

PCRLB-based Waveforms Design for Target Tracking in Multistatic Radar Systems

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Abstract—This paper investigates waveform synthesis for target tracking in a multistatic radar operating in colored Gaussian interference. The waveform design at each tracking frame is formulated as the minimization of the trace of the Posterior Cramér Rao Lower Bound (PCRLB), subject to power budget requirements and a similarity constraints. To address this challenging optimization problem, an efficient approximation solution technique is proposed to enhance tracking accuracy over reference codes. Specifically, the original problem is approximated via Taylor series expansion, yielding a tractable problem that are efficiently solved using a customized block Majorization–Minimization (block-MM) algorithm. Simulation results demonstrate improved target state estimation accuracy and robust tracking performance under target state uncertainty.

Index Terms—Code design, majorization-minimization, multistatic radar, posterior Cramér Rao lower bound, target tracking.

I. INTRODUCTION

In active sensing systems, the transmitted waveform plays a significant role in determining both detection performance and measurement accuracy [1]. Recently, considerable efforts have been devoted to waveform selection and design, leveraging the cognitive situational awareness provided by the tracker through the predicted target state and by environmental awareness, to maintain high-quality trajectory reconstruction [2], [3]. Two main research lines have emerged. The former is focused on optimizing detection probability and measurement estimation accuracy [4]–[8]. The latter aims at obtaining an accurate estimate of the target state at the next time step by selecting the optimal waveform from a finite library of pre-configured options or by designing bespoke transmitted signals [9]–[11]. Notably, only a few studies have focused on the problem of waveform synthesis to enhance the accuracy of target state estimation for tracking in distributed radar systems [12], [13].

This paper focuses on synthesizing slow-time coded waveforms for a multistatic radar tracking a point-like target in a colored Gaussian interference environment. Specifically, each radar node is equipped with a uniform linear array, and emits a slow-time coded pulse train of linear chirps. In this context, by capitalizing on the one-step-ahead predicted target state, the

theoretical detection probability and the measurement noise covariance matrix of the time-delay, Doppler shift and arrival angle observations at each radar node are first derived. Then, the trace of the PCRLB (evaluated under the assumption of negligible process noise [14], [15]) is used as the waveform optimization criterion. Moreover, a transmit power budget limitation, along with a similarity constraint that controls relevant waveform characteristics, are imposed on the probing waveforms. To address this challenging non-convex problem, an approximate solution technique is proposed. The approach relies on reformulating the trace of the PCRLB into a more tractable expression to enable the optimization of the radars degrees of freedom. Specifically, a second-order Taylor expansion is employed to achieve this approximation. The resulting problem is then solved using a customized block Majorization–Minimization (block-MM) algorithm with guaranteed convergence. Finally, numerical results are provided to assess the performance of the new waveform resource allocation strategy for multistatic radars in terms of convergence and target state estimation accuracy.

Notation: The following notations have been adopted. $\mathbb{R}^N$, $\mathbb{C}^N$, $\mathbb{C}^{N \times M}$, $\mathbb{S}_{++}^N$, $\mathbb{H}_{++}^N$ are, respectively, the set of N -dimensional vectors of real numbers, complex numbers, $N \times M$ complex matrices, $N \times N$ symmetric positive matrices, and $N \times N$ Hermitian positive matrices. Scalars, vectors, and matrices are denoted by standard lowercase letter a , lower case boldface letter $\mathbf{a}$, and upper case boldface letter $\mathbf{A}$, respectively. $\odot$ and $\otimes$ denote, respectively, the Hadamard product and the Kronecker product. For any complex number x , $\Re\{x\}$, $\Im\{x\}$, and $|x|$ indicate, respectively, its real part, imaginary part, and modulus. Letter j represents the imaginary unit (i.e., $j = \sqrt{-1}$). The symbols $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, $(\cdot)^{-1}$, $\text{Tr}(\cdot)$, $\|\cdot\|$, $\lambda_{\min}(\cdot)$, and $\lambda_{\max}(\cdot)$ denote the transpose, complex conjugate, conjugate transpose, inverse, trace, the Euclidean norm of a vector, the smallest and the largest eigenvalue of a matrix, respectively.

II. SYSTEM MODEL & PROBLEM FORMULATION

Consider a 2D multistatic radar comprising N widely separated radar nodes. Each node has transceiver capabilities and is equipped with a uniform linear array of N_r elements. The location of the n -th radar is denoted as $\mathbf{p}_n = [x^n, y^n]^T \in \mathbb{R}^2$ for $n \in \mathcal{N} = \{1, 2, \dots, N\}$. In the following, it is assumed

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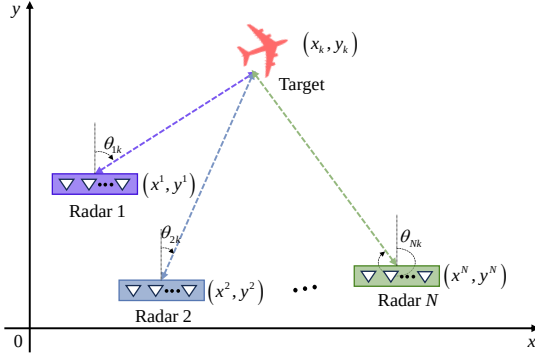


Fig. 1: Spatial geometry relationship between the radar systems and target at frame k .

that the radar nodes transmit frequency orthogonal signals, each tuned to its own transmitted frequency.

