land

MDPI

Article

Envisaging the Intrinsic Departure from Zipf's Law as an Indicator of Economic Concentration along Urban–Rural Gradients

Adele Sateriano [1], Giovanni Quaranta [2], Rosanna Salvia [2], Francisco Escrivà Saneugenio [3], Alvaro Marucci [4], Luca Salvati [5,*], Barbara Zagaglia [6] and Francesco Chelli [6,*]

[1] Independent Researcher, 00184 Rome, Italy; adele.sateriano.pul@gmail.com
[2] Department of Mathematics, Computer Science and Economics, University of Basilicata, 85100 Potenza, Italy; rosanna.salvia@unibas.it (R.S.)
[3] Department of Geography, University of Valencia, Av. Blasco Ibanez, 46010 Valencia, Spain; francisco.escriva@uv.es
[4] Department of Agriculture and Forest Sciences (DAFNE), University of Tuscia, Via S. Camillo de Lellis snc, 01100 Viterbo, Italy; marucci@unitus.it
[5] Department of Methods and Models for Economics, Territory and Finance (MEMOTEF), Faculty of Economics, Sapienza University of Rome, Via del Castro Laurenziano 9, 00161 Rome, Italy
[6] Department of Social and Economic Sciences, Polytechnic University of Marche, Piazzale Martelli 8, 60121 Ancona, Italy; b.zagaglia@univpm.it
* Correspondence: luca.salvati@uniroma1.it (L.S.); f.chelli@univpm.it (F.C.); Tel.: +39-06-49766418 (L.S.)

Abstract: A rank-size rule following Zipf's law was tested along a complete urban–rural hierarchy in Greece using 2021 census data released at different administrative levels. Testing five econometric specifications (linear, quadratic, and cubic forms, together with refined logistic and Gompertz forms) on log-transformed population numbers, deviations from the rank-size rule were assumed as an indicator of economic concentration (considering settlements, population, and activities jointly) along the density gradient in Greece. This hypothesis was verified using progressively disaggregated population numbers at (i) regional units (n = 75), (ii) 'Kallikratis' municipalities (n = 333), (iii) 'Kapodistrian' municipalities (n = 1037), and (iv) local communities (n = 6126). Econometric results were stable across geographical levels and indicate a relatively poor fit of linear specifications, the classical formulation of Zipf's law. Quadratic specifications displayed a good fit for all territorial levels outperforming cubic specifications. Gompertz specifications outperformed logistic specifications under aggregate partitions (e.g., regional units and 'Kallikratis' municipalities). Quadratic specifications outperformed both logistic and Gompertz specifications under disaggregated levels of investigation ('Kapodistrian' municipalities and local communities). Altogether, these findings indicate the persistence of non-linear rank-size relationships estimated over a cross-section of population data at progressively detailed observational units. Such evidence enriches the recent literature on Zipf's law, demonstrating the inherent complexity of rank-size rules tested on real data along the whole density gradient in a given country.

Keywords: rank-size rule; settlement structure; regional economics; indicators; Mediterranean

check for updates

Citation: Sateriano, A.; Quaranta, G.; Salvia, R.; Saneugenio, F.E.; Marucci, A.; Salvati, L.; Zagaglia, B.; Chelli, F. Envisaging the Intrinsic Departure from Zipf's Law as an Indicator of Economic Concentration along Urban–Rural Gradients. *Land* **2024**, *13*, 415. https://doi.org/10.3390/land13040415

Academic Editor: Dagmar Haase

Received: 2 March 2024
Revised: 21 March 2024
Accepted: 22 March 2024
Published: 24 March 2024

1. Introduction

Understanding the intimate pattern of population distribution and accounting for the intrinsic socioeconomic dynamics of urban–rural hierarchies, are key arguments of applied economics, regional science, spatial planning, and political geography [1–3]. By considering population, settlements, and activities, both separately and together, comparative analyses of urban–rural hierarchies that use population (or economic) data at different aggregation levels were rather infrequent in both advanced economies and emerging countries [4–6]. The need for coherent, high-quality statistics limits the practical applicability of theoretical exercises referring to the rank-size relationship and, more specifically, to the empirical verification of Zipf's law for specific variables, basically population or, less frequently, settlements and activities [7–9]. While maintaining full comparability over time and space, results of official surveys, such as general censuses, seem to be particularly appropriate for this type of analysis [10–12].

Until now, urban–rural hierarchies were mainly investigated using key variables such as population, regarded as the basic information for assessing the structure and dynamics of demographic systems, settlement morphology, and activity dynamics [13]. Broadly speaking, population is an economically relevant variable—being frequently regarded as the driver of almost all social processes—and it is commonly used as the denominator of economically relevant variables or indicators, including per-capita income—a basic measure of the affluence of a given place, community, or geographical entity [14]. Demographic growth rates are also intended as economic indicators since they assess trends over time in the short-term (or long-term) evolution of any productive system [15–17].

With this perspective in mind, total population is considered a basic factor driving regional development in both urban and rural locations, being a powerful engine of economic growth and metropolitan transformations through urbanization channels, especially—but not exclusively—in advanced economies [18]. Stability or change in population hierarchies over small (or relatively small) districts is especially interesting in Europe [19], a continent with 'sticky people and implicit boundaries' [20]. Adopting total population as a key variable when exploring the process underlying consolidation (or modification) of urban–rural gradients across regions and countries thus remains a relevant task in the economic analysis [21]. The rank-size relationship was extensively tested for this variable, providing evidence of the inverse proportionality between the rank of settlements and the related population [22].

The rank-size relationship between settlements and cities was investigated in the present study, assuming the predictions of Zipf's law as the basis for the interpretation and discussion of urban, metropolitan, and regional development processes [23]. The rank-size rule was also assumed as a source of demographic heterogeneity and economic complexity [24–26]. Based on stylized facts repeatedly observed in real cases [27–29], Zipf's law postulates the existence of a significant relationship (fitted with a straight line having a slope around -1) between the logarithm of city rank against the logarithm of its population [30–32]. In other words, the frequency of a given event that is part of a set was modeled as a function of the position (rank) in the decreasing ordering with respect to the frequency of that event [33]. However, the existence of (more or less intense) deviations from a purely linear rank-size relationship suggests the appropriateness of the combined use of mixed (e.g., linear and non-linear) specifications with the aim of better capturing the inherent complexity in the underlying economic processes over time and across space [34], as well as cross-sectional heterogeneities and systematic errors associated with official data.

Another aspect of the empirical verification of population rank-size relationships applied to urban–rural systems are the primary focus on cities, towns, and high-density settlements [35]. As a matter of fact, the rank-size relationship for cities is one of the most explored applications of Zipf's law in real-world systems [36]. It predicts, with sufficient precision, the distribution of total population along a gradient of city sizes irrespective of the geographical coverage, being positively verified for relatively small regions and countries [37], while also maintaining significance at broader scales, namely continental or global [38]. Based on these premises and considering the broad literature on Zipf's law and the related criticisms [39–41], the present study proposes an empirical verification of the rank-size relationship of total population along a complete urban–rural hierarchy in Greece, a small and peripheral European country with intense socioeconomic divides and a marked primacy of large cities over the rest of the settlements. As a contribution to the empirical study of rank-size rules applied to cities [42], the exercise proposed here was aimed at addressing three issues less investigated in the current literature.

First, we focused on a complete population gradient (from the largest cities to the smaller villages), not censored at a specific demographic threshold, in line with Calderín-Ojeda [33]. This practical choice means managing data heterogeneity because of the presence of a relatively long tie of (medium-small) settlements, more or less sparse across the country, that contribute to the formation and consolidation of the metropolitan hierarchy in both accessible locations and more peripheral places [43]. In other words, with this choice, we are not exclusively interested in the goodness-of-fit of the rank-size relationship estimated on real data, but also in the inherent departure from the fit along the whole density gradient [44]. In this perspective, we assumed local heterogeneity as a proxy of economic systems with distinctive functioning because of different factors that determine urban growth and social change [45–47].

