Improving Delay Estimation in Underwater Acoustic Applications by the Additional Use of Cross-Cross-Correlation

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***Abstract*—Next generation space-terrestrial-ocean integrated mobile networks providing global internet access that extend to the undersea are based on heterogeneous networks. In underwater applications, a key role is played by the acoustic positioning. In particular, this task can be accomplished making use of multiple passive sensors that estimate the differential signal delays employed for positioning. This paper exploits a methodology aimed at improving delay estimation by means of cross-cross-correlation, i.e., the cross-correlation between all the multi-sensor cross-correlations. The resulting equation system is formulated as a least squares (LS) minimization problem, whose solution is efficiently found resorting to the pseudo-inverse technique, ensuring a fast execution of the algorithm, without using statistical information on random signal spectra. The performance of the devised method is numerically analyzed for an extensive range of operating parameters to demonstrate the validity of the proposed approach in comparison with classic counterparts and theoretical optimum bounds.**

***Index Terms*—Cross-cross-correlation, delay estimation, heterogeneous network, multiple sensors, sensor signal processing, underwater acoustics.**

I. Introduction

An important vision of next generation mobile system is to provide global internet access. The Space-Terrestrial Integrated Network [1] has been proposed and intensively studied to tackle this challenge. To tackle various underwater applications, where radio signals suffer severe attenuation, it is necessary to extend the current space-terrestrial-ocean integrated network to underwater space to provide underwater internet access. Therefore, this work can be framed in a heterogeneous network system that allows to extend the radio coverage to the undersea [2]. In underwater applications, such as maritime surveillance, environmental monitoring, etc, a key role is played by the acoustic positioning [3]–[6]. This task can be accomplished making use of multiple passive sensors that estimate the differential signal delays then used for target localization. In fact, passive technologies utilize a multitude of sensors to record a number of replicas of the acoustic signal emitted by the target/source to detect and localize it by processing the time delays estimated by each signal at the receiver side. A classic way to obtain an estimate of the time delay between two replicas of a stationary signal received at two different spatial locations consists in computing the cross-correlation between them and evaluating the time instant at which its maximum value arises. A way to improve the estimation accuracy was developed in the seminal Knapp and Carter's paper [7]. It consists in applying a filter to the incoming signals before the computation of the cross-correlation. By doing so, the maximum value of such a filtered cross-correlation, referred to as generalized cross-correlation (GCC), provides the optimum delay estimation between the two involved signals [8]–[11]. However, the GCC is strictly related to the availability of a priori information about both signal and noise spectral statistics [7]. Therefore its implementation is difficult in those applications for which the spectral properties of the incoming signals are not fully known or cannot be effectively estimated. In this respect, in [12], a new delay estimator that does not require a priori spectral signal knowledge has been developed and tailored for radio-frequency scenarios. In particular, it refers to the problem of passive radar location, performing the computation of the cross-cross-correlations (CCC), i.e., the cross-correlation between each couple of cross-correlations [12], of the received signals in the complex domain. Additionally, in [13]–[15] the second-order time differences are used for black-box localization.

Differently from [12], in this paper we make use of the cross-cross- and conv-cross-correlation for the delay estimation of real acoustic (unmodulated) signals in underwater applications. Moreover, even if in [13]–[15] the second-order time differences are already used for localization purposes, herein it is also shown how to obtain them together with also the sum of the time differences (obtained through the conv-cross-correlations). It is also worth to note that, the unknown time delays can be derived from the positions of the maximum values of the cross-cross-correlations through the formalization of a least squares (LS) linear problem. Hence, it would be expected that this estimator performs better than the conventional one in the presence of a correlated signal and a relevant level of random noise for two basic reasons. Firstly, the random errors of cross-correlation peak's noisy estimates can be reduced by the higher number of equations of the proposed method. In addition, the true cross-correlation is just a shifted and scaled signal's auto-correlation. Results obtained by simulations prove the effectiveness of the proposed method with respect to its standard counterpart.