The sensing scenario of interest subsumes the multistatic radar tracking a moving point-like target within its surveillance region. Let T denote the update tracking interval between the two contiguous frames. The kinematic target state at time kT is specified by its location $[x_k, y_k]^T \in \mathbb{R}^2$ and velocity $[\dot{x}_k, \dot{y}_k]^T \in \mathbb{R}^2$. A notional illustration of the sensing configuration at the discrete time (frame) k is depicted in Fig. 1. Let $\mathbf{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^T \in \mathbb{R}^4$ be the target state vector at frame k . Suppose that the target motion follows a linear CV model: $\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{u}_k$, where $\mathbf{F}$ is the transition matrix. The term $\mathbf{u}_k$ is the process noise associated with the target motion, which obeys a zero-mean Gaussian distribution [16].

1) *Signal Model*: Suppose that at frame k , each radar node emits a burst of M coherent pulses, modulated by a slow-time code $\mathbf{c}_{nk} = [c_{nk}(0), c_{nk}(1), \dots, c_{nk}(M-1)]^T \in \mathbb{C}^M$. Each pulse is a linear chirp signal $s(t)$ with bandwidth B and pulsewidth T_p . The echo signal backscattered from the moving point-like target and received by the n -th radar node is downconverted and sampled with a temporal step-size Δt . Ignoring the intra-pulse Doppler shift, the space-time observation vector of the n -th radar can be expressed as [17]

$$\mathbf{r}_{nk} = \alpha_{nk} \mathbf{a}_{st,nk} + \mathbf{v}_{nk}, \quad (1)$$

where $\mathbf{a}_{st,nk} = (\mathbf{a}_t(f_{d,nk}) \odot \mathbf{c}_{nk}) \otimes \mathbf{s}(\tau_{nk}) \otimes \mathbf{a}_s(\theta_{nk})$, and α_{nk} is the target's complex amplitude related to the n -th radar node at the k -th frame accounting for both the target and channel propagation effects. Moreover, $\tau_{nk} = 2r(\mathbf{x}_k, \mathbf{p}_n)/c$ is the round-trip time with c being the speed of light and $r(\mathbf{x}_k, \mathbf{p}_n) = \sqrt{(x_k - x^n)^2 + (y_k - y^n)^2}$ being the distance between the target and the n -th radar node at frame k . $f_{d,nk} = 2v_d(\mathbf{x}_k, \mathbf{p}_n)/\lambda_n$ is the Doppler shift of the target on the n -th radar node at frame k , where λ_n is the n -th central operating wavelength and $v_d(\mathbf{x}_k, \mathbf{p}_n) = -[(x_k - x^n)\dot{x}_k + (y_k - y^n)\dot{y}_k]/r(\mathbf{x}_k, \mathbf{p}_n)$ is the radial velocity. $\theta_{nk} = \arctan[(x_k - x^n)/(y_k - y^n)]$ represents the azimuthal angle of the target to the n -th radar node at frame k as specified in Fig. 1. $\mathbf{a}_t(f_d) \in \mathbb{C}^M$ and $\mathbf{a}_s(\theta) \in \mathbb{C}^{N_r}$ denote, respectively, the temporal steering vector and spatial

steering vector. $\mathbf{s}(\tau_{nk}) \in \mathbb{C}^{N_p}$ is the vector collecting the samples of the fast-time pulse $s(t)$ delayed by τ_{nk} , where N_p is the number of samples. $\mathbf{v}_{nk} \in \mathbb{C}^{M N_p N_r}$ denotes the signal-independent interference vector of the n -th radar node at frame k , including noise, intentional and non intentional interference, and hot clutter. It is modeled as a zero-mean complex circular Gaussian vector with positive definite covariance matrix $\mathbb{E}[\mathbf{v}_{nk} \mathbf{v}_{nk}^H] = \boldsymbol{\Sigma}_{v,nk}$, expressed as $\boldsymbol{\Sigma}_{v,nk} = \boldsymbol{\Sigma}_{t,nk} \otimes \mathbf{I}_{N_p} \otimes \boldsymbol{\Sigma}_{s,nk}$, where $\mathbf{I}_{N_p}$ is the covariance matrix of the signal-independent interference in the fast-time domain, whereas, $\boldsymbol{\Sigma}_{t,nk} \in \mathbb{H}_{++}^M$ and $\boldsymbol{\Sigma}_{s,nk} \in \mathbb{H}_{++}^{N_r}$ are the covariance matrices in slow-time and space, respectively.

2) *Detection Model*: Assuming that the interference covariance matrix $\boldsymbol{\Sigma}_{v,nk}$ is known, and α_{nk} is an unknown deterministic parameter with uniformly distributed phase, the Generalized Likelihood Ratio Test (GLRT) detector is equivalent to the optimum test according to the Neyman-Pearson criterion. Hence, the detection probability of the n -th radar node at frame k can be written as

$$P_{d,nk}(\mathbf{c}_{nk}) = Q_1\left(\sqrt{2\text{SINR}(\mathbf{c}_{nk}, \tau_{nk}, f_{d,nk}, \theta_{nk})}, \sqrt{2b_0}\right), \quad (2)$$

where $Q_1(\cdot, \cdot)$ is the Marcum Q function of order 1, $b_0 = -\ln P_{fa}$, and the output $\text{SINR}(\mathbf{c}_{nk}, \tau_{nk}, f_{d,nk}, \theta_{nk})$ is

$$\text{SINR}(\mathbf{c}_{nk}, \tau_{nk}, f_{d,nk}, \theta_{nk}) = \varepsilon_{\alpha,nk} \mathbf{c}_{nk}^H \mathbf{M}_{0,nk} \mathbf{c}_{nk}, \quad (3)$$

where $\mathbf{M}_{0,nk} = \boldsymbol{\Sigma}_{t,nk}^{-1} \odot (\mathbf{a}_t(f_{d,nk}) \mathbf{a}_t^H(f_{d,nk})) \in \mathbb{H}^M$, and $\varepsilon_{\alpha,nk} = N_p |\alpha_{nk}|^2 \mathbf{a}_s^H(\theta_{nk}) \boldsymbol{\Sigma}_{s,nk}^{-1} \mathbf{a}_s(\theta_{nk})$.