Second, we tested the implicit role of the data disaggregation level, assuming departures from a traditional Zipf's law depending on the size and number of elementary analysis units [48]. In the current literature, there is no consensus on the use of a unique aggregation level for the empirical testing of rank-size rules for cities [49]. Heterogeneity derives from the partial availability of comparable (official) statistics from censuses and/or sampling surveys [50]. In Europe, the most common variable in official statistics (namely, total population across a given area) was released at various aggregation levels, from regions to prefectures and from municipalities to local communities [51]. Despite a common classification of the various aggregation levels for statistical purposes (namely, the Nomenclature Units of Territorial Statistics, NUTS, adopted by Eurostat, the statistical office of the European Commission), the use of these different data aggregation typologies when testing Zipf's law is still not completely codified and discussed [52]. With this perspective in mind, our study compares the rank-size rule at four different aggregation levels, from prefectures to local communities [53], controlling for sample sizes and the implicit influence of the statistical distribution of the target variable, namely the resident population [54].

Third, we assumed linear and non-linear rank-size relationships as implicit signals of homogeneous and diversified economic systems, respectively, with a given study area, possibly with different functions and underlying drivers [55]. Our hypothesis is that a non-linear rank-size rule reflects the existence of local systems governed by distinctive economic drivers [56]. This implies the existence of at least two systems, namely a strictly urban system responding to scale and agglomeration economies [57] and a less dense system where agglomeration and scale perform less effectively [58]. Therefore, we tested the rank-size rule on the real data described above by comparing the statistical fit of five econometric specifications: (i) linear, (ii) quadratic, and (iii) cubic forms, augmented with (iv) logistic and (v) Gompertz laws [59]. Common diagnostics were implemented to study the fit of such specifications and discuss the eventual diversification of the economic system within the study area [60], together, addressing the global adherence to a rank-size relationship and the local departure from such trends [61].

Regarding the selection of the study area, our empirical work evaluated Greece as a peripheral and marginal European country characterized by (i) important socioeconomic gaps, (ii) an urban primacy of the capital city without equality in the old continent (35% of the Greek population lives in metropolitan Athens), and (iii) important disparities, both of an infrastructural and telecommunication nature, in the accessibility of urban and rural districts [62]. These gaps, typically of an economic nature, have already been reflected in the path of regional and local development for many decades, consolidating the so-called 'leading' areas (urban, agricultural, and tourism-specialized) and depressing internal and remote areas, the so-called 'laggers' [63]. Such a model, consolidated over time, appears difficult to reverse with top-down policies and requires targeted approaches addressing the problems of urban and rural areas, which are often totally divergent [64]. From this perspective, a more precise classification of urban and rural territories based on the specific functioning of local and regional economic systems, instead of simple official statistical indicators (e.g., population density, per capita income, and unemployment rate), proved to be an operational tool informing any development policy [65]. Analyzing a metropolitan hierarchy through the assumption of a rank-size relationship that follows Zipf's law allows us to verify the existence of one, two, or more economic systems that are spatially organized and that function with different economic speeds, social attributes, and demographic dynamics [66]. Our hypothesis is that different coefficient slopes of Zipf's law identify local systems characterized by different economic drivers, e.g., scale/agglomeration vs. accessibility/amenities vs. subsistence [67]. The empirical verification proposed in this work and based on elementary econometric techniques allows us to answer this research question [68].

2. Methodology

2.1. Study Area

We studied the population hierarchy in Greece (nearly 130,000 km^2) taken as a representative example of social and economic structures typical of peripheral and internally centralized countries in South-Eastern Europe [69]. Greek economy displays an evident legacy with tourism, traditional industries (including construction), the dominance of basic services (commerce, public administration, and real estate), and extensive agricultural systems [5]. More than 30% of the Greek population gravitated around Athens, the capital city, showing a consolidated urban primacy since World War I [70].

In the present study, we adopted a selection of elementary (polygonal) analysis domains reflecting progressively more detailed levels of local governance in Greece. These administrative levels correspond with the Nomenclature of Territorial Statistical Units (NUTS) adopted by Eurostat, the European Statistical Office. More precisely, the partitions selected here are (i) NUTS-3 prefectures (n = 75 units), (ii) NUTS-5 'Kallikratis' municipalities (n = 333 units), (iii) LAU-1 (Local Administrative Unit) 'Kapodistrian' municipalities (n = 1037 units), and (iv) LAU-2 communities (n = 6126 units). Maps illustrating the boundaries of these administrative levels are presented in Figure 1. Moreover, these four analysis levels reflect a gradient of governance centralization–decentralization when moving from prefectures to local communities. These units are also assumed to be a suitable domain when investigating the distribution and concentration of population and economic activities in Greece [62] since they represent—better than other economic indicators—territorial gradients and urban–rural divides [18]. More specifically, such units identify (i) urban nodes, (ii) accessible coastal areas and dynamic lowlands attracting tourists and the working population, and (iii) peripheral districts experiencing depopulation and land abandonment.

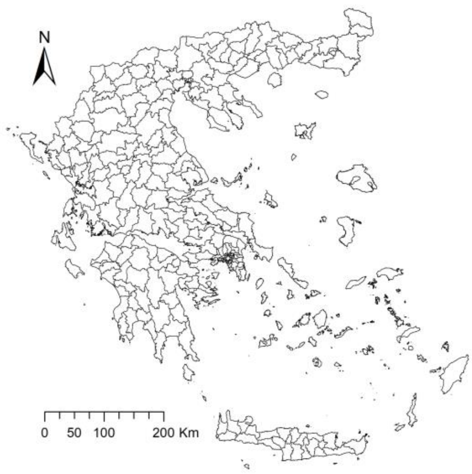

Figure 1. *Cont.*

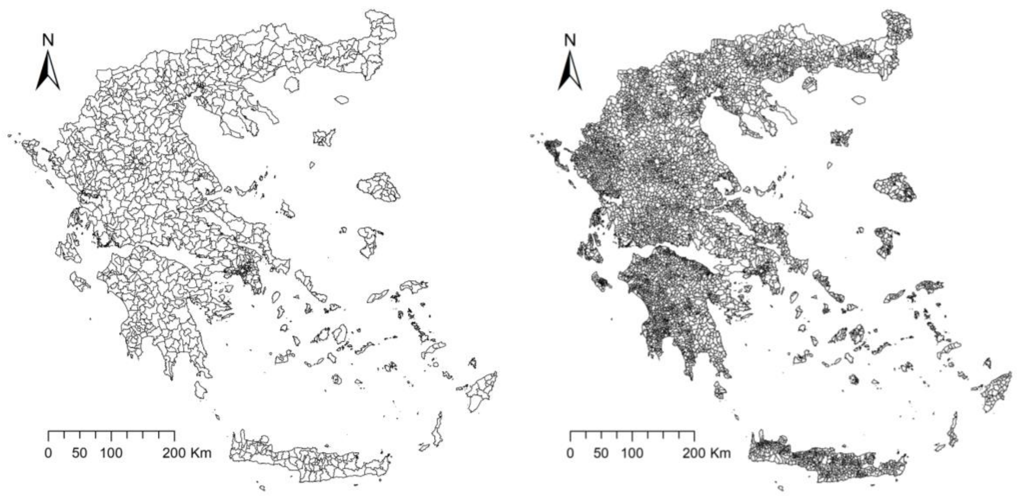

Figure 1. Spatial distribution of the observational units reflecting different population partitions based on administrative boundaries in Greece; upper left: regional units (n = 75); upper right: 'Kallikratis' municipalities (n = 333); lower left: 'Kapodistrian' municipalities (n = 1037); lower right: local communities (n = 6126); maps generated through ArcGIS release 10 (ESRI, Redwoods, CA).

