

Sensor Failure Detection for TDOA-based Localization Systems

Gaetano Giunta

Dep. of Ind., Elect., and Mech. Eng.
University of Roma Tre
Rome, Italy
gaetano.giunta@uniroma3.it

Daniilo Orlando

Università degli Studi
"Niccolò Cusano"
Rome, Italy
daniilo.orlando@unicusano.it

Luca Pallotta

School of Engineering
University of Basilicata
Potenza, Italy
luca.pallotta@unibas.it

Abstract—This paper outlines a strategy for identifying sensors in a passive locating system that are not functioning properly. The framework is based on the information extracted from delay estimation errors, obtained from solving a system of equations in which the cross- and cross-cross-correlation methods are both used. Hence, we remove equations with the highest errors and use a statistical test to identify which sensor is experiencing failure. Our approach is analyzed through numerical simulations and real-recorded data, and compared to heuristic and conventional methods to prove its advantages.

Index Terms—cross-cross-correlation, failure detection, fourth-order moments, outlier measurements, SDR, TDOA.

I. INTRODUCTION

Passive radars often rely on time difference of arrival (TDOA) to estimate the position of non-cooperative targets [1]–[4]. This method involves computing the time of arrivals by identifying the time at which the highest correlation between the incoming signal and a reference one occurs [5]–[7].

Sometimes, in realistic contexts, outliers can impact the accuracy of certain measurements, which can result in a reduction of performance. A sensor may fail due to interference or physical damage. Therefore, it is crucial to ensure that the entire system correctly works. This involves not only removing outliers from measurements (as done in several references, e.g., [8], [9]) but also identifying if any sensors are faulty and excluding all measurements from them.

In this respect, the methodology followed in this paper allows to detect faulty sensors in passive radars. More precisely, this framework involves a two-stage algorithm that firstly identifies outliers and then determines whether a sensor is in failure. To identify outliers, we compute multiple cross-correlations (CCs) and cross-cross-correlations (CCCs) of signal replicas acquired at all receiving nodes. The given moments are utilized to create a set of equations, and their solutions are the delay estimates. Afterward, an approach for detecting and removing outliers, on the bases of the highest values of mean square error (MSE), is applied sequentially. A warning score is generated for each sensor based on the eliminated equations associated with it. These scores are then used to create a decision statistic that asserts the sensor under consideration is experiencing a failure. At last, the equations pertaining to the malfunctioning sensors are eliminated from the position estimation process.

Several tests are performed on simulated data as well as utilizing real-recorded data of frequency modulated (FM) signals acquired with a software defined radio (SDR) device. The presented numerical examples demonstrate the validity of the proposed approach also with respect to its competitor based on heuristic assumptions as well as the conventional estimator.

II. TDOA ESTIMATION IN MULTIPLE RECEIVERS PASSIVE RADAR

The considered system model refers to a localization system with $M > 2$ noncolocated sensors operating in an anechoic scenario. The signal samples collected at time instants nT by the i -th detection node is $r_i(nT) = \gamma_i s(nT - t_i) + w_i(nT)$, $i = 0, \dots, M - 1$, where t_i is the unknown delay at receiver, $s(\cdot) \in \mathbb{C}$ is the (not known) signal radiated by target, $\gamma_i \in \mathbb{C}$ is a parameter accounting for the transmitted power and channel effects, and $w_i(\cdot) \in \mathbb{C}$ is the noise component.

Resorting to a compact matrix form, the TDOA estimation problem can be written as [7]

$$\mathbf{X} \begin{bmatrix} t_1 \\ \vdots \\ t_{M-1} \end{bmatrix} = \mathbf{X} \mathbf{t} = \begin{bmatrix} \hat{b}_1 \\ \vdots \\ \hat{b}_Q \end{bmatrix}, \quad (1)$$

where $\hat{b}_1, \dots, \hat{b}_Q$ are Q measurements (with $Q > M$) evaluated from the position of the peak of the magnitude of all the possible correlations between the M signals, including the classical second-order CC and (optionally) the fourth-order CCC recently proposed in [7]. Moreover, the model matrix $\mathbf{X} \in \mathbb{R}^{Q \times (M-1)}$ has nonzero elements at the positions denoted by the indices of the signals involved in each correlation. Obviously, the value assumed by Q depends on the specific algorithm, viz. CC, CCC, or both of them (denoted as CCC2) [7]. The solution of the resulting overdetermined system (1) is obtained through the least squares (LS) approach.