3) *Measurement Model*: Assuming that the false alarm probability $P_{fa} \approx 0$, the measurement model for range (time-delay), velocity (Doppler shift) and arrival angle is

$$\mathbf{z}_{nk} = \mathbf{h}_n(\mathbf{x}_k) + \mathbf{n}_{nk}, \quad (4)$$

where $\mathbf{h}_n(\mathbf{x}_k) = [r(\mathbf{x}_k, \mathbf{p}_n), v_d(\mathbf{x}_k, \mathbf{p}_n), \theta(\mathbf{x}_k, \mathbf{p}_n)]^T$ denotes the ground-truth measurement of the n -th radar node at frame k , and $\mathbf{n}_{nk}$ is the measurement noise vector with covariance matrix $\mathbf{R}_{nk}(\mathbf{c}_{nk}) = \mathbf{T}_n \mathbf{CRLB}_{nk}(\mathbf{c}_{nk}) \mathbf{T}_n^T$, where $\mathbf{T}_n = \text{Diag}([c/2, \lambda_n/2, 1]^T)$ is a matrix that allows to convert time-delay and Doppler shift to physical distance and velocity. $\mathbf{CRLB}_{nk}(\mathbf{c}_{nk})$ is the CRLB matrix for the estimation of τ_{nk} , $f_{d,nk}$, and θ_{nk} at radar node n in frame k , which is a diagonal matrix whose diagonal elements are

$$[\mathbf{CRLB}_{nk}(\mathbf{c}_{nk})]_{11} = \frac{1}{\varepsilon_{\tau,nk}} \frac{1}{\mathbf{c}_{nk}^H \mathbf{M}_{0,nk} \mathbf{c}_{nk}}, \quad (5a)$$

$$[\mathbf{CRLB}_{nk}(\mathbf{c}_{nk})]_{22} = \frac{1}{\varepsilon_{f_d,nk}} \frac{\mathbf{c}_{nk}^H \mathbf{M}_{0,nk} \mathbf{c}_{nk}}{\phi(\mathbf{c}_{nk})}, \quad (5b)$$

$$[\mathbf{CRLB}_{nk}(\mathbf{c}_{nk})]_{33} = \frac{1}{\varepsilon_{\theta,nk}} \frac{1}{\mathbf{c}_{nk}^H \mathbf{M}_{0,nk} \mathbf{c}_{nk}}, \quad (5c)$$

where $\phi(\mathbf{c}_{nk}) = \mathbf{c}_{nk}^H \mathbf{M}_{0,nk} \mathbf{c}_{nk} \mathbf{c}_{nk}^H \mathbf{M}_{2,nk} \mathbf{c}_{nk} - |\mathbf{c}_{nk}^H \mathbf{M}_{1,nk} \mathbf{c}_{nk}|^2$ with $\mathbf{M}_{1,nk} = \text{Diag}(\mathbf{b}_t^*) \mathbf{M}_{0,nk} \in \mathbb{C}^{M \times M}$, $\mathbf{M}_{2,nk} = \mathbf{M}_{0,nk} \odot (\mathbf{b}_t \mathbf{b}_t^H) \in \mathbb{H}^M$, $\mathbf{b}_t = [0, j2\pi T_r, \dots, j2\pi T_r(M-1)]^T \in \mathbb{C}^M$, and

$$\varepsilon_{\tau,nk} = \frac{2N_p |\alpha_{nk}|^2 \pi^2 B^2}{3} \mathbf{a}_s^H(\theta_{nk}) \boldsymbol{\Sigma}_{s,nk}^{-1} \mathbf{a}_s(\theta_{nk}), \quad (6a)$$

$$\varepsilon_{f,nk} = 2N_p |\alpha_{nk}|^2 \mathbf{a}_s^H(\theta_{nk}) \boldsymbol{\Sigma}_{s,nk}^{-1} \mathbf{a}_s(\theta_{nk}), \quad (6b)$$

$$\varepsilon_{\theta,nk} = 2N_p |\alpha_{nk}|^2 \left((\mathbf{a}_s(\theta_{nk}) \odot \mathbf{b}_s(\theta_{nk}))^H \boldsymbol{\Sigma}_{s,nk}^{-1} (\mathbf{a}_s(\theta_{nk}) \odot \mathbf{b}_s(\theta_{nk})) - \frac{|(\mathbf{a}_s(\theta_{nk}) \odot \mathbf{b}_s(\theta_{nk}))^H \boldsymbol{\Sigma}_{s,nk}^{-1} \mathbf{a}_s(\theta_{nk})|^2}{\mathbf{a}_s^H(\theta_{nk}) \boldsymbol{\Sigma}_{s,nk}^{-1} \mathbf{a}_s(\theta_{nk})} \right). \quad (6c)$$