II. System model and proposed solution

An underwater acoustic passive locating system composed by M sensors (depicted in Figure 1), whose physical displacements in the area of interest are not known, is herein considered. Each receiving sensor acquires a delayed copy of the signal transmitted by the source (or target) to be localized. Then, the reference node elaborates all received signals to provide an estimate of the target position starting from the delay estimates.

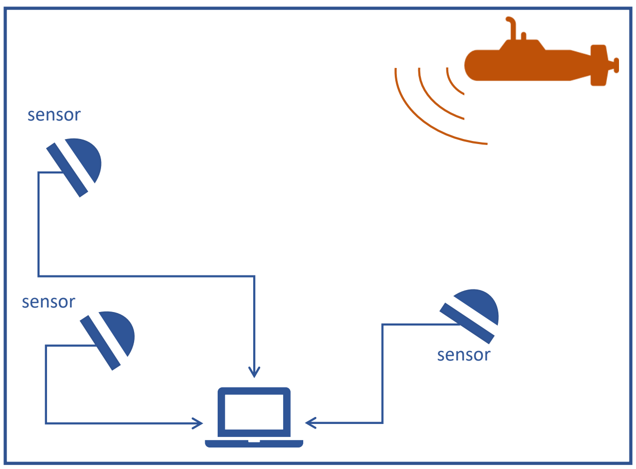

Figure 1. Illustration of passive location system for underwater acoustic applications.

Therefore, indicating with $s(t)$ the unknown acoustic signal transmitted by the object under identification, the signal received at the i-th node can be described by means of the following equation

$$r_i(t) = \zeta_i s(t - t_i) + w_i(t), \quad i = 0, \dots, M-1, \tag{1}$$

where $\zeta_i \in \mathbb{R}$, $i = 0 \dots, M-1$, is an unknown scaling factor accounting for the channel attenuation essentially related to the distance between the transmitter and the i-th sensing node, whereas $w_i(t)$ is the thermal noise contribution at each receiving sensor assumed to be uncorrelated with the signal. Additionally t_i, $i = 0 \dots, M-1$, indicates the time delay at each receiving node to be estimated, evaluated with respect to the delay of the first sensor, say t_0, assumed in the following, without loss of generality, equal to 0 s. Moreover, due to the random displacement of sensors, there is no a priori known functional dependence between time instants t_i.

Indicating with τ the generic delay variable (i.e., the respective delay between two signals), the cross-correlation estimation between each couple of signals, $r_i(t)$ and $r_j(t)$, at the sensing nodes (i, j), $i, j = 0, \dots, M-1$, is given by

$$R_{ij}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} r_i(t) r_j(t - \tau) dt, \tag{2}$$

having assumed the validity of the ergodicity property and having indicated with T the observation time. Moreover, even tough τ depends on the considered couple, the subscripts ij on it are omitted for sake of simplicity.

Now, the classic way to obtain an estimate of the delay difference consists in evaluating the peak position of cross-correlation magnitude (in the ideal noise-free case) derived by (2), that is

$$\hat{\tau} = \arg\max_{\tau} \left\{ |R_{ij}(\tau)| \right\}. \tag{3}$$

Now, considering all couples of sensors (i, j), $i, j = 0, \dots, M-1$, all the cross-correlation maxima can be properly used to estimate the relative signals' delays acquired by the M sensors, viz. $\tau = t_i - t_j$. We can observe that $R_{ij}(\tau)$ produces some redundant estimates due to its intrinsic symmetric definition as well as reduces to the auto-correlation for $i = j$. Therefore, (2) is evaluated only for $j > i$ to eliminate all the above-mentioned redundant information. By doing so, the total number of admitted combinations of M sensors is $Q = 1/2 \left(M^2 - M \right)$.

