



COMPLEX
NETWORKS

COMPLEX NETWORKS 2021

THE 10TH INTERNATIONAL CONFERENCE
ON COMPLEX NETWORKS
AND THEIR APPLICATIONS

November 30 - December 02 , 2021
Madrid, Spain

BOOK OF ABSTRACTS

COMPLEX NETWORKS 2020

The 10th International Conference on Complex Networks & Their Applications

November 30 -December 2, 2021 Madrid, Spain - Online

Published by the International Conference on Complex Networks & Their Applications

Editors

Rosa María Benito

Universidad Politécnica de Madrid, Spain

Hocine Cherifi

University of Burgundy, France

Chantal Cherifi

University of Lyon, France

Esteban Moro

Universidad Carlos III, Spain

Luis Mateus Rocha

Indiana University, USA

Marta Sales-Pardo

Universitat Rovira i Virgili, Spain

COMPLEX NETWORKS 2020

e-mail: hocine.cherifi@u-bourgogne.fr

Copyright Notice COMPLEX NETWORKS 2021 and the Authors

This publication contributes to the Open Access movement by offering free access to its articles and permitting any users to read, download, distribute, print, search, or link to the full texts of these articles, crawl them for indexing, pass them as data to software. The copyright is shared by authors and the 10th International Conference on Complex Networks & Their Applications (COMPLEX NETWORKS 2021) to control over the integrity of their work and the right to be properly acknowledged and cited.

To view a copy of this license, visit <http://www.creativecommons.org/licenses/by/4.0/>

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use. While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained her.

ISBN: 978-2-9557050-5-6

Table of Contents

Tutorials

Psychology-informed Recommender Systems	2
<i>Elisabeth Lex¹ and Markus Schedl²</i>	
A crash-course in TDA and higher-order dynamics	4
<i>Giovanni Petri</i>	

Invited Speakers

Challenges in spatial networks	6
<i>Marc Barthélémy</i>	
Higher-order networks and their dynamics	7
<i>Ginestra Bianconi</i>	
Mining Evolving Large-Scale Networks	9
<i>João Gama</i>	
How Networks Can Change Everything for Better or for Worse	10
<i>Dirk Helbing</i>	
Graph-based Neural ODEs for Learning Dynamical Systems	11
<i>Yizhou Sun</i>	
Computational Epidemiology at the time of COVID-19	12
<i>Alessandro Vespignani</i>	

I Biological Networks

A systematic evaluation of the impact of environmental exposures on the human interactome network	14
<i>Salvo Danilo Lombardo and Jörg Menche</i>	
Percolation on the gene regulatory network	18
<i>Giuseppe Torrisi, Reimer Kühn and Alessia Annibale</i>	
The effective graph: a weighted graph that captures nonlinear logical redundancy in biochemical systems	22
<i>Alexander Gates, Rion Brattig Correia, Xuan Wang and Luis M. Rocha</i>	

ETMM: Egocentric temporal motifs miner	291
<i>Antonio Longa, Giulia Cencetti, Andrea Passerini and Bruno Lepri</i>	

X Network Analysis

Variance and covariance of probability distributions on networks	295
<i>Karel Devriendt, Samuel Martin-Gutierrez and Renaud Lambiotte</i>	
The first-mover advantage explains gender disparities in Physics citations	298
<i>Hyunsik Kong, Samuel Martin-Gutierrez and Fariba Karimi</i>	
Spectral Pruning of Fully Connected Layers: Ranking the Nodes Based on the Eigenvalues	301
<i>Lorenzo Giambagli, Lorenzo Chicchi, Lorenzo Buffoni, Enrico Civitelli and Duccio Fanelli</i>	
A network approach to identification of individual traits in animal groups	305
<i>Fanqi Zeng, Martin Homer and Thilo Gross</i>	
Complex Networks Analysis of Solar Magnetograms along the 23rd Solar Cycle.....	308
<i>Víctor Muñoz and Eduardo Flández</i>	
Empirical Analysis of Acknowledgment Network and Citations.....	311
<i>Keigo Kusumegi and Yukie Sano</i>	
Graph signal processing and wavelet packets.....	314
<i>Iulia Martina Bulai and Sandra Saliiani</i>	
Recommender systems increase exposure diversity. Or do they? A complex networks approach.....	318
<i>Augustin Godinot and Fabien Tarissan</i>	
Analyzing the physical dependence of scale-free critical exponent in seismic complex networks	321
<i>Fernanda Martin and Denisse Pasten</i>	
Assigning Degree of Irreversibility to Light Curves from Blazars through the Horizontal Visibility Graph Method	324
<i>Belén Acosta-Tripailao, Walter Max-Moerbeck, Denisse Pastén and Pablo S. Moya</i>	
On the use of Null Models to Assess Statistical Significance in Bipartite Networks.....	327
<i>María Palazzi, Aniello Lampo, Albert Sole and Javier Borge-Holthoefer</i>	