4) *Target State Estimation Problem Formulation:* Suppose the target state $\mathbf{x}_k$ is estimated by an unbiased estimator $\hat{\mathbf{x}}_k$. The PCRLB for its covariance is defined as the inverse of the Information Matrix (IM) [14], i.e.,

$$\mathbb{E}[(\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^H] \geq \mathbf{J}_k^{-1}, \quad (7)$$

where $\mathbf{J}_k$ is the IM at frame k . Assuming that the target's process noise is sufficiently weak to be neglected, the IM at frame k can be calculated via a Riccati-like recursion [15] as

$$\begin{aligned} \mathbf{J}_k(\mathbf{c}_{1k}, \mathbf{c}_{2k}, \dots, \mathbf{c}_{Nk}) \\ = (\mathbf{F}^{-1})^T \mathbf{J}_{k-1}(\mathbf{c}_{1(k-1)}, \mathbf{c}_{2(k-1)}, \dots, \mathbf{c}_{N(k-1)}) \mathbf{F}^{-1} \\ + \sum_{n \in \mathcal{N}} P_{d,nk}(\mathbf{c}_{nk}) \mathbf{H}_{nk}^T \mathbf{R}_{nk}^{-1}(\mathbf{c}_{nk}) \mathbf{H}_{nk}, \end{aligned} \quad (8)$$

where $\mathbf{J}_{k-1}$ is the IM at frame $k-1$, and $\mathbf{H}_{nk}$ is the Jacobian of $\mathbf{h}_n(\mathbf{x}_k)$. Without loss of generality, $\mathbf{J}_0$ is initialized as a diagonal matrix with small positive entries to signify the lack of informative prior knowledge and ensure invertibility. Notice that the IM at frame k depends on the radar codes transmitted at the previous $k-1$ frames as well as the current one. The functional dependence of (8) on $\mathbf{c}_{1\bar{k}}, \mathbf{c}_{2\bar{k}}, \dots, \mathbf{c}_{N\bar{k}}, \bar{k} = 1, \dots, k$, highlights that a tailored design of these sequences can be exploited to enhance the accuracy of the target state estimation. In this context, the trace of the PCRLB is used as the figure of merit to minimize, that is

$$f_k(\mathbf{c}_{1k}, \mathbf{c}_{2k}, \dots, \mathbf{c}_{Nk}) = \text{Tr}(\mathbf{J}_k^{-1}(\mathbf{c}_{1k}, \mathbf{c}_{2k}, \dots, \mathbf{c}_{Nk})). \quad (9)$$

Additionally, to regulate the power budget and maintain some desired characteristics of the probing signal, energy and similarity constraints [18] are forced on the radar sequence: $\|\mathbf{c}\|^2 = 1$, $\Re\{\mathbf{c}_0^H \mathbf{c}_{nk}\} \geq \zeta_1$, where $\mathbf{c}_0$ denotes a reference code with unit energy $\|\mathbf{c}_0\|^2 = 1$, and $\zeta_1 = 1 - \zeta/2 \geq 0$ is a real parameter ruling the degree of similarity.

Based on the above discussion, the radar code design problem at frame k can be formulated as

$$\mathcal{P}_k \begin{cases} \min_{\{\mathbf{c}_{nk}\}} & f_k(\mathbf{c}_{1k}, \mathbf{c}_{2k}, \dots, \mathbf{c}_{Nk}) \\ \text{s.t.} & \|\mathbf{c}_{nk}\|^2 = 1, n \in \mathcal{N} \\ & \Re\{\mathbf{c}_0^H \mathbf{c}_{nk}\} \geq \zeta_1, n \in \mathcal{N} \end{cases}. \quad (10)$$

Note that $\mathcal{P}_k$ is a challenging optimization problem due to its nonlinear and non-convex objective function and non-convex constraints.

III. RADAR NETWORK CODE SYNTHESIS FOR SINGLE TARGET TRACKING

In this section, a solution technique is proposed to handle $\mathcal{P}_k$. The approach involves approximating the objective function with a more tractable expression, thereby simplifying the associated optimization problem. This approximated code design problem is then solved to generate candidate radar waveforms. Only if the resulting PCRLB is smaller than the value ensured by the reference codes, the waveforms on the nodes are updated; otherwise, the reference code is retained.

To construct the approximation, the real-valued formulation of $\mathcal{P}_k$ is considered first. Letting $\tilde{\mathbf{c}}_{nk} = [\Re\{\mathbf{c}_{nk}\}^T, \Im\{\mathbf{c}_{nk}\}^T]^T$ and $\tilde{\mathbf{c}}_0 = [\Re\{\mathbf{c}_0\}^T, \Im\{\mathbf{c}_0\}^T]^T$, the quantities $P_{d,nk}(\mathbf{c}_{nk})$ and the diagonal entries of $\mathbf{R}_{nk}(\mathbf{c}_{nk})$ can be, respectively, expressed as functions of $\tilde{\mathbf{c}}_{nk}$, i.e., $\tilde{P}_{d,nk}(\tilde{\mathbf{c}}_{nk})$ and $\tilde{\mathbf{R}}_{nk}(\tilde{\mathbf{c}}_{nk})$ [19]. Then, defining $\mathbf{S}_{nk}(\tilde{\mathbf{c}}_{nk}) = \tilde{\mathbf{R}}_{nk}(\tilde{\mathbf{c}}_{nk}) / \tilde{P}_{d,nk}(\tilde{\mathbf{c}}_{nk})$, the functional dependency of the matrix $\mathbf{J}_k(\mathbf{c}_{1k}, \mathbf{c}_{2k}, \dots, \mathbf{c}_{Nk})$ on the radar codes can be specified via $\tilde{\mathbf{c}}_{nk}$, namely,

$$\begin{aligned} \tilde{\mathbf{J}}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) \\ = (\mathbf{F}^{-1})^T \tilde{\mathbf{J}}_{k-1} \mathbf{F}^{-1} + \sum_{n \in \mathcal{N}} \mathbf{H}_{nk}^T \mathbf{S}_{nk}^{-1}(\tilde{\mathbf{c}}_{nk}) \mathbf{H}_{nk}, \end{aligned} \quad (11)$$

where $\tilde{\mathbf{J}}_{k-1}$ is used to synthetically denote $\tilde{\mathbf{J}}_{k-1}(\tilde{\mathbf{c}}_{1(k-1)}, \tilde{\mathbf{c}}_{2(k-1)}, \dots, \tilde{\mathbf{c}}_{N(k-1)})$.