To simplify the used notation, in the following, each cross-correlation considered in (2) is numbered by a single subscript $q = 0, \dots, Q-1$, that is

$$R_q(\tau) = R_{ij}(\tau), \tag{4}$$

with $q = 0, \dots, Q-1$, and $i, j = 0, \dots, M-1$ $(j > i)$. Then, it is possible to estimate the $M-1$ delays in the minimum mean square error (MMSE) sense by computing the apex of each cross-correlation magnitude and writing the corresponding equation as a linear combination of $M-1$ unknowns (i.e., the signal delays). More precisely, we can solve the overdetermined system made by the Q equations consisting of a linear combination of the $M-1$ unknowns equal to the index of the maximum of the cross-correlations considered in (2), that is

$$t_i - t_j = \hat{\tau}_{ij}, \tag{5}$$

with $i, j = 0, \dots, M-1$ $(j > i)$, and

$$\hat{\tau}_{ij} = \arg\max_{\tau} \left\{ |R_{ij}(\tau)| \right\}. \tag{6}$$

Resorting to a more compact matrix form, (5) can be rewritten as

$$\boldsymbol{A}\boldsymbol{t} = \boldsymbol{\tau}, \tag{7}$$

with

$$\boldsymbol{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_{M-1} \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\tau} = \begin{bmatrix} \hat{\tau}_{01} \\ \vdots \\ \hat{\tau}_{(M-2)(M-1)} \end{bmatrix}.$$

The model matrix $\boldsymbol{A}$ of size $Q \times (M-1)$ can be built as described in [12]. Then, the solution to (7) is given by the pseudo-inverse of $\boldsymbol{A}$.

A. Cross-cross-correlation

This section describes the solution based on the cross-cross-correlation to improve the delay estimate in the presence of $M > 2$ receiving sensors casually arranged in the area of interest. To this end, let us first introduce the CCC estimate, that is

$$C_{ijlm}(\delta) = \frac{1}{2T} \int_{-T}^{T} R_q(\tau) R_p(\tau - \delta) d\tau, \tag{8}$$

with $q, p = 0, \ldots, Q-1$ ($p > q$). Once again, the choice $p > q$ is performed to avoid redundant equations. In addition, similarly to (8), the flipped cross-cross-correlation (or conv-cross-correlation) estimate can be also considered, that is

$$F_{ijlm}(\delta) = \frac{1}{2T} \int_{-T}^{T} R_q(\tau) R_p(\delta - \tau) d\tau, \tag{9}$$

with $q, p = 0, \ldots, Q-1$ ($p > q$). Note that, in (9), the second cross-correlation is time-reversed.

Now, representing all the combinations of (q, p) as row and column indices of a $Q \times Q$ square matrix, the combinations so that $p > q$ in (8)-(9) are located under the main diagonal of the matrix. Then, the total number of admitted combinations of all (both direct and flipped) cross-cross-correlations is $L = 1/4M^4 - 1/2M^3 - 1/4M^2 + 1/2M$.

As observed before, in the noise-free case the apex of the magnitude of the cross-cross-correlation, $|C_{ijlm}(\delta)|$, should be at the index $t_i - t_j - t_l + t_m$, while that of $|F_{ijlm}(\delta)| = |C_{ijml}(\delta)|$ should be at the index $t_i - t_j + t_l - t_m$. Hence, we are now able to estimate the $M-1$ delays in the MMSE sense solving the overdetermined system made by the L equations, consisting of the linear combination of the $M-1$ unknowns equal to the index of the maximum of the standard and flipped cross-cross-correlations considered in (8) and (9), that is

$$t_i - t_j - t_l + t_m = \bar{\delta}_{ijlm}, \tag{10}$$

with $i, j, l, m = 0, \ldots, M-1$ ($j > i$ and $m > l$), and

$$t_i - t_j + t_l - t_m = \breve{\delta}_{ijlm}, \tag{11}$$

with $i, j, l, m = 0, \ldots, M-1$ ($j > i$ and $m > l$), where

$$\bar{\delta}_{ijlm} = \arg\max_{\delta} \left\{ |C_{ijlm}(\delta)| \right\}, \tag{12}$$

and

$$\breve{\delta}_{ijlm} = \arg\max_{\delta} \left\{ |F_{ijlm}(\delta)| \right\}. \tag{13}$$