2.2. Data and Variables

We adopted a homogenized collection of population data prepared and released by the Hellenic Statistical Authority (ELSTAT) from the last population census (2021). The log-transformed total population based on the absolute number of inhabitants in a given analysis unit was used as the target variable. Based on population size, the unit's rank was log-transformed as well. The population size was made available in the same survey year at the four administrative partitions of Greece illustrated above. The population data used here are the most updated in the country since administrative registers releasing annual demographic estimates of the total population provided suitable and comparable data only at NUTS-0, 1, 2, and 3 levels. This means that population data at more granular domains (such as NUTS-5, LAU-1, and LAU-2 units) were (and still are) available only from population censuses held every ten years in Greece [71]. Despite having slight differences in the administrative structure of the country since 1991, the spatial distribution of the population was rather stable over time in the last three decades, as demonstrated in the corresponding population census [72]. Earlier studies documented the persistence of marked population divides across Greek regions, resulting in huge density gaps between the metropolitan regions of Athens and Thessaloniki and the less populated, rural, and peripheral districts of Epirus, Trace, and Peloponnese [73]. A previous work [74] also documented the intrinsic temporal stability (1961–2011) of the rank-size rule for the Greek population explored at the administrative level of LAU-1 municipalities. Based on these results, we focused on a comparative analysis of the importance of the spatial scale and administrative partition of the country [75], assuming rather modest changes in the rank-size relationship depicting the distribution of the Greek population since World War II.

2.3. Econometric Analysis

The statistical distribution of the total population across the selected Greek units (see above) was analyzed using metrics of central tendency, dispersion, and shape (arithmetic mean, standard error, minimum and maximum values, median, 25th and 75th percentiles, skewness, and kurtosis). These metrics were calculated separately at each administrative partition and provided a preliminary description of the target variable, namely population size at a given territorial unit [62]. As a second step, the following equation was considered, with the aim of predicting the rank-size relationship across the selected administrative units (see above):

$$Y_i = AX_i^{-b} \tag{1}$$

where X_i is the rank of the i-th administrative unit, Y_i is the population size of each unit, and A is a constant term [72]. According to the specification proposed by Lotka [73], the population size of the i-th unit and its rank were log-transformed [74]. Thus, the impact of rank on population size or density was assessed through the estimate of the regression coefficient (slope) in a (linear) specification [$\log(Y_i) = a - b\log(X_i) + e$], where $a = \log(A)$ is a scale factor reflecting the size (or density) of the largest city (rank = 1) and e is the (random) error term [75]. In other words, this specification means that the frequency of a given event being part of a target data set was modeled as a function of the position (rank) in the decreasing ordering with respect to the frequency of that event [76].

This approach means that an ordered hierarchy of cities and villages, from the larger to the smaller settlement measured through population size, correlated negatively with the respective hierarchical rank, and these findings reflect the assumption at the base of Zipf's law [77]. This law hypothesizes a linear proportionality between population size and rank, the so-called rank-size rule, with a negative slope coefficient reaching -1. Empirical exercises run on real data (population along density gradients from larger to smaller cities) in various socioeconomic contexts basically confirm such assumptions [78]. The stylized facts at the base of Zipf's law thus constitute well-known empirical evidence that is mostly stable over time and space, encompassing different industrial cycles, historical time periods, social phases, and demographic transitions in both advanced countries and emerging economies [79].

The average effect of independent (external) factors (namely, the overall level of the process, irrespective of the impact of the predictor) was estimated with the intercept regression coefficient [80]. The linear form was augmented with the separate estimation of quadratic and cubic forms. Additionally, two complex specifications, less frequently used in rank-size studies, namely the logistic equation and the Gompertz law, were also estimated, in line with specific indications from previous exercises [81]. Regression diagnostics (adjusted R^2, Fisher-Snedecor F, and Akaike Information Criterion, AIC) were used to quantify the goodness-of-fit of each model to real data and to identify the specification adhering most to the population hierarchy in Greece, considering the related heterogeneity. Graphical scrutiny of the rank-size relationship by administrative partition and fitted specification contributed to this issue.

3. Results

Table 1 reports a selection of descriptive metrics of central tendency, dispersion, and symmetry that were run on the statistical distribution of the total population at the four administrative levels considered in this study. Based on the increasing sample size, the metrics indicate substantial heterogeneity in the distribution of the population by administrative partition. The indicators of central tendency decreased, moving from aggregate to disaggregate administrative partitions of Greece. This was observed coherently for simple averages and more robust indices such as the median, being less sensitive to outliers. The coefficient of variation increased with the same direction and intensity, being relatively small at the level of both NUTS-3 prefectures and NUTS-5 municipalities and rising significantly at the level of LAU-1 and LAU-2 communes. This may indicate a particularly high heterogeneity when using granular data such as LAU-1 and LAU-2 municipalities and local communities. A standardized measure of range (namely, the normalized range, i.e., maximum–minimum divided by the arithmetic average) followed the same pattern. Considering more robust metrics, such as the 25th and 75th percentiles, the interquartile range (i.e., the value of 75th–25th percentile divided by the median) maintained an internal coherency across different partitions, rising from 1.33 (prefectures) to 1.84 (local communities). Finally, kurtosis, asymmetry, and the basic metric obtained by dividing the median by the arithmetic mean all indicate an increased skewness, moving from aggregate to disaggregate administrative units.

Table 1. Descriptive statistics of population distribution in Greece (2021 census, total inhabitants) by observational unit, from aggregated (regional units) to disaggregated ones (local communities).

Metric	Regional Units	Kallikratis Municipalities	Kapodistrian Municipalities	Local Communities
Observational units	75	333	1037	6126
Mean	139,766	31,479	10,108	1711
Median	84,866	17,610	3470	286
Median/mean	0.607	0.559	0.343	0.167
Coefficient of variation	1.419	1.554	2.783	6.969
Normalized range	7.81	20.44	63.65	376.0
Percentile, 25th	28,980	8274	1759	117
Percentile, 75th	142,195	38,033	7608	644
Interquartile range	1.33	1.69	1.69	1.84
Kurtosis	11	77	264	1471
Asymmetry	3	7	13	31

The empirical results of the econometric estimation of the rank-size relationship reflecting population hierarchy at the level of regional units (n = 75) in Greece (2021) are illustrated by specification in Table 2. The goodness-of-fit of the tested models improved from linear to more complex specifications. The linear form has a relatively modest fit, increasing substantially with square forms. Based on AIC, the Gompertz law provided the highest goodness-of-fit. In general, quadratic relationships offer better results (capturing the largest part of local heterogeneity along the density gradient). The population at the lowest-rank locations (mostly rural, peripheral, and depopulated) was estimated with less steep fits; the reverse pattern was observed for populations at higher-rank locations, corresponding to urban and suburban spaces.

Table 2. Estimation of the rank-size relationship along the population hierarchy at the level of regional units (n = 75) in Greece by econometric specification (2021).