Graph signal processing and wavelet packets

Iulia Martina Bulai and Sandra Saliani

Dipartimento di Matematica, Informatica ed Economia
Università degli Studi della Basilicata, ITALY
iulia.bulai@unibas.it, sandra.saliani@unibas.it

1 Introduction

Nowadays graphs became of significant importance given their use to describe complex system dynamics, with important applications to real world problems, e.g. graph representation of the brain, social networks, biological networks, spreading of a disease, etc., [1]-[5].

What is missing in graph signal processing is a general definition of Spectral Graph Wavelet Packets Transform in the same fashion as for the classical framework [6] (see also [7]), where the equivalent of frequency is represented by the eigenvalues of the Laplacian matrix. Bremer and coauthors in [8] introduced diffusion wavelet packets transforms starting from diffusion wavelet definition [9], based on a diffusion operator T on a manifold or a graph. Cloninger et al. in [10] defined the natural graph wavelet packet dictionaries by introducing a set of novel multiscale basis transforms by considering the distance between graph Laplacian eigenvectors.

In this paper we introduce a novel graph wavelet packets construction, to our knowledge different from the ones known in literature. Our work is inspired by the Spectral Graph Wavelet Transform (SGWT) defined by Hammond et al. in [11] and can be viewed as a generalization of their work. The result is a dictionary of frames particularly suitable for analyzing signals defined on graphs with a large number of nodes.

We will give some concrete examples on how the wavelet packets can be used for compressing, denoising and reconstruction by considering a signal, given by the fMRI (functional magnetic resonance imaging) data, on the nodes of voxel-wise brain graph \mathcal{G} with 900.760 nodes (representing the brain voxels) defined in [1]-[2].

2 Main Results

2.1 Spectral wavelet operator and wavelet spaces

Let \mathcal{G} denote an undirected connected weighted graph with N nodes, A the $N \times N$ (symmetric) adjacency matrix with non-negative real entries a_{ij} , where $a_{ij} > 0$ if there is an edge between vertices i and j . D denotes the diagonal $N \times N$ matrix with diagonal entries given by the degrees $d(i) = \sum_j a_{ij}$, and $\mathcal{L} = D - A$ is the non-normalized Laplacian. Alternatively one can consider the *normalized* Laplacian defined as in [1].

We shall denote by $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_{N-1}$ the eigenvalues of \mathcal{L} ordered in ascending order, including multiplicities, and by χ_ℓ , $\ell = 0, \dots, N-1$ a set of corresponding orthonormal eigenvectors. Let $g : [0, +\infty) \rightarrow \mathbb{R}$ be a continuous function. In

[11] the authors call the operator $T_g : \mathbb{R}^N \rightarrow \mathbb{R}^N$, $T_g = g(\mathcal{L})$, the *wavelet operator*, and for $s > 0$, $T_g^s = g(s\mathcal{L})$ is called the *wavelet operator at scale s*, i.e. the wavelet operator corresponding to the dilation of g by $s : g(s \cdot)$.