Again, resorting to a compact matrix form, (10)-(11) can be rewritten as

$$\boldsymbol{B}\boldsymbol{t} = \boldsymbol{\delta}, \tag{14}$$

with

$$\boldsymbol{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_{M-1} \end{bmatrix}, \qquad \boldsymbol{\delta} = \begin{bmatrix} \bar{\delta}_{0102} \\ \vdots \\ \bar{\delta}_{(M-3)(M-1)(M-2)(M-1)} \\ \breve{\delta}_{0102} \\ \vdots \\ \breve{\delta}_{(M-3)(M-1)(M-2)(M-1)} \end{bmatrix}.$$

The procedure to construct the model matrix $\boldsymbol{B}$ of size $L \times (M-1)$ is given in [12]. Therefore, the solution to (14) is obtained applying the pseudo-inverse of $\boldsymbol{B}$. It is important to note that $\boldsymbol{B}$ depends only on the number of sensors M, therefore it can be computed and a priori stored. Indeed, its pseudo-inverse can be computed off-line, with the matrix $\boldsymbol{B}^T\boldsymbol{B}$ of size $(M-1) \times (M-1)$ that can be easily inverted due to its reduced dimension. These considerations allow to frame the proposed method as a fast algorithm useful from practical applications.

Finally, in addition to the above procedure, a second version of the proposed algorithm, indicated as cross-cross-correlation 2 (CCC2), is considered. It exploits all the linear equations of both the systems above defined

$$\boldsymbol{C}\boldsymbol{t} = \boldsymbol{\xi}, \tag{15}$$

where

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{B} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\tau} \\ \boldsymbol{\delta} \end{bmatrix}.$$

III. Performance Analysis

In this section the performance of the proposed methodology for estimating the time of arrival of signals received at M passive locating sensors is assessed. To do this, the considered figure of merit is the root mean square error (RMSE) of the estimated time of arrivals, which is theoretically given by

$$\text{RMSE} = \sqrt{\mathbb{E}\left[\left| \hat{t} - t \right|^2 \right]}. \tag{16}$$

Since a closed-form expression for the RMSE in (16) is not available, it has been extensively studied by performing Monte Carlo simulations made of 10^3 independent trials. In our tests, the transmitted signal $s(t)$ is assumed to be a zero-mean stationary Gaussian random process with unit variance and a Gaussian-shaped auto-correlation function given by [10]

$$\rho_s(\tau) = \exp\left(-\tau^2 / \sigma_a^2 \right),$$

where σ_a^2 is the variance of the auto-correlation function, corrupted by white Gaussian noise with the same variance $\sigma_i^2 = \sigma^2$ for all the M sensors. An observation period of

$T = 10^3$ s is used for the numerical tests, having assumed a unitary sampling time. The study cases considered herein comprise M sensors, and the $M-1$ time of arrivals have been picked-up at each Monte Carlo run as a realization of a uniform random variable within the interval $[0, 1]$ s, namely $t_i = \mathcal{U}[0, 1]$, $i = 1, \ldots, M-1$. The trials were parametrically run to investigate the performance versus the number M of sensors, the autocorrelation width σ_a, and the SNR level $1/\sigma^2$.

The analyses are conducted comparing the proposed technique based on the use of the cross-cross-correlation of the received signals (indicated with CCC in the following) with the classic method that exploits the cross-correlation only (referred to as CC). In addition, a further algorithm that utilizes both the systems of equations given by the two methods above (called CCC2) is also considered. The Cramèr Rao lower bound (CRLB) devised within the context of passive time delay estimation [9] is used as performance benchmark, referring to the expression developed by Schultheiss in [8]. Moreover, the ideal GCC [7], [11] which almost draws the CRLB is also considered as performance benchmark. In fact, it is based on the ideal assumption of a perfect a priori knowledge of the signal and noise spectra to derive the prefilter and hence hard to realize in many practical contexts.

In Figure 2 the RMSE is plotted versus the number of sensors for the above illustrated simulating scenario setting the auto-correlation width to $\sigma_a = 2$ and SNR $= 0$ dB.