Therefore, $\mathcal{P}_k$ can be equivalently expressed as

$$\tilde{\mathcal{P}}_k \begin{cases} \min_{\{\tilde{\mathbf{c}}_{nk}\}} & \tilde{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) \\ \text{s.t.} & \|\tilde{\mathbf{c}}_{nk}\|^2 = 1, n \in \mathcal{N} \\ & \tilde{\mathbf{c}}_0^T \tilde{\mathbf{c}}_{nk} \geq \zeta_1, n \in \mathcal{N} \end{cases}, \quad (12)$$

where the function $\tilde{f}_k$ is $\text{Tr}(\tilde{\mathbf{J}}_k^{-1}(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}))$.

Notably, the diagonal matrix $\mathbf{S}_{nk}(\tilde{\mathbf{c}}_{nk})$ is the only term in the objective function of $\tilde{\mathcal{P}}_k$ that depends on the radar codes $\tilde{\mathbf{c}}_{nk}$. The positivity of its diagonal elements, coupled with their dependence on the radar codes, introduces nonlinearities that complicate the optimization. To circumvent this drawback, these elements are approximated a Taylor series expansion, yielding the resulting approximation $\tilde{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk})$ and the approximated objective function $\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$. Since the ideal cost function is a sum of strictly positive terms, the approximation must preserve positivity, which may require restricting the feasible set. Accordingly, a modified approximated objective function with a correction term is introduced as

$$\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) + \mathbb{I}_+(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}), \quad (13)$$

where

$$\mathbb{I}_+(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) = \begin{cases} 0, & \min_{\substack{n \in \mathcal{N}, \\ l=1,2,3}} [\hat{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk})]_{ll} \geq \varepsilon, \\ +\infty, & \text{otherwise} \end{cases}$$

with ε being a quite small positive constant.

Following these guidelines, the solution technique developed to solve the approximated version of $\tilde{\mathcal{P}}_k$ is reported in Algorithm 1. Inspection of the steps reveals that the

two crucial tasks are the construction of the approximated objective function $\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$ and the solution of the corresponding approximated problem $\hat{\mathcal{P}}_k$. The following subsections focus on these two aspects.

Algorithm 1 The approximate solution technique to solve $\tilde{\mathcal{P}}_k$.

Input: $\tilde{\mathbf{c}}_0, \zeta, \mathbf{F}, \tilde{\mathbf{J}}_{k-1}$, and $\{\mathbf{H}_{nk}\}_{n \in \mathcal{N}}$.

Output: Optimized radar codes $\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}$ and the achieved IM $\tilde{\mathbf{J}}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$.

- 1: Determine an approximation $\hat{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk})$ to $\mathbf{S}_{nk}(\tilde{\mathbf{c}}_{nk})$;
- 2: Construct the corresponding approximated objective function $\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$ replacing $\mathbf{S}_{nk}(\tilde{\mathbf{c}}_{nk})$ in (11) with $\hat{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk})$;
- 3: Solve the following problem to get the candidates $\{\tilde{\mathbf{c}}_{nk}^*\}$,

$$\hat{\mathcal{P}}_k \begin{cases} \min_{\{\tilde{\mathbf{c}}_{nk}\}} & \hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) + \mathbb{I}_+(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) \\ \text{s.t.} & \|\tilde{\mathbf{c}}_{nk}\|^2 = 1, n \in \mathcal{N} \\ & \tilde{\mathbf{c}}_0^\top \tilde{\mathbf{c}}_{nk} \geq \zeta_1, n \in \mathcal{N}; \end{cases}$$

- 4: **if** $\hat{f}_k(\tilde{\mathbf{c}}_{1k}^*, \tilde{\mathbf{c}}_{2k}^*, \dots, \tilde{\mathbf{c}}_{Nk}^*) < \hat{f}_k(\tilde{\mathbf{c}}_0, \tilde{\mathbf{c}}_0, \dots, \tilde{\mathbf{c}}_0)$ **then**
- 5: Let $\tilde{\mathbf{c}}_{nk} = \tilde{\mathbf{c}}_{nk}^*, n \in \mathcal{N}$;
- 6: **else**
- 7: Let $\tilde{\mathbf{c}}_{nk} = \tilde{\mathbf{c}}_0, n \in \mathcal{N}$;
- 8: **end if**
- 9: Compute $\tilde{\mathbf{J}}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$ defined in (11).

1) *Approximation of $\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$:* According to Lemma 1 in [19], the second order Taylor expansion of the l -th diagonal elements of $\mathbf{S}_{nk}(\tilde{\mathbf{c}}_{nk})$ in the neighborhood of the reference code $\tilde{\mathbf{c}}_0$ is given by

$$[\hat{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk})]_{ll} = \tilde{\mathbf{c}}_{nk}^\top \mathbf{A}_{l,nk} \tilde{\mathbf{c}}_{nk} + \mathbf{a}_{l,nk}^\top \tilde{\mathbf{c}}_{nk} + a_{l,nk}, \quad (14)$$

where $\mathbf{A}_{l,nk}$, $\mathbf{a}_{l,nk}$, and $a_{l,nk}$ are specified in [19]

By approximating $\mathbf{S}_{nk}(\tilde{\mathbf{c}}_{nk})$ in $\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$ via the expansion formula in (14), the resulting function $\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})$ can be expressed as

$$\hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) = \text{Tr}(\hat{\mathbf{J}}_k^{-1}(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk})), \quad (15)$$

where $\hat{\mathbf{J}}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) = (\mathbf{F}^{-1})^\top \tilde{\mathbf{J}}_{k-1} \mathbf{F}^{-1} + \sum_{n \in \mathcal{N}} \mathbf{H}_{nk}^\top \hat{\mathbf{S}}_{nk}^{-1}(\tilde{\mathbf{c}}_{nk}) \mathbf{H}_{nk}$.