III. PROPOSED FRAMEWORK FOR FAILURE DETECTION

Notice that each equation in (1) comes from the correlations between a group of receiving nodes, and an outlier equation provides a *warning* of a possible failure of the sensors associated with it. The warning score of each sensor is defined as

the number of equations associated with it and classified as outliers. When all sensors correctly work, the wrong equations (caused by noise) generate randomly distributed outliers; such distribution is expected uniform. On the other hand, when there is a faulty sensor, a higher value in its warning score is observed. This behavior motivates the idea of performing a test on the warning score for failure detection even under noisy scenarios.

Shortly, after the signal emitted by the target is acquired at the sensor nodes, their second- and fourth-order correlations are computed. They are then used to build-up a system of equations whose solution are the delay estimates of the incoming replicas. However, in practical scenarios, a high delay estimation errors could be observed for some of the equations, that appear to significantly differ from the remainders. This effect can be the consequence for instance of intentional or unintentional interference, or malfunctions of sensors/communication links. Then, the equations showing the highest errors are canceled from the equations system and the new solution is derived. In particular, firstly, the weak (low warning scores) and then the strong outlier equations are rejected. Finally, after the outlier equations cancellation, the detection of a failed sensor is performed as detailed in Subsection III-B. So, if a failure is declared, all the related measurements are rejected and the final delay estimate is derived. Conversely, in the absence of failures, only the outliers are removed.

A. Outliers cancellation

The applied algorithm evaluates the absolute error associated with the estimated delay vector \mathbf{t} , that is

$$\mathbf{e} = |\mathbf{X}\hat{\mathbf{t}} - \mathbf{b}|, \quad (2)$$

where \mathbf{b} is the measurements vector involved in the LS equations and detailed in [7], $\hat{\mathbf{t}}$ is the vector whose entries are the delay estimates, whereas $|\cdot|$ returns a vector containing the absolute values of each element in its input vector.

Outliers identification is performed comparing each value of \mathbf{e} with a threshold to decide for the presence of outlier. Hence, after these tests, all the equations in the system labeled as outliers (i.e., those for which the threshold is overcome) are removed from the entire set and a new reduced-size LS problem is derived. More details about the cancellation procedure can be found in [10].

B. Statistical method for failure detection

For each sensor, the warning score is computed by summing all warnings to it assigned. A sensor receives a warning when the removed equation derives from the computation of the CC and CCC in which its measurements are involved. Specifically, the following situations can arise:

- 2 sensors take a warning when the removed equation is related to a cross-correlation;
- 3 (resp. 4) sensors take a warning when the removed equation is related to a cross-cross-correlation derived from 3 (resp. 4) measurements.

It should be noticed now that, when no failures occur, all the warning scores are expected to be close to each others, with approximately the same amount of rejected equations. Differently, in the presence of a faulty receiver, its corresponding warning score should be much higher than the others. Therefore, under the null hypothesis H_0 , a fully-random model for the warning scores is assumed, where all the errors are independent of each other and due to random noise only (hence, the warnings can be assumed uniformly distributed). Denoting by N_2 (resp., N_3 and N_4) the number of equations labeled as outliers related to 2 (resp., 3 or 4) sensors, we can define the probability mass function (pmf) of the number of warning scores associated with each sensing node. More precisely, the warning scores associated to the rejection of an equation involving 2 (resp., 3 or 4) sensors is a binomial random variable with parameters N_2 and $2/M$ (resp., N_3 and $3/M$, or N_4 and $4/M$). As a matter of fact, $2/M$ (resp., $3/M$ and $4/M$) is the probability of selecting one sensor). As a consequence, for each sensor, the total number of warning scores is obtained as the sum of the three above that, under the assumption of independence, is a random variable whose pmf is given by the convolution between their pmfs [10].