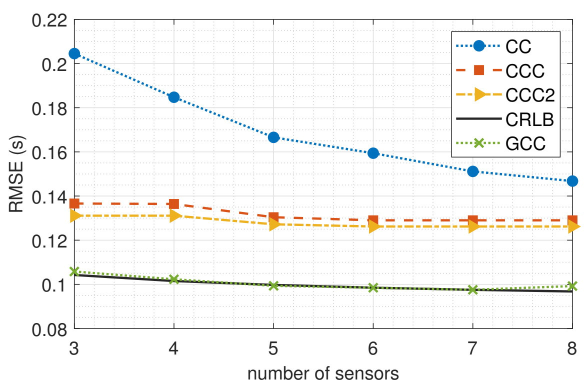

Figure 2. RMSE (s) of the delay estimate versus number of passive receiving sensors M.

The curves clearly show how a higher number of receiving sensors permits to reduce the delay estimation error. This trend is observed for both the new algorithms (CCC and CCC2) that gain over their classic counterpart being closer to the CRLB. This result is not surprising observing that the proposed algorithm acts an implicit prefiltering approximating that of the GCC. Moreover, the higher number of equations in the LS problem allows to obtain a more accurate solution with respect to the CC. In fact, the random errors of the peak estimates in the cross-correlation results to be reduced by the higher number of equations employed in the pseudosolution.

Similarly, Figure 3 shows the RMSE as a function of the signal's auto-correlation width for a given number of sensors ($M = 4$) and for SNR $= 0$ dB.

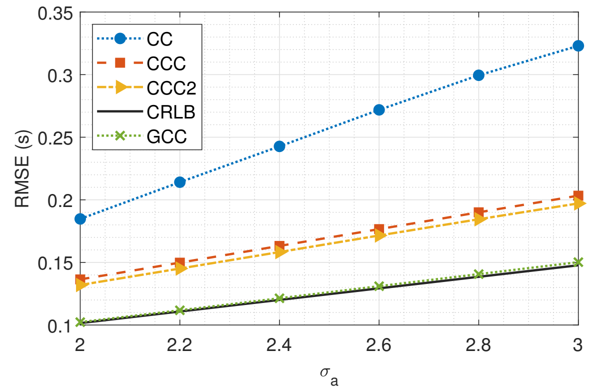

Figure 3. RMSE (s) of the delay estimate versus signal's auto-correlation standard deviation σ_a.

Interestingly, in this situation all estimators tend to experience almost a linear increment in the estimation error, but with a lower slope for the proposed techniques with respect to the classic one. Moreover, both the proposed version of the algorithm ensure almost the same performance.

The last analysis, whose results are depicted in Figure 4, shows the behavior of the quoted estimation methods as a function of SNR. As expected, the evidence is that all estimators tend to reduce their errors getting closer and closer to the CLRBs as the SNR increases. Moreover, as observed in the previous analysis, the proposed algorithms share better performance than their classic competitor.

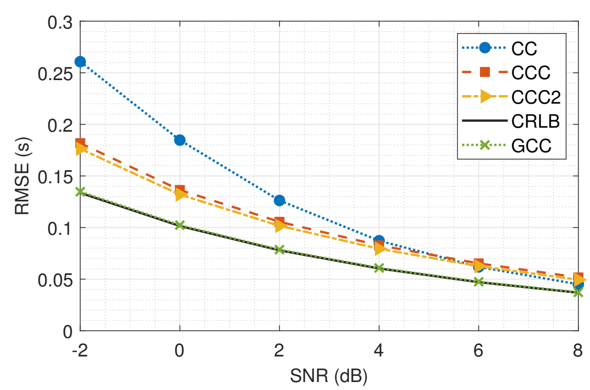

Figure 4. RMSE (s) of the delay estimate versus SNR.

Before concluding, it is worth to underline that the computational cost of the proposed method can be severely reduced if the cross-cross-correlations are computed from the cross-correlations windowed around their maxima. Moreover, also the cross-cross-correlations can be computed only for a limited number of lags in a neighborhood of the expected position of its maximum. By doing so, the resulting cost of the cross-cross-correlation method becomes close to that of the conventional one, and hence this algorithm can be defined fast.