2) *Solution to Problem $\hat{\mathcal{P}}_k$:* Before proceeding further, let us focus on an equivalent expression of $\hat{\mathcal{P}}_k$, i.e.,

$$\hat{\mathcal{P}}_k \begin{cases} \min_{\{\tilde{\mathbf{c}}_{nk}\}} & \hat{f}_k(\tilde{\mathbf{c}}_{1k}, \tilde{\mathbf{c}}_{2k}, \dots, \tilde{\mathbf{c}}_{Nk}) \\ \text{s.t.} & \|\tilde{\mathbf{c}}_{nk}\|^2 = 1, n \in \mathcal{N} \\ & \tilde{\mathbf{c}}_0^\top \tilde{\mathbf{c}}_{nk} \geq \zeta_1, n \in \mathcal{N} \\ & [\hat{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk})]_{ll} \geq \varepsilon, l = 1, 2, 3, n \in \mathcal{N} \end{cases}. \quad (16)$$

Inspired by the block-MM framework [20], [21], which combines the Block Coordinate Descent (BCD) paradigm with the MM framework, a novel solution technique for Problem (16) is developed. In order to proceed, let $\{\tilde{\mathbf{c}}_{nk}^i\}_{n \in \mathcal{N}}$ be the

optimized codes up to the i -th iteration, and let $\tilde{\mathbf{c}}_{pk}, p \in \mathcal{N}$ be the block to be updated at $(i+1)$ -th iteration. The restriction of the objective function with respect to the p -th block is given by

$$\hat{f}_{pk}(\tilde{\mathbf{c}}_{pk}) = \text{Tr}((\mathbf{B}_{pk}^i + \mathbf{H}_{pk}^\top \hat{\mathbf{S}}_{pk}^{-1}(\tilde{\mathbf{c}}_{pk}) \mathbf{H}_{pk})^{-1}), \quad (17)$$

where

$$\mathbf{B}_{pk}^i = (\mathbf{F}^{-1})^\top \tilde{\mathbf{J}}_{k-1}(\tilde{\mathbf{c}}_{1(k-1)}, \tilde{\mathbf{c}}_{2(k-1)}, \dots, \tilde{\mathbf{c}}_{N(k-1)}) \mathbf{F}^{-1} + \sum_{\substack{n \in \mathcal{N}, \\ n \neq p}} \mathbf{H}_{nk}^\top \hat{\mathbf{S}}_{nk}^{-1}(\tilde{\mathbf{c}}_{nk}^i) \mathbf{H}_{nk}. \quad (18)$$

It is worth noting that $\mathbf{B}_{pk}^i \in \mathbb{S}_{++}^M$, since $\hat{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk}^i) \in \mathbb{S}_{++}^M$ and the positive definite initial IM $\mathbf{J}_0$.

Proposition 1: Let $\mathbf{B} \in \mathbb{S}_{++}^M$, the function $h(\mathbf{R}) = \text{Tr}((\mathbf{B} + \mathbf{H}^\top \mathbf{R}^{-1} \mathbf{H})^{-1})$ for $\mathbf{R} \in \mathbb{S}_{++}^M$ is concave.

Following the principles of the block-MM framework, a surrogate function for $\hat{f}_{pk}(\tilde{\mathbf{c}}_{pk})$ is now derived. At first, $\hat{f}_{pk}(\tilde{\mathbf{c}}_{pk})$ is concave with $\mathbf{S}_{pk}(\tilde{\mathbf{c}}_{pk})$ according to Proposition 1. Based on the first order condition for concave functions, a majorizing function for $\hat{f}_{pk}(\tilde{\mathbf{c}}_{pk})$ at $\tilde{\mathbf{c}}_{pk} = \tilde{\mathbf{c}}_{pk}^i$ is obtained as

$$\hat{f}_{pk}(\tilde{\mathbf{c}}_{pk}) \leq \tilde{\mathbf{c}}_{pk}^\top \mathbf{D}_{pk}^i \tilde{\mathbf{c}}_{pk} + (\mathbf{d}_{pk}^i)^\top \tilde{\mathbf{c}}_{pk} + d_{pk}^i, \quad (19)$$

where $\mathbf{D}_{pk}^i = \sum_{l=1}^3 [\hat{\mathbf{D}}_{pk}^i]_{ll} \mathbf{A}_{l,pk}$, $\mathbf{d}_{pk}^i = \sum_{l=1}^3 [\hat{\mathbf{D}}_{pk}^i]_{ll} \mathbf{a}_{l,pk}$, $d_{pk}^i = \hat{f}_{pk}(\tilde{\mathbf{c}}_{pk}^i) - \text{Tr}(\hat{\mathbf{D}}_{pk}^i \mathbf{R}_{pk}^i) + \sum_{l=1}^3 [\hat{\mathbf{D}}_{pk}^i]_{ll} a_{l,pk}$, and $\hat{\mathbf{D}}_{pk}^i = (\mathbf{R}_{pk}^i)^{-1} \mathbf{H}_{pk} (\mathbf{B}_{pk}^i + \mathbf{H}_{pk}^\top (\mathbf{R}_{pk}^i)^{-1} \mathbf{H}_{pk})^{-2} \mathbf{H}_{pk}^\top (\mathbf{R}_{pk}^i)^{-1}$.