Metric	Linear	Quadratic	Cubic	Logistic	Gompertz
Diagnostics					
Akaike Information Criterion	6.83	6.66	8.82	6.68	6.63
Adjusted R^2	0.77	0.97	0.98		
Fisher Snedecor F	246	1261	1093		
Estimated parameters					
a_0	4.37	−4.22	4.29	1.88	1.88
a_1	−0.60	3.10	−2.50	2.6×10^{-7}	-1.5×10^{-6}
a_2		−0.39	0.82	−2.80	2.39
a_3			−0.09		

An estimation of the rank-size relationship along the density gradient (2021) at the level of LAU-1 municipalities (n = 333), corresponding to the new structure of local governments enforced by the 'Kallikratis' reform of 2011, is illustrated in Table 3 by econometric specification. The goodness-of-fit of the tested models increased substantially from linear to more complex specifications, reaching the maximum with quadratic specifications. While the linear form showed a rather modest fit, in line with the results illustrated above, the Gompertz law provided the highest goodness-of-fit based on AIC and confirmed by visual, comparative scrutiny of the rank-size relationship estimated on real data (Figure 2). It seems clear that the negligible heterogeneity associated with the rank-size relationship along the whole population gradient may indicate an intermediate partition with moderate disaggregation, such as LAU-1 municipalities, as the most performing partition for the empirical verification of population rank-size rules. Similarly, with the findings illustrated above, the population at the lowest-rank locations (mostly rural, peripheral, and depopulated) was estimated with less steep functional forms. The reverse was observed for populations at higher-rank locations, corresponding to urban and peri-urban spaces (Figure 2).

Table 3. Estimation of the rank-size relationship along the population hierarchy at the level of 'Kallikratis' municipalities (n = 333) in Greece by econometric specification (2021).

Metric	Linear	Quadratic	Cubic	Logistic	Gompertz
Diagnostics					
Akaike Information Criterion	22.02	8.36	9.27	6.90	6.25
Adjusted R^2	0.69	0.96	0.98		
Fisher Snedecor F	748	4074	5478		
Estimated parameters					
a_0	4.61	−1.91	4.68	2.52	2.52
a_1	−0.60	2.76	−2.56	2.3×10^{-6}	-5.2×10^{-6}
a_2		−0.42	0.97	−2.59	2.37
a_3			−1.12		

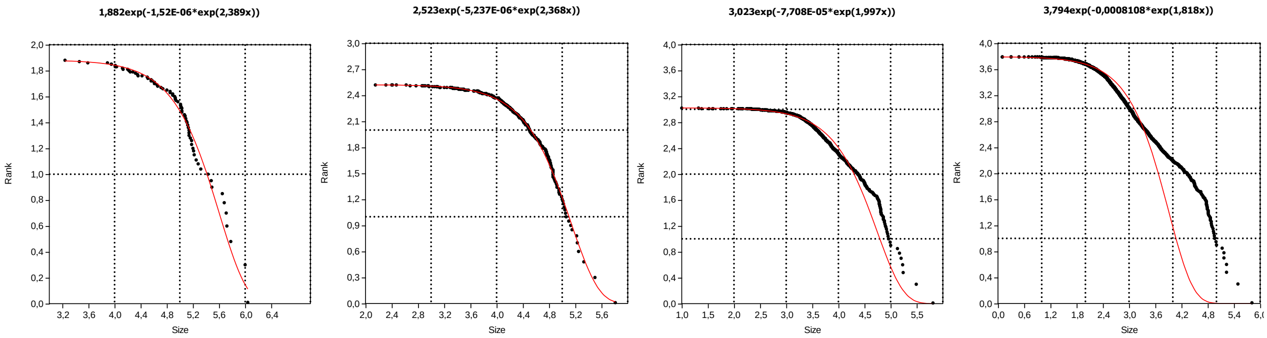

Figure 2. Gompertz fit (red line) of the rank-size relationship along the population hierarchy of Greece (black dots represent real data) by observational unit (left: NUTS-3 regional units; center-left: NUTS-5 'Kallikratis' municipalities; center-right: LAU-1 'Kapodistrian' municipalities; right: LAU-2 local communities) in 2021.

An estimation of the rank-size relationship along the density gradient (2021) at the level of LAU-1 municipalities corresponding to the old administrative structure of local governance in Greece (the so-called 'Kapodistrian', n = 1037), is illustrated in Table 4 by econometric specification. As in previous exercises, the goodness-of-fit of the tested models increased when moving from linear to more complex specifications. The linear form displays a relatively modest fit while improving substantially in comparison to what has been observed at more aggregate partitions. The goodness-of-fit increased substantially with second-order polynomial forms, totalizing the lowest AIC and thus being considered the optimal fit within the tested models. In contrast, with previous results, both logistic and Gompertz laws performed relatively badly, and, in particular, the Gompertz law was unable to capture the intrinsic heterogeneity of population distribution at lower ranks (in both peri-urban and urban locations). From this perspective, a systematic deviation from the expected curve was observed for population amounts between 5000 and 10,000 inhabitants (Figure 2). This demographic class reflects medium-small settlements but rather dynamic agricultural centers responding slowly to urban (agglomeration and scale) economies. However, this class and those with an even higher rank were relatively heterogeneous in comparison to strictly rural populations.

An estimation of the rank-size relationship along the density gradient (2021) at the level of LAU-2 communities in Greece (n = 6126) is illustrated by econometric specification in Table 5. While increasing from linear to more complex specifications, the goodness-of-fit of both quadratic and cubic forms was the highest in the sample (R^2 = 0.98 and 0.99, respectively), and the linear form gave the best result in this study, with R^2 approaching 0.9, despite the larger (and possibly, heterogeneous) sample size. Both logistic and Gompertz laws performed badly; the Gompertz law was unable to describe the intrinsic heterogeneity of the population distribution at settlements above 1000 inhabitants, suggesting that rural-dynamic, peri-urban, and urban locations require a more flexible estimation of the rank-size rule at this (so-called granular) investigation level, as shown in Figure 2.

Table 4. Estimation of the rank-size relationship along the density gradient at the level of 'Kapodistrian' municipalities (n = 1037) in Greece, by econometric specification (2021).

Metric	Linear	Quadratic	Cubic	Logistic	Gompertz
Diagnostics					
Akaike Information Criterion	39.09	8.02	9.73	12.57	22.30
Adjusted R^2	0.82	0.99	0.99		
Fisher Snedecor F	4568	48,661	38,349		
Estimated parameters					
a_0	4.86	1.46	2.24	3.023	3.023
a_1	−0.64	1.34	0.59	4.51×10^{-5}	-7.7×10^{-5}
a_2		−0.28	−0.05	−2.17	2.00
a_3			−0.02		

Table 5. Estimation of the rank-size relationship along the population hierarchy at the level of local communities (n = 6126) in Greece by econometric specification (2021).

Metric	Linear	Quadratic	Cubic	Logistic	Gompertz
Diagnostics					
Akaike Information Criterion	114.5	29.1	21.1	198.1	410.6
Adjusted R^2	0.90	0.98	0.99		
Fisher Snedecor F	57,462	1.49×10^5	1.76×10^5		
Estimated parameters					
a_0	4.94	3.85	3.11	3.79	3.79
a_1	−0.64	0.23	1.15	0.001	−0.001
a_2		−0.16	−0.52	−1.94	1.82
a_3			0.04		

4. Discussion

Focusing on the whole hierarchy from urban to rural locations in Greece, our contribution has verified the rank-size relationship in the total population, comparing the econometric performance of linear and non-linear specifications of Zipf's law [82]. To achieve this objective, we tested different representations of the same population hierarchy partitioning the total population into diversified (administrative) spatial units, from more centralized ones (NUTS-3 prefectures) to more decentralized ones (LAU-2 local communities). The consideration of the whole population hierarchy from the largest cities to the smallest villages contributes to overcoming the issue of selecting a population threshold to include (or exclude) locations from the statistical estimation of the rank-size relationship, as performed in earlier studies. In other words, the total population in each observational unit was intended as the result of the complex interactions between urban and rural economies in the country [83].