IV. Conclusions

In this paper, a novel algorithm, suited for the undersea access in heterogeneous networks, has been applied and analyzed to accurately estimate the time delays from multiple passive underwater receivers. The advantages of the proposed method are its fast computation and the capability to operate without any a priori spectral knowledge. The effectiveness of its performance in comparison with classic counterparts and theoretical bounds is proved by simulations in several underwater acoustic operating cases. As a possible future work, it would be interesting to the extend the localization algorithm to scenarios characterized by the presence of faulty sensors [16].

References

[1] H. Yao, L. Wang, X. Wang, Z. Lu, and Y. Liu, "The Space-Terrestrial Integrated Network: An Overview," *IEEE Communications Magazine*, vol. 56, no. 9, pp. 178–185, 2018.

[2] F. Chen, Z. Jiang, F. Ji, H. Yu, Q. Guan, and F. Liu, "Radio-Acoustic Integrated Network for Ocean Information Transmission: Framework and Enabling Technologies," *China Communications*, vol. 18, no. 9, pp. 62–70, 2021.

[3] I. F. Akyildiz, D. Pompili, and T. Melodia, "Underwater Acoustic Sensor Networks: Research Challenges," *Ad Hoc Networks*, vol. 3, no. 3, pp. 257–279, 2005.

[4] D. Sun, J. Ding, C. Zheng, and W. Huang, "An Underwater Acoustic Positioning Algorithm for Compact Arrays With Arbitrary Configuration," *IEEE Journal of Selected Topics in Signal Processing*, vol. 13, no. 1, pp. 120–130, 2019.

[5] Q. Liang, B. Zhang, C. Zhao, and Y. Pi, "TDoA for Passive Localization: Underwater versus Terrestrial Environment," *IEEE Transactions on Parallel and Distributed Systems*, vol. 24, no. 10, pp. 2100–2108, 2012.

[6] Y. Cao, W. Shi, L. Sun, and X. Fu, "Frequency Diversity based Underwater Acoustic Passive Localization," *IEEE Internet of Things Journal*, 2021.

[7] C. Knapp and G. Carter, "The Generalized Correlation Method for Estimation of Time Delay," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 24, no. 4, pp. 320–327, 1976.

[8] P. Schultheiss, "Locating a Passive Source with Array Measurements: a Summary of Results," in *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 4. IEEE, 1979, pp. 967–970.

[9] A. Quazi, "An Overview on the Time Delay Estimate in Active and Passive Systems for Target Localization," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, no. 3, pp. 527–533, 1981.

[10] L. Miller and J. Lee, "Error Analysis of Time Delay Estimation using a Finite Integration Time Correlator," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, no. 3, pp. 490–496, 1981.

[11] M. Azaria and D. Hertz, "Time Delay Estimation by Generalized Cross Correlation Methods," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 32, no. 2, pp. 280–285, 1984.

[12] L. Pallotta and G. Giunta, "Accurate Delay Estimation for Multisensor Passive Locating Systems Exploiting the Cross-Correlation Between Signals Cross-Correlations," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 58, no. 3, pp. 2568–2576, 2022.

[13] S. Sun, X. Zhang, C. Zheng, J. Fu, and C. Zhao, "Underwater Acoustical Localization of the Black Box Utilizing Single Autonomous Underwater Vehicle based on the Second-Order Time Difference of Arrival," *IEEE Journal of Oceanic Engineering*, vol. 45, no. 4, pp. 1268–1279, 2019.

[14] S. Sun, C. Zhao, C. Zheng, C. Zhao, and Y. Wang, "High-Precision Underwater Acoustical Localization of the Black Box Based on an Improved TDOA Algorithm," *IEEE Geoscience and Remote Sensing Letters*, vol. 18, no. 8, pp. 1317–1321, 2020.

[15] S. Sun, S. Qin, Y. Hao, G. Zhang, and C. Zhao, "Underwater Acoustic Localization of the Black Box Based on Generalized Second-Order Time Difference of Arrival (GSTDOA)," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 59, no. 9, pp. 7245–7255, 2020.

[16] G. Giunta, L. Pallotta, and D. Orlando, "Detecting Sensor Failures in TDOA-based Passive Radars: A Statistical Approach based on Outlier Distribution," *IEEE Transactions on Aerospace and Electronic Systems*, 2022.