\sigma d \zeta_o >> 1$, being d the characteristic dimension of the shield (i.e. in the limit of an almost complete reflection of the incident field). The space is subdivided into two regions: region 1 is the vacuum region outside the shield, while region 2 includes the shield and the vacuum bounded by the shield (Fig. 8). The technique goes through the following steps:

1) set **E**×**n**=**0** at the interface S between region 1 and region 2;
2) solve the wave propagation in region 1 with **E**×**n** assigned on S, obtaining **H**×**n** on S.
3) solve the field diffusion (magnetoquasistatic limit) in region 2 with **H**×**n** assigned on S, obtaining **E**×**n** on S
4) repeat steps 2 and 3 until convergence is achieved.

It is worth noticing that the iteration can be applied also in the presence of nonlinear materials. It should also be noted that inside the conducting nonlinear material the displacement current can be neglected and in the vacuum inside the screen the wavelength is usually several order of magnitude larger than the size of the screen, so that the magnetoquasistatic model is adequate.

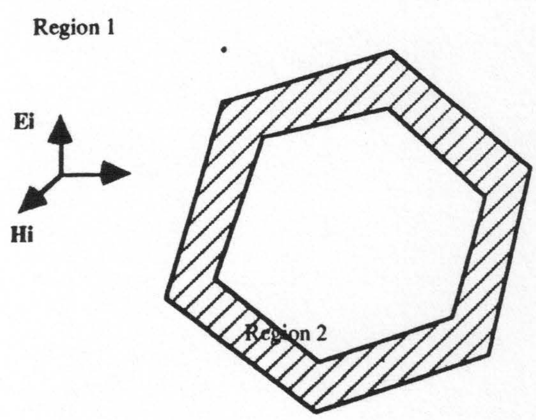

Fig. 8 – The reference geometry.

As an example, the procedure is applied to the solution of a linear steady state problem, where a sinusoidal incident field of amplitude E_{imax}= 10 kV/m and frequency f=1kHz is shielded by a conducting slab ($\sigma=10^7$ S/m, $\mu=10^4\mu_o$, $d=1.26\cdot10^{-4}$). The above procedure is used. In this case the solution of the wave propagation problem in region 1 is given by (14)-(15) whereas the solution of the diffusion in the conducting region is obtained analytically. The results of the iterative procedure, in SI units, are reported in Table II.

Table II – Linear slab: results of the iterative method.

iteration		Real part	Imaginary part
1			
	E(0) =	0	0
	E(d) =	0	0
	H(0) =	-53.08837459	0
	H(d) =	0	0
2			
	E(0) =	0.1045283420	0.1072401147
	E(d) =	-0.0034339276	-0.0242086716
	H(0) =	-53.088097127	0.0002846602
	H(d) =	0.0000091151	0.0000642600
3			
	E(0) =	0.1045283994	0.1072389854
	E(d) =	-0.0034341513	-0.0242083817
	H(0) =	-53.088097127	0.0002846572
	H(d) =	0.0000091157	0.0000642592
4			
	E(0) =	0.1045283994	0.1072389854
	E(d) =	-0.0034341513	-0.0242083817
	H(0) =	-53.088097127	0.0002846572
	H(d) =	0.0000091157	0.0000642592

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