Next, exploiting $\|\tilde{\mathbf{c}}_{pk}\|^2 = 1$, the objective function can be further tightly upperbounded as [22]

$$\hat{f}_{pk}(\tilde{\mathbf{c}}_{pk}) \leq (\mathbf{g}_{pk}^i)^\top \tilde{\mathbf{c}}_{pk} + g_{pk}^i, \quad (20)$$

where $\mathbf{g}_{pk}^i = 2(\mathbf{D}_{pk}^i - \lambda_{\max}(\mathbf{D}_{pk}^i) \mathbf{I}_{2M}) \tilde{\mathbf{c}}_{pk}^i + \mathbf{d}_{pk}^i$ and $g_{pk}^i = 2\lambda_{\max}(\mathbf{D}_{pk}^i) - (\tilde{\mathbf{c}}_{pk}^i)^\top \mathbf{D}_{pk}^i \tilde{\mathbf{c}}_{pk}^i + d_{pk,t}$.

$\tilde{\mathcal{P}}_k$ remains non-convex and NP-hard in general due to the energy requirement and the last non-convex quadratic constraint. Using $\|\tilde{\mathbf{c}}_{pk}\|^2 = 1$, the last quadratic constraints in $\tilde{\mathcal{P}}_k$ can be approximated by

$$(\mathbf{h}_{l,pk}^i)^\top \tilde{\mathbf{c}}_{pk} + h_{l,pk}^i \geq \varepsilon, l = 1, 2, 3, \quad (21)$$

where $\mathbf{h}_{l,pk}^i = 2(\mathbf{A}_{l,pk} - \lambda_{\min}(\mathbf{A}_{l,pk}) \mathbf{I}_{2M}) \tilde{\mathbf{c}}_{pk}^i + \mathbf{a}_{l,pk}$ and $h_{l,pk}^i = 2\lambda_{\min}(\mathbf{A}_{l,pk}) - (\tilde{\mathbf{c}}_{pk}^i)^\top \mathbf{A}_{l,pk} \tilde{\mathbf{c}}_{pk}^i + a_{l,pk}$.

Therefore, $\tilde{\mathbf{c}}_{pk}^{i+1}$ can be obtained by solving the following problem

$$\begin{cases} \min_{\tilde{\mathbf{c}}_{pk}} & (\mathbf{g}_{pk}^i)^\top \tilde{\mathbf{c}}_{pk} + g_{pk}^i \\ \text{s.t.} & \|\tilde{\mathbf{c}}_{pk}\|^2 = 1 \\ & \tilde{\mathbf{c}}_0^\top \tilde{\mathbf{c}}_{pk} \geq \zeta_1 \\ & (\mathbf{h}_{l,pk}^i)^\top \tilde{\mathbf{c}}_{pk} + h_{l,pk}^i \geq \varepsilon, l = 1, 2, 3 \end{cases}. \quad (22)$$

Problem (22) is feasible and can be recast into a convex form by equivalently rewriting the unit energy constraint as a unit sphere constraint, which enables efficient solution via standard convex solvers such as CVX [23].

IV. NUMERICAL RESULTS

Numerical experiments are conducted to evaluate the performance of the proposed method for multistatic radar code design. Consider a multistatic radar with $N = 4$ nodes, operating at X band (the carrier frequency is 10 GHz), using frequency orthogonal waveforms. The antenna of each node consists of a uniform linear array of $N_r = 8$ elements spaced at half-wavelength intervals. Their transmitted trains utilize $M = 8$ pulses with a PRI of $T_r = 250 \mu\text{s}$. The chirp signal has a bandwidth of $B = 5 \text{ MHz}$ and a pulsewidth of $T_p = 10 \mu\text{s}$. The sampling time is $\Delta t = 1/2B$ leading to $N_p = 100$. The observation sampling interval between consecutive tracking frames is set to $T = 1 \text{ s}$.

Consider a single point-like target moving in the radar system surveillance area, as depicted in Fig. 1. The four radar nodes are located at (20, 10) km, (25, 16) km, (35, 16) km, and (40, 10) km, respectively. It is assumed that the antenna beams of the radar nodes consistently point towards the target. The initial location and velocity of the target are (30, 55) km and (80, 240) m/s, respectively. The target power $|\alpha_{nk}|^2$ is assumed to remain constant across all frames, and are set as $|\alpha_{1k}|^2 = 0.035$, $|\alpha_{2k}|^2 = 0.099$, $|\alpha_{3k}|^2 = 0.176$, $|\alpha_{4k}|^2 = 0.051$. The signal-independent interference covariance matrix [17], [24] in the slow-time domain $\Sigma_{t,nk}$, and spatial domain $\Sigma_{s,nk}$ are modeled as $\Sigma_{t,nk}(i, j) = \rho_t^{|i-j|}$, $\Sigma_{s,nk}(i, j) = \rho_s^{|i-j|}$, $n \in \mathcal{N}$, where the one-lag correlation coefficients are $\rho_t = 0.8$ and $\rho_s = 0.8$, respectively. The similarity sequence is considered as a P3 code [25]. The initial IM $\mathbf{J}_0$ is initialized as a diagonal matrix whose diagonal entries are all 10^{-10} . The regularization parameter for the approximation $\hat{\mathbf{S}}_{nk}(\tilde{\mathbf{c}}_{nk})$ is set to $\varepsilon = 10^{-8}$. The stopping parameter of the developed block-MM procedure is fixed to $\xi = 10^{-3}$.