It follows from the definition that if χ_ℓ is an eigenvector of \mathcal{L} , corresponding to the eigenvalue λ_ℓ , then $T_g(\chi_\ell) = g(\lambda_\ell)\chi_\ell$. Also, for any $f \in \mathbb{R}^N$,

$$(T_g f)(m) = \sum_{\ell=0}^{N-1} g(\lambda_\ell) \hat{f}(\ell) \chi_\ell(m), \quad m \in \mathbb{R}^N. \quad (1)$$

In the classic case, wavelets are usually defined starting from a Multiresolution Analysis (MRA), $V_j \subset V_{j+1} \subset L^2(\mathbb{R})$. Translations of the scaling function φ span the core space V_0 , and the translations of the wavelet ψ span the orthogonal complement W_0 of V_0 in V_1 . In analogues way we define this spaces for the graph theory framework:

Definition 1. Let $(s_j)_{j \geq 1}$ be a strictly increasing sequence of positive real numbers (thought as scales), set $s_0 = 1$. The space V_0 is spanned by the finite linear combinations of translates of the low-pass filter, h , $V_0 = \langle T_h \delta_n, n = 1, \dots, N \rangle$. Hence

$$f \in V_0 \Leftrightarrow \hat{f}(\ell) = h(\lambda_\ell) \sum_{n=1}^N a_n \chi_\ell^*(n), \quad a_n \in \mathbb{C}.$$

The space W_0 is spanned by the finite linear combinations of translates of the band-pass filter, g , $W_0 = \langle T_g \delta_n, n = 1, \dots, N \rangle$.

Next, we define the spaces V_{-j} and W_{-j} for any $j \geq 1$. The space $V_{-j} = \langle T_h^{s_j} T_h^{s_{j-1}} \dots T_h \delta_n, n = 1, \dots, N \rangle$. The space $W_{-j} = \langle T_g^{s_j} T_g^{s_{j-1}} \dots T_g \delta_n, n = 1, \dots, N \rangle$.

Hence, following the same notation and the reasoning used for the wavelet spaces, we can define the *wavelet packets spaces*.

Definition 2. We define $W_{0,0} = V_0$, $W_{1,0} = W_0$, and, for any $j \geq 1$, $W_{0,-j} = V_{-j}$, $W_{1,-j} = W_{-j}$. For any $j \geq 1$, we write $n \in \mathbb{N}$ in finite dyadic expansion $n = \sum_{i=0}^j \varepsilon_i 2^i$, $\varepsilon_i = 0, 1$ we define (set $s_0 = 1$) $W_{n,-j} = \langle T_{M_{\varepsilon_0}}^{s_0} T_{M_{\varepsilon_1}}^{s_1} \dots T_{M_{\varepsilon_j}}^{s_j} \delta_k, k = 1, \dots, N \rangle$, where

$$M_{\varepsilon_i} = \begin{cases} h, & \text{if } \varepsilon_i = 0, \\ g, & \text{if } \varepsilon_i = 1. \end{cases}$$

and $T_{M_{\varepsilon_i}}^{s_i} = M_{\varepsilon_i}(s_i \mathcal{L})$.

2.2 Frames of spectral graph wavelet packets

The tuple $\varepsilon = (\varepsilon_0, \varepsilon_1, \dots, \varepsilon_j)$ in the dyadic expansion of $n = 0, \dots, 2^{j+1} - 1$ corresponds to the dyadic interval

$$I_{j,n} = \left[\frac{\varepsilon_j}{2} + \dots + \frac{\varepsilon_0}{2^{j+1}}, \frac{\varepsilon_j}{2} + \dots + \frac{\varepsilon_0}{2^{j+1}} + \frac{1}{2^{j+1}} \right) \subset [0, 1)$$

Proposition 1. Let \mathcal{F} be a finite set of indices $(j, n) \in \mathbb{N} \times \{0, \dots, 2^{j+1} - 1\}$, such that the collection $\{I_{j,n}, (j, n) \in \mathcal{F}\}$ forms a partition of $[0, 1) : [0, 1) = \bigcup_{(j,n) \in \mathcal{F}} I_{j,n}$.