Our study implements standard econometric techniques using linear, quadratic, cubic, and more complex (square) relationships between the rank and size of cities (and villages) in Greece. These specifications (especially the linear one) were extensively used and verified in previous studies dealing with Zipf's law [74]. In this paper, we reported the empirical outcome of the econometric estimation run over four administrative scales partitioning the Greek territory into more or less granular units, with the aim of testing the stability of the rank-size rule along a density gradient from aggregated to disaggregated observation domains. The empirical results demonstrate that the use of different (more or less aggregated) partitions of the Greek territory does not significantly influence the estimation of Zipf's law over real population data [84]. With this perspective in mind, testing real data under the rank-size rule means controlling the appropriateness of Zipf's law in different economic systems representative of urban and rural locations and the intermediate contexts in between [85].

More specifically, the empirical results of the econometric estimation run in this study on real population data at four disaggregation levels (moving from an aggregate level, such as the NUTS-3 prefectures, to a mostly disaggregated level, such as the LAU-2 local communities in Greece) document the highest fit of a quadratic relationship in the estimation of the rank-size rule in Greece against more classical linear relationships [86]. These findings were collected irrespective of the aggregation level characteristic of the elementary observational units and, thus, may demonstrate the existence of distinctive dynamics along the density hierarchy that are independent of data aggregation [87]. Moreover, the results of the econometric estimation have documented the appropriateness of a quadratic relationship identifying two rank-size relationships [75], i.e., with distinctive regression coefficients, along the investigated population hierarchy from the largest cities to the smallest villages in Greece [88].

A reasonable explanation for this observed pattern is the co-existence of two settlement structures reflective of socioeconomic systems responding to different drivers of growth and change: (i) a strictly urban system influenced by agglomeration/scale economies and translated into a settlement structure with a 'more steep'—and less homogeneous—rank-size rule and (ii) a typical rural system possibly influenced by agricultural economies and translated into a more dispersed—less steep and homogeneous—settlement structure [76,77,79]. The inherent stability of econometric estimation (namely, the goodness-of-fit (adjusted R^2) and Akaike Information Criterion (AIC), as well as the individual regression coefficients) reflects the appropriateness of such an interpretation of the diversified economic dynamics along the Greek density gradient [22].

The heterogeneous goodness-of-fit of quadratic and more complex square relationships (e.g., logistic and Gompertz) is instead reflective of even more diversified dynamics, possibly depending on the level of data aggregation. Logistic and, in particular, Gompertz laws run well in the case of aggregate observational units (prefectures and NUTS-5 municipalities, with 75 and 333 observational units, respectively). Quadratic estimations seem to give better fits for population hierarchy under more disaggregated observational units (e.g., LAU-2 local communities). These differences are in line with the economic divide discussed above [65–67]. More specifically, urban locations were characterized by a relatively high departure from a rank-size rule [83], possibly associated with accelerated economic dynamics. Less evident heterogeneities were observed in rural locations characterized by smaller population numbers and more dispersed settlements [26].

Altogether, our results outline the substantial stability of Zipf's regression coefficients, irrespective of the econometric specification or administrative partition adopted [62]. By suggesting the importance of non-linear rank-size specifications [68], deviations from Zipf's law were reframed as an intrinsic characteristic of largely divided economic systems [65], identifying the locations where the impact of economic forces is more (or less) intense [32]. These results imply economic interactions at different organizational scales [34] and may reflect the role of articulated networks mediated by accessibility, congestion externalities, amenities, and other background factors [53].

Focusing more on the results' comparison across spatial scales, instead of deepening the results' comparison across temporal scales, is justified by the findings of earlier studies that demonstrate how changes in the spatial distribution of the Greek settlements and population over the last three decades were relatively mild compared to what has been observed in previous decades, namely 1951–1991 [10]. As a matter of fact, it was documented that the distribution of resident populations remained substantially asymmetric and divided into high-density metropolitan areas (Athens and Thessaloniki) and low-density rural districts in Trace, Peloponnese, Epirus, and other peripheral regions in the country [64]. Moreover, the intense rates of population growth observed in the immediate aftermath of World War II have slowed down since the 1980s, marking a demographic decline associated with fertility reduction and aging that has persisted until now [61]. While international immigration contributed to containing population deceleration, demographic rates remained substantially stable (or moderately positive in some locations), slightly altering the population geography of Greece [86]. Based on these considerations, a specific study testing Zipf's law for the Greek population at the level of LAU-1 municipalities demonstrated evident stability of the estimated rank-size relationship over a prolonged time interval between 1961 and 2011 [74].

These findings—together with more general results documenting the stability of empirically estimated Zipf's laws over time in several advanced countries [23]—justify limiting the empirical analysis to the most recent data for Greece (2021 census), without any significant loss of information. However, empirical trials running the five econometric specifications presented in this study on previous census data (2011, 2001, and 1991) confirmed the main outcomes of the estimation of real 2021 data (unpublished results available from the authors upon request). A refined analysis of the intrinsic stability of the rank-size relationship over sufficiently long time intervals in Greece is, however, relevant to regional science [34]. The impact of exogenous shocks (including intense recessions and health crises, such as the recent COVID-19 pandemic) on the population geography in advanced economies deserves further investigation [89]. In these regards, extending the empirical design to other European countries, with the aim of verifying the negligible role of using different administrative levels and territorial partitions when estimating the rank-size relationship, seems to be particularly appropriate from a comparative perspective [20].

Another issue explored in this study is using the econometric estimation of rank-size rules to discriminate urban from rural systems with distinctive economic functioning [21]. This evidence stems from the empirical observation that Zipf's law in Greece, as in other socioeconomic contexts, may assume a curvilinear shape if a complete density hierarchy is considered, not censored at any specific value of population size (or density). Differential slope coefficients are supposed to be the intrinsic characteristics of urban and rural systems [57]. This finding may also imply the existence of a 'buffering zone' with an economic behavior between urban and rural systems [55]. With this perspective in mind, we fitted Zipf's law in the context of total population dynamics across urban and rural areas in Greece, identifying (i) different slopes for low-hierarchy and high-hierarchy locations (corresponding to urban and rural places) and (ii) a distinctive deviation regime (e.g., more or less heterogeneity) from the estimated Zipf's law corresponding to urban and rural locations [51]. These deviations from the empirically observed rank-size rule for Greece were intended as a fundamental (and likely stable over scale) attribute of both systems; more specifically, urban systems seem to exhibit higher heterogeneity, especially in correspondence to the largest cities in the hierarchy, while rural systems display lower heterogeneity [58]. Taken together, these disparities (both slope coefficients and heterogeneity regime) may reflect diversified growth drivers and a spatially varying impact of agglomeration and scale economies when comparing urban to rural systems [12].

Based on these premises, the empirical framework proposed here can be adopted in other socioeconomic conditions and generalized to divided countries in both advanced and emerging contexts [48]. The framework may benefit from additional research efforts clarifying the role of urban and rural economic systems in quadratic rank-size relationships [56]. As for the Greek experience, the rural–urban dualism that emerges from the estimated (quadratic) rank-size rule is not new in the recent literature [74]. This persistent divide may stimulate some reflections on the estimation of rank-size relationships on real population data when classifying high-density and low-density settlements or, better, to identify urban and rural economies governed by different drivers of change, socio-demographic transformations, and territorial contexts [73]. The distinction between urban and rural areas is a key aspect of regional studies, and several distinctive methodologies were developed to allow such classification based on positive and normative approaches [70]. Spatial planning may also benefit from an operational definition of urban and rural locations [69].