In Fig. 2(a), the approximated objective $\hat{f}_k$ in (15) versus iterations is plotted for $\zeta = \{0.01, 0.05, 0.1, 0.15\}$. Inspection of the curves highlight that the approximated objective value monotonically decreases along the iterations. Besides, the larger the similarity parameter ζ the lower the achieved objective values due to the enlarged feasible set of $\hat{\mathcal{P}}_1$. In Fig. 2(b), the PCRLB trace $\hat{f}_k$ is shown versus the frame time for the designed sequences with distinct values of ζ , respectively. For comparison, Fig. 2(b) also reports the PCRLB achieved using the reference code throughout all frames. Inspection of the curves highlights that $\hat{f}_k$ decreases monotonically over the frames, and the designed sequences consistently outperform the reference code. In particular, at the 30-th frame, the codes obtained under $\zeta = 0.15$ realize a remarkable PCRLB gain of 3.5 dB compared to the reference.

The developed design procedure relies on the predicted target state at the next frame. The behavior of the procedure in the presence of a mismatch between the predicted (nominal) and actual target states is examined. The predicted target state $\hat{\mathbf{x}}_k$ is assumed to follow a multivariate distribution with mean vector $\mathbf{x}_k$ and covariance matrix $\Sigma_k = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \otimes \mathbf{I}_2$, where σ_r^2 and σ_v^2 represent covariances for target position and

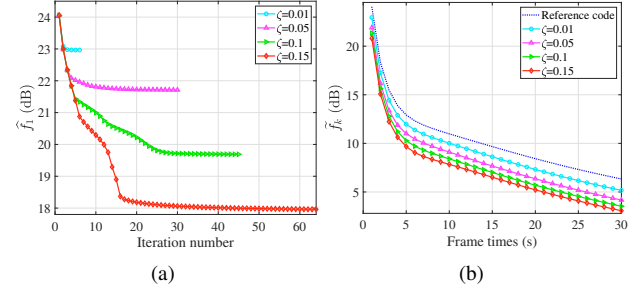


Fig. 2: (a) The approximated PCRLB performances at the first frame versus iteration number. (b) The PCRLB performances versus frame time achieved by the designed codes and the reference sequence.

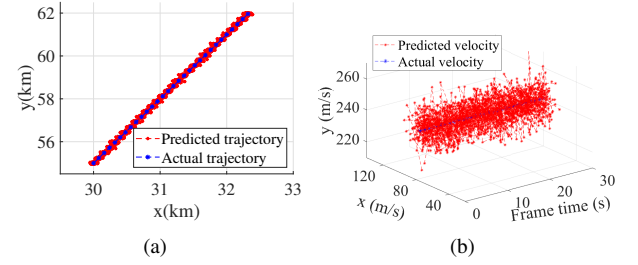


Fig. 3: Predicted target state and actual state over 50 trials: (a) trajectory and (b) velocity.

velocity, respectively. The radar codes designed based on the predicted target state are used to evaluate the mismatch loss when applied to the actual target state. The average PCRLB is adopted as performance metric, and is computed over 50 Monte Carlo trials on the predicted target state. For these tests, the values of σ_r^2 and σ_v^2 are set to the squares of the range and velocity resolutions, i.e., 900 m^2 and $56.25 \text{ m}^2/\text{s}^2$, respectively. The predicted and actual target trajectories and velocities across the 50 trials are shown in Fig. 3(a) and Fig. 3(b).

Fig. 4 depicts the average PCRLB achieved by the devised code with $\zeta = 0.15$ obtained using the predicted state as

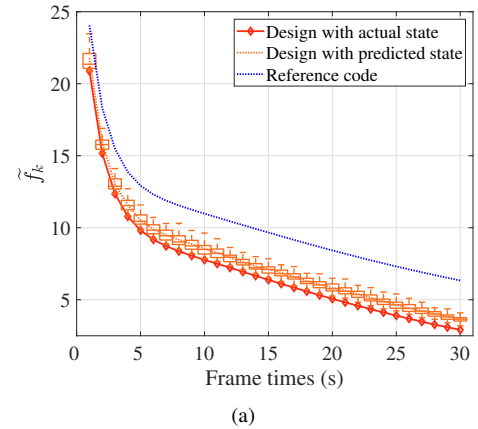


Fig. 4: Average PCRLB with inaccurate predicted target state.

well as the actual state. Moreover, the average PCRLB for the reference code, associated with the actual target state, is also plotted as a term of comparison. The box for each frame represents the range of the trace PCRLB trace values obtained across the conducted trials, while the curves illustrate the corresponding average values. Under the given uncertainty in the target state, the average PCRLB performance is close to that of the error-free condition exhibiting only minor fluctuations. The curves also emphasize that the devised code significantly outperforms the P3 sequence in terms of target state estimation accuracy.

V. CONCLUSION

A novel approach for synthesizing slow-time codes for single target tracking in multistatic radars operating under colored Gaussian interference has been presented. The optimization framework was based on minimizing the trace of the PCRLB for target state estimation while adhering to power constraints and waveform similarity requirements. By approximating the objective function through a second order Taylor expansion and employing a bespoke block-MM algorithm, an efficient approximation solution technique with guaranteed convergence has been developed. The optimized waveforms monotonically reduce the PCRLB trace over frames, and improve target state estimation accuracy compared to reference sequences. Furthermore, the design demonstrates robustness to moderate mismatches in the assumed target state, thus confirming its practical effectiveness.

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