Assume $G(\lambda) = h^2(\lambda) + g^2(\lambda) > 0$ for all $\lambda \in \mathbb{R}^+$. Let $L \geq 0$ be the maximum of the set $\{j \in \mathbb{N}, (j, n) \in \mathcal{F}, \text{ for some } n\}$. Then the system $\{T_{M_{(j,n)}} \delta_k, (j, n) \in \mathcal{F}, k = 1, \dots, N\}$ is a frame for \mathbb{R}^N with lower and upper frame bound given respectively by

$$\min\{1, \min_{\lambda \in [0, \lambda_{\max})} G^{L+1}(\lambda)\} \quad \text{and} \quad \max\{1, \max_{\lambda \in [0, \lambda_{\max})} G^{L+1}(\lambda)\}. \quad (2)$$

Hence if $\{T_{M_i} \delta_k, i = 0, 1, k = 1, \dots, N\}$ is a Parseval frame for \mathbb{R}^N , then the introduced system is a Parseval frame for \mathbb{R}^N , too.

3 Conclusion

As an application we have used a brain graph with 900.760 nodes, an fMRI signal and its noisy version, represented on the top row of Figure 1. After computing the wavelet packet coefficients we processed the signals, e.g. reconstruction, compressing and/or denoising. For lack of space, here, we report only the results for denoising using two different filters, bottom row of Figure 1.

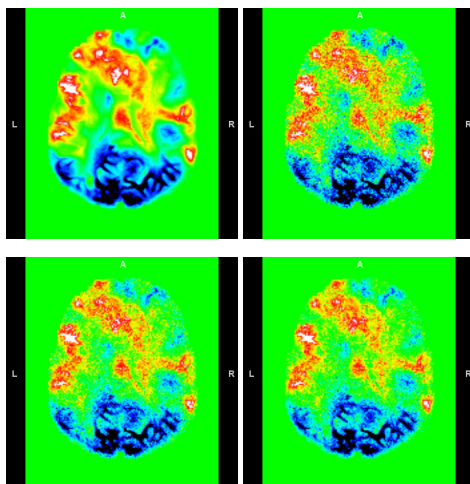


Fig. 1. Top row: (left panel) axial slices of the representation of the fMRI signal; (right panel) noisy fMRI signal on the brain graph. Bottom row: Reconstruction of the signal, from the left to the right after denoising at 33% for type-Meyer filter, and 54% spline filter, thresholding, respectively. The color spectrum assumes values from -0.002 black to 0.002 white.

References

1. A. Tarun, D. Abramian, M. Larsson, H. Behjat, and D. Van De Ville, Voxel-Wise Brain Graphs from Diffusion-Weighted MRI: Spectral Analysis and Application to Functional MRI, preprint (2021).
2. A. Tarun, H. Behjat, T. Bolton, D. Abramian, D. Van De Ville, (2021) Structural mediation of human brain activity revealed by white-matter interpolation of fMRI, *NeuroImage* 213 (2020) 116718.
3. F. V. Farahani, W. Karwowski, N. R. Lighthall, *Application of Graph Theory for Identifying Connectivity Patterns in Human Brain Networks: A Systematic Review*, *Frontiers in Neuroscience* **13** (2019) 585.
4. A. Fornito, A. Zalesky, E.T. Bullmore, *Fundamentals of Brain Network Analysis*, Academic Press (2016) .
5. S. P. Borgatti, M. G. Everett, J. C. Johnson, *Analyzing Social Networks*, SAGE Publications Ltd; Second edition (February 27, 2018).
6. R. R. Coifman, Y. Meyer, and V. Wickerhauser, *Size properties of wavelet-packets*, in *Wavelets and their applications*, Jones and Bartlett, Boston, MA, 453–470 (1992).
7. S. Saliani, *The solution of a problem of Coifman, Meyer, and Wickerhauser on wavelet packets*, *Constr. Approx.*, **33** (1) 15–39 (2011).
8. J. C. Bremer, R. R. Coifman, M. Maggioni, A. D. Szlam, *Diffusion wavelet packets*, *Applied and Computational Harmonic Analysis*, **2**(1) 95–112 (2006).
9. R. R. Coifman, M. Maggioni, *Diffusion wavelets*, *Applied and Computational Harmonic Analysis*, **21**(1) 53–94 (2006).
10. A. Cloninger, H. Li, and N. Saito, *Natural Graph Wavelet Packet Dictionaries*, *J Fourier Anal Appl* **27** 41 (2021).
11. D. K. Hammond, P. Vandergheynst , and R. Gribonval, *Wavelets on graphs via spectral graph theory*, *Appl. Comput. Harmon. Anal.* **30** (2011) 129–150.