Policy-oriented classifications of urban and rural areas based on the results of a curvilinear estimation of a rank-size rule *à la* Zipf seem to be particularly appropriate because (i) the real effectiveness of spatial planning and (ii) the intrinsic response of local systems to development policies can be vastly different at urban and rural locations governed by diverse economic functioning [80]. Being estimated directly from the rank-size rule of real population data, local heterogeneities in metropolitan hierarchies, both high-density and low-density conditions, provide additional and relevant information about spatial planning and policy measures addressing rural development, economic growth in urban and suburban areas, and the reduction of the inherent disparities typical of socially divided countries and/or regions [11].

In the case of Greece, heterogeneity in urban areas possibly derives from the evident differences in urban size among the ten largest cities in the country, with two leaders (Athens and Thessaloniki) and some laggers with a completely different (lower) economic size and governance power [70]. Heterogeneity in rural areas is lower because of the intrinsic conditions of remoteness and economic backwardness typical of inland locations with an economic system dominated by low-value-added activities such as traditional farming and unspecialized forestry [71]. Heterogeneity in the intermediate locations—e.g., agricultural districts specialized in tourism and intensive crop productions frequently linked with urban markets—may explain, at least in part, the differential rank-size rule compared to strictly urban and strictly rural locations [9]. Intermediate locations in Greece also include Ionian and Aegean islands, taken as another (specific) case of semi-rurality, since they feature low-density settlements with seasonally high-density populations because of sea tourism and natural amenities [18].

In other words, the different slope coefficients of the empirical estimation of Zipf's law and the varying heterogeneity regime along the density gradient may delineate two or more territorial systems with a possibly distinctive economic functioning. With more specific modeling and assessment frameworks [88], these attributes can be routinely used to classify urban, intermediate, and rural locations from an economic point of view. Based on all these considerations, the intrinsic linkage between rural–urban discourses in general and the application of Zipf's law in highly divided countries thus deserves further research efforts [89], reconnecting methodological approaches with planning and policy for sustainable development.

5. Conclusions

The results of linear and non-linear econometric estimates of Zipf's law for total population suggest different socioeconomic dynamics along the urban–rural hierarchy in Greece. A quadratic relationship was considered suitable to capture non-linear cross-sectional patterns, basically highlighting two settlement structures. These structures reflect distinctive hierarchies associated with concentrated (or dispersed) settlement patterns. A comparative analysis of the rank-size relationship over urban–rural gradients, considering multiple population partitions from the same information source (e.g., demographic census), sheds more light on the statistical significance (and the substantive meaning) of real data deviations from Zipf's law or other specifications predicting the geo-economic structure of regions and countries.

In other words, the intrinsic deviation from a Zipfian pattern characteristic of rank-size rules seems to be the basis of such hierarchy, with higher heterogeneity typical of urban systems and lower heterogeneity typical of rural systems. Such disparities may be reflective of different drivers of growth and change or, better, of the different roles of agglomeration and scale economies in urban and rural locations. From this perspective, population size remains an honest descriptor of such a gradient. Unraveling the role of heterogeneity in population hierarchies as a basic, distinctive aspect of urban and rural locations is an issue that merits further study. From an applied economics perspective, these empirical results may finally support policies that promote a spatially balanced system of cities and villages finely tuned with intrinsic (e.g., accessibility) and extrinsic (e.g., latent networking and interactions) forces typical of any region or country.

Author Contributions: Conceptualization, L.S. and A.S.; methodology, L.S. and R.S.; software, F.E.S. and B.Z.; validation, G.Q. and B.Z.; formal analysis, R.S. and A.M.; investigation, R.S. and G.Q.; resources, F.C. and A.S.; data curation, A.M. and B.Z.; writing—original draft preparation, L.S. and A.S.; writing—review and editing, G.Q. and R.S.; visualization, A.M.; supervision, F.E.S. and B.Z.; project administration, F.E.S. and B.Z.; funding acquisition, A.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partially funded by Sapienza University of Rome with the research project entitled "UnRaveling the inherent complexity in spatio-temporal patterns of urBanization: Theoretical and empirical contributions from global to local observation scaleS" (URBS: progetto medio di ateneo 2023).

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Cai, B.; Shao, Z.; Fang, S.; Huang, X.; Tang, Y.; Zheng, M.; Zhang, H. The Evolution of urban agglomerations in China and how it deviates from Zipf's law. *Geo-Spatial Inf. Sci.* **2022**, *27*, 38–48. [CrossRef]
2. Bettencourt, L.M.; Lobo, J. Urban scaling in Europe. *J. R. Soc. Interface* **2016**, *13*, 20160005. [CrossRef]
3. Carlucci, M.; Grigoriadis, E.; Rontos, K.; Salvati, L. Revisiting an Hegemonic Concept: Long-term 'Mediterranean Urbanization' in between city re-polarization and metropolitan decline. *Appl. Spat. Anal. Policy* **2017**, *10*, 347–362. [CrossRef]
4. Hackmann, A.; Klarl, T. The evolution of Zipf's Law for U.S. cities. *Pap. Reg. Sci.* **2020**, *99*, 841–852. [CrossRef]
5. Grekousis, G.; Manetos, P.; Photis, Y.N. Modeling urban evolution using neural networks, fuzzy logic and GIS: The case of the Athens metropolitan area. *Cities* **2013**, *30*, 193–203. [CrossRef]
6. González-Val, R. The Spanish spatial city size distribution. *Environ. Plan. B Urban Anal. City Sci.* **2021**, *48*, 1609–1631. [CrossRef]
7. Düben, C.; Krause, M. Population, light, and the size distribution of cities. *J. Reg. Sci.* **2021**, *61*, 189–211. [CrossRef]
8. Duran, H.E.; Cieślik, A. The distribution of city sizes in Turkey: A failure of Zipf's law due to concavity. *Reg. Sci. Policy Pract.* **2021**, *13*, 1702–1719. [CrossRef]
9. Duvernoy, I.; Zambon, I.; Sateriano, A.; Salvati, L. Pictures from the Other Side of the Fringe: Urban Growth and Peri-urban Agriculture in a Post-industrial City (Toulouse, France). *J. Rural. Stud.* **2018**, *57*, 25–35. [CrossRef]
10. Naude, W.A.; Krugell, W.F. Are South Africa's cities too small? *Cities* **2003**, *20*, 175–180. [CrossRef]
11. Shao, J.; Ivanov, P.C.; Urošević, B.; Stanley, H.E.; Podobnik, B. Zipf rank approach and cross-country convergence of incomes. *Europhys. Lett.* **2011**, *94*, 48001. [CrossRef]
12. Kleynhans, E.P.J.; Coetzee, C.E. The rank-size distribution of cities in South Africa. *GeoJournal* **2022**, *87*, 4775–4790. [CrossRef]
13. Ciccone, A. The law of population concentration. *Environ. Plan. B Urban Anal. City Sci.* **2023**, *50*, 290–298. [CrossRef]
14. Morelli, V.G.; Rontos, K.; Salvati, L. Between suburbanisation and re-urbanisation: Revisiting the urban life cycle in a Mediterranean compact city. *Urban Res. Pract.* **2014**, *7*, 74–88. [CrossRef]
15. Turok, I. Cities, regions and competitiveness. *Reg. Stud.* **2004**, *38*, 1061–1075. [CrossRef]
16. Solon, J. Spatial context of urbanization: Landscape pattern and changes between 1950 and 1990 in the Warsaw metropolitan area, Poland. *Landsc. Urban Plan.* **2009**, *93*, 250–261. [CrossRef]
17. Venanzoni, G.; Carlucci, M.; Salvati, L. Latent sprawl patterns and the spatial distribution of businesses in a southern European city. *Cities* **2017**, *62*, 50–61. [CrossRef]
18. Salvati, L.; Carlucci, M. Patterns of sprawl: The socioeconomic and territorial profile of dispersed urban areas in Italy. *Reg. Stud.* **2016**, *50*, 1346–1359. [CrossRef]
19. Oueslati, W.; Alvanides, S.; Garrod, G. Determinants of urban sprawl in European cities. *Urban Stud.* **2015**, *52*, 1594–1614. [CrossRef] [PubMed]
20. Cheshire, P.; Magrini, S. Urban Growth Drivers in a Europe of Sticky People and Implicit Boundaries. *J. Econ. Geogr.* **2009**, *9*, 85–115. [CrossRef]
21. Chen, Y. Exploring the level of urbanization based on Zipf's scaling exponent. *Phys. A Stat. Mech. Its Appl.* **2021**, *566*, 125620. [CrossRef]
22. Rodríguez-Pose, A.; Fratesi, U. Between development and social policies: The impact of European Structural Funds in Objective 1 regions. *Reg. Stud.* **2004**, *38*, 97–113. [CrossRef]
23. Arshad, S.; Hu, S.; Ashraf, B.N. Zipf's law and city size distribution: A survey of the literature and future research agenda. *Phys. A Stat. Mech. Its Appl.* **2018**, *492*, 75–92. [CrossRef]
24. Gabaix, X. Zipf's law for cities: An explanation. *Q. J. Econ.* **1999**, *114*, 739–767. [CrossRef]
25. Gabaix, X. Zipf's Law and the Growth of Cities. *Am. Econ. Rev.* **1999**, *89*, 129–132. [CrossRef]
26. Wan, G.; Zhu, D.; Wang, C.; Zhang, X. The size distribution of cities in China: Evolution of urban system and deviations from Zipf's law. *Ecol. Indic.* **2020**, *111*, 106003. [CrossRef]

27. Dijkstra, L.; Florczyk, A.J.; Freire, S.; Kemper, T.; Melchiorri, M.; Pesaresi, M.; Schiavina, M. Applying the degree of urbanisation to the globe: A new harmonised definition reveals a different picture of global urbanisation. *J. Urban Econ.* **2021**, *125*, 103312. [CrossRef]
28. De Marzo, G.; Gabrielli, A.; Zaccaria, A.; Pietronero, L. Dynamical approach to Zipf's law. *Phys. Rev. Res.* **2021**, *3*, 013084. [CrossRef]
29. De Marzo, G.; Attili, F.; Pietronero, L. Growing inequality in systems showing Zipf's law. *J. Phys. Complex.* **2023**, *4*, 015014. [CrossRef]
30. Bergs, R. The detection of natural cities in the Netherlands—Nocturnal satellite imagery and Zipf's law. *Rev. Reg. Res.* **2018**, *38*, 111–140. [CrossRef]
31. Brakman, S.; Garretsen, H.; Van Marrewijk, C.; Berg, M.V.D. The return of Zipf: Towards a further understanding of the rank-size distribution. *J. Reg. Sci.* **2002**, *39*, 183–213. [CrossRef]
32. Budde, R.; Neumann, U. The size ranking of cities in Germany: Caught by a MAUP? *GeoJournal* **2019**, *84*, 1447–1464. [CrossRef]
33. Calderín-Ojeda, E. The distribution of all French communes: A composite parametric approach. *Phys. A Stat. Mech. Its Appl.* **2016**, *450*, 385–394. [CrossRef]
34. Cartone, A.; Postiglione, P.; Hewings, G.J. Does economic convergence hold? A spatial quantile analysis on European regions. *Econ. Model.* **2021**, *95*, 408–417. [CrossRef]
35. Colantoni, A.; Grigoriadis, E.; Sateriano, A.; Sarantakou, E.; Salvati, L. Back to Von Thunen: A Southern European perspective on mono-centric urban growth, economic structure and non-urban land decline. *Int. Plan. Stud.* **2017**, *22*, 173–188. [CrossRef]
36. Corral, Á.; Serra, I.; Ferrer-I-Cancho, R. Distinct flavors of Zipf's law and its maximum likelihood fitting: Rank-size and size-distribution representations. *Phys. Rev. E* **2020**, *102*, 052113. [CrossRef] [PubMed]
37. Cristelli, M.; Batty, M.; Pietronero, L. There is more than a power law in Zipf. *Sci. Rep.* **2012**, *2*, srep00812. [CrossRef] [PubMed]
38. Furceri, D. Zipf's law and world income distribution. *Appl. Econ. Lett.* **2008**, *15*, 921–923. [CrossRef]
39. Gan, L.; Li, D.; Song, S. Is the Zipf law spurious in explaining city-size distributions? *Econ. Lett.* **2006**, *92*, 256–262. [CrossRef]
40. Gao, H.; Wu, K. Zipf's law and influential factors of the Pareto exponent of the city size distribution: Evidence from China. *Front. Econ. China* **2008**, *3*, 137–149. [CrossRef]
41. Giesen, K.; Südekum, J. Zipf's law for cities in the regions and the country. *J. Econ. Geogr.* **2011**, *11*, 667–686. [CrossRef]
42. Gomez-Lievano, A.; Youn, H.; Bettencourt, L.M.A. The statistics of urban scaling and their connection to Zipf's law. *PLoS ONE* **2012**, *7*, e40393. [CrossRef] [PubMed]
43. Gong, J.; Li, S.; Ye, X.; Peng, Q.; Kudva, S. Modelling impacts of high-speed rail on urban interaction with social media in China's mainland. *Geo-Spatial Inf. Sci.* **2021**, *24*, 638–653. [CrossRef]
44. González-Val, R. Historical urban growth in Europe (1300–1800). *Pap. Reg. Sci.* **2019**, *98*, 1115–1136. [CrossRef]
45. Di Feliciantonio, C.; Salvati, L. 'Southern' Alternatives of Urban Diffusion: Investigating Settlement Characteristics and Socio-Economic Patterns in Three Mediterranean Regions. *Tijdschr. Voor Econ. Soc. Geogr.* **2015**, *106*, 453–470. [CrossRef]
46. Cuadrado-Ciuraneta, S.; Durà-Guimerà, A.; Salvati, L. Not only tourism: Unravelling suburbanization, second-home expansion and "rural" sprawl in Catalonia, Spain. *Urban Geogr.* **2017**, *38*, 66–89. [CrossRef]
47. Gavalas, V.S.; Rontos, K.; Salvati, L. Who becomes an unwed mother in Greece? Socio-demographic and geographical aspects of an emerging phenomenon. *Popul. Space Place* **2014**, *20*, 250–263. [CrossRef]
48. González-Val, R. Deviations from Zipf's law for American cities: An empirical examination. *Urban Studies* **2011**, *48*, 1017–1035. [CrossRef]
49. Hordijk, W. Snooker Statistics and Zipf's Law. *Stats* **2022**, *5*, 985–992. [CrossRef]
50. Nota, F.; Song, S. Further analysis of the Zipf's law: Does the rank-size rule really exist? *J. Urban Manag.* **2012**, *1*, 19–31. [CrossRef]
51. Jiang, B.; Jia, T. Zipf's law for all the natural cities in the United States: A geospatial perspective. *Int. J. Geogr. Inf. Sci.* **2011**, *25*, 1269–1281. [CrossRef]
52. Kazemzadeh-Zow, A.; Shahraki, S.Z.; Salvati, L.; Samani, N.N. A Spatial Zoning Approach to Calibrate and Validate Urban Growth Models. *Int. J. Geogr. Inf. Sci.* **2017**, *31*, 763–782. [CrossRef]
53. Kinoshita, T.; Kato, E.; Iwao, K.; Yamagata, Y. Investigating the rank-size relationship of urban areas using land cover maps. *Geophys. Res. Lett.* **2008**, *35*. [CrossRef]
54. Kosmopoulou, G.; Buttry, N.; Johnson, J.; Kallsnick, A. Suburbanization and the rank-size rule. *Appl. Econ. Lett.* **2007**, *14*, 1–4. [CrossRef]
55. Lalanne, A. Zipf's law and Canadian urban growth. *Urban Stud.* **2014**, *51*, 1725–1740. Available online: https://www.jstor.org/stable/26145821 (accessed on 29 February 2024). [CrossRef]
56. Aurélie, L.; Martin, Z. From Gibrat's law to Zipf's law through cointegration? *Econ. Lett.* **2020**, *192*, 109211. [CrossRef]
57. Li, Z.; Jiao, L.; Zhang, B.; Xu, G.; Liu, J. Understanding the pattern and mechanism of spatial concentration of urban land use, population and economic activities: A case study in Wuhan, China. *Geo-Spatial Inf. Sci.* **2021**, *24*, 678–694. [CrossRef]
58. Mulligan, G.F.; Partridge, M.D.; Carruthers, J.I. Central place theory and its reemergence in regional science. *Ann. Reg. Sci.* **2012**, *48*, 405–431. [CrossRef]
59. Nitsch, V. Zipf zipped. *J. Urban Econ.* **2012**, *57*, 86–100. [CrossRef]
60. Peña, G.; Sanz-Gracia, F. Zipf's exponent and Zipf's law in the BRICS: A rolling sample regressions approach. *Econ. Bull.* **2021**, *41*, 2543–2549.

61. Peng, G. Zipf's law for Chinese cities: Rolling sample regressions. *Phys. A Stat. Mech. Its Appl.* **2010**, *389*, 3804–3813. [CrossRef]
62. Pili, S.; Grigoriadis, E.; Carlucci, M.; Clemente, M.; Salvati, L. Towards Sustainable Growth? A Multi-criteria Assessment of (Changing) Urban Forms. *Ecol. Indic.* **2017**, *76*, 71–80. [CrossRef]
63. Pérez-Campuzano, E.; Guzmán-Vargas, L.; Angulo-Brown, F. Distributions of city sizes in Mexico during the 20th century. *Chaos Solitons Fractals* **2015**, *73*, 64–70. [CrossRef]
64. Petrakos, G.; Rodríguez-Pose, A.; Rovolis, A. Growth, integration, and regional disparities in the European Union. *Environ. Plan. A Econ. Space* **2005**, *37*, 1837–1855. [CrossRef]
65. Pilgrim, C.; Hills, T.T. Bias in Zipf's law estimators. *Sci. Rep.* **2021**, *11*, 17309. [CrossRef] [PubMed]
66. Rastvortseva, S.; Manaeva, I. Zipf's law appearance in the Russian cities. *Reg. Sci. Inq.* **2016**, *8*, 51–59.
67. Rastvortseva, S.; Manaeva, I. Zipf's Law for Russian Cities: Analysis of New Indicators. *Econ. Reg.* **2020**, *3*, 935–947. [CrossRef]
68. Reggiani, A.; Nijkamp, P. Did Zipf anticipate spatial connectivity structures? *Environ. Plan. B Plan. Des.* **2015**, *42*, 468–489. [CrossRef]
69. Salvati, L. Bridging the divide: Demographic dynamics and urban–rural polarities during economic expansion and recession in Greece. *Popul. Space Place* **2019**, *25*, e2267. [CrossRef]
70. Salvati, L. Population growth and the economic crisis: Understanding latent patterns of change in Greece, 2002–2016. *Lett. Spat. Resour. Sci.* **2018**, *11*, 105–126. [CrossRef]
71. Salvia, R.; Quaranta, G.; Rontos, K.; Cudlin, P.; Salvati, L. Investigating metropolitan hierarchies through a spatially explicit (local) approach. *ISPRS Int. J. Geo-Inf.* **2023**, *12*, 315. [CrossRef]
72. Schluter, C. On Zipf's law and the bias of Zipf regressions. *Empir. Econ.* **2021**, *61*, 529–548. [CrossRef]
73. Lotka, A.J. *Elements of Physical Biology*; William and Wilkins: Baltimore, MD, USA, 1925.
74. Semboloni, F. Hierarchy, cities size distribution and Zipf's law. *Eur. Phys. J. B* **2008**, *63*, 295–301. [CrossRef]
75. Rozenfeld, H.D.; Rybski, D.; Gabaix, X.; Makse, H.A. The area and population of cities: New insights from a different perspective on cities. *Am. Econ. Rev.* **2011**, *101*, 2205–2225. [CrossRef]
76. Saichev, A.; Malevergne, Y.; Sornette, D. *Theory of Zipf's Law and Beyond (Lecture Notes in Economics and Mathematical Systems)*; Springer: Berlin/Heidelberg, Germany, 2009; p. 632. [CrossRef]
77. Sokołowski, D.; Jażdżewska, I. Zipf's Law for cities: Estimation of regression function parameters based on the weight of American urban areas and Polish towns. *Bull. Geogr. Socio-Econ. Ser.* **2021**, *53*, 147–156. [CrossRef]
78. Soo, K.T. Zipf's Law for cities: A cross-country investigation. *Reg. Sci. Urban Econ.* **2005**, *35*, 239–263. [CrossRef]
79. Sun, X.; Yuan, O.; Xu, Z.; Yin, Y.; Liu, Q.; Wu, L. Did Zipf's Law hold for Chinese cities and why? Evidence from multi-source data. *Land Use Policy* **2021**, *106*, 105460. [CrossRef]
80. Van Der Burg, A.J.; Dieleman, F.M. Dutch urbanization policies: From 'compact city' to 'urban network'. *Tijdschr. Econ. Soc. Geogr.* **2004**, *95*, 108–116. [CrossRef]
81. Verbavatz, V.; Barthelemy, M. The growth equation of cities. *Nature* **2020**, *587*, 397–401. [CrossRef]
82. Rai, K.; Garg, B. Demographic transition and inflation. *Econ. Syst.* **2024**, 101214. [CrossRef]
83. Wang, Q.A. Principle of least effort vs. maximum efficiency: Deriving Zipf-Pareto's laws. *Chaos Solitons Fractals* **2021**, *153*, 111489. [CrossRef]
84. Wei, J.; Zhang, J.; Cai, B.; Wang, K.; Liang, S.; Geng, Y. Characteristics of carbon dioxide emissions in response to local development: Empirical explanation of Zipf's law in Chinese cities. *Sci. Total. Environ.* **2021**, *757*, 143912. [CrossRef] [PubMed]
85. Wu, Y.; Jiang, M.; Chang, Z.; Li, Y.; Shi, K. Does China's urban development satisfy Zipf's law? A multiscale perspective from the NPP-VIIRS nighttime light data. *Int. J. Environ. Res. Public Health* **2020**, *17*, 1460. [CrossRef]
86. Xu, Z.; Harriss, R. A Spatial and Temporal Autocorrelated Growth Model for City Rank—Size Distribution. *Urban Studies* **2010**, *47*, 321–335. [CrossRef]
87. Sotiropoulou, K.F.; Vavatsikos, A.P. A decision-making framework for spatial multicriteria suitability analysis using PROMETHEE II and k nearest neighbor machine learning models. *J. Geovis. Spat. Anal.* **2023**, *7*, 20. [CrossRef]
88. Chettry, V. A critical review of urban sprawl studies. *J. Geovis. Spat. Anal.* **2023**, *7*, 28. [CrossRef]
89. Hsu, W.-T. Central place theory and city size distribution. *Econ. J.* **2012**, *122*, 903–932. [CrossRef]