Now, the resulting problem to discriminate between the presence or not of a failure, is defined for each receiving node as the following binary hypothesis test

$$\begin{cases} H_{F,0} : & \text{absence of failure} \\ H_{F,1} : & \text{presence of failure} \end{cases} \quad (3)$$

The above binary hypothesis testing problem rejects the null hypothesis by comparing the warning score x for sensor i (say $x^{(i)}$) to a specific significance level α , i.e.,

$$x^{(i)} \begin{cases} > \\ < \end{cases} \begin{matrix} H_{F,1} \\ H_{F,0} \end{matrix} \alpha, \quad (4)$$

with α a threshold set in the null hypothesis to ensure a desired percentile of false failure detections.

Another simple way to perform failure detection could consist in comparing the warning score of each sensor with a heuristic threshold, ζP , with P the overall number of equations associated with the quoted sensor and $\zeta \in [0, 1)$ a parameter tuning the capability of the method to inhibit a given sensor. This method is referred to as heuristic sensor failure (HSF) in the next analyses.

IV. PERFORMANCE ANALYSES

This section evaluates the effectiveness of the devised pipeline in detecting failures of the i -th sensor. To do this, the considered figure of merit is represented by the probability of correct failure detection ($P_{CFD}^{(i)}$). The simulations are carried out using a signal transmitted by the target ($s[n]$) consisting of 10^3 samples. The designed approach makes no assumptions about the nature of incoming signals, so the simulations use random Gaussian signals, which are the most common in practical scenarios. Therefore, the signal is generated as a

stationary zero-mean complex Gaussian random process with unit variance and a Gaussian-shaped auto-correlation function [11] with a variance of σ_a^2 , set equal to 2 in the experiments. At each of the $M - 1$ receiving nodes, a copy of the signal $s[n]$ is produced with a delay randomly chosen from the interval $[0, 1]$ second. Moreover, without loss of generality, the delay at the first sensor is set to 0 seconds for all the conducted experiments. The noise term is modeled as a white circularly symmetric complex Gaussian vector with the same variance of σ^2 for all M sensors, and $\gamma_i = 1$ for convenience, resulting in an SNR level of $1/\sigma^2$. Moreover, to account for a sensor failure, its received replica is modeled with only the noise component. Since we don't have closed-form expressions for the performance metric $P_{\text{CFD}}^{(i)}$, we adopt the Monte Carlo counting technique to estimate it reproducing $M_c = 10^3$ independent tests. The analysis of detection performance begins with Figure 1, which shows the plot of $P_{\text{CFD}}^{(i)}$ versus the SNR. Among the total of 8 available sensors, sensor $i = 3$ is selected to be under failure, i.e., its corresponding signals are enforced to contain the noise contribution only. The figure compares the statistical sensor failure (SSF) procedure of Section III applied to CC, CCC, and CCC2 (shortly denoted as SSF-CC, SSF-CCC, and SSF-CCC2), with respect to the HSF counterparts (denoted with the acronyms HSF-CC, HSF-CCC, and HSF-CCC2). For the SSF-based algorithms, the detection thresholds are chosen so as the nominal false failure detection probability is 10^{-2} , whereas for the HSF-based algorithms, ζ is set equal to $1/2$ for all simulations.

The results demonstrate that both SSF-CCC and SSF-CCC2 can accurately estimate failures. Even their heuristic counterparts perform well at low SNR values, even with fewer sensors (e.g., $M = 8$ sensors). However, as the SNR increases, the capabilities of these methods in correctly detecting a failure reduce. This can be explained by the fact that as the SNR increases, the LS provides a more accurate sub-optimal solution due to a better estimate of the correlation peaks, resulting in a reduction in the number of rejected equations. The observed detection losses are hence a result of the joint combination of the above phenomenon together with the fixed heuristic thresholds. On the other hand, the SSF-CCC and SSF-CCC2 can ensure the same level of detection regardless of the SNR by adaptively setting their thresholds based on the number of deleted equations. However, the SSF-CC produces the worst estimation. This is because the SSF-CC provides a low number of equations, which does not significantly differ from the number of rejected outliers across different sensors. This issue can be resolved by increasing the number of used sensors, which will result in a higher number of equations overall.

In the following tests, we create a partially-simulated environment utilizing a signal recorded through real measurements to emulate a passive radar system. To accomplish this, we exploit real-recorded data collected from a SDR device "R820T2 RTL2832U RTL-SDR MCX" [12]. The device is capable of recording signals in the frequency interval $[25, 1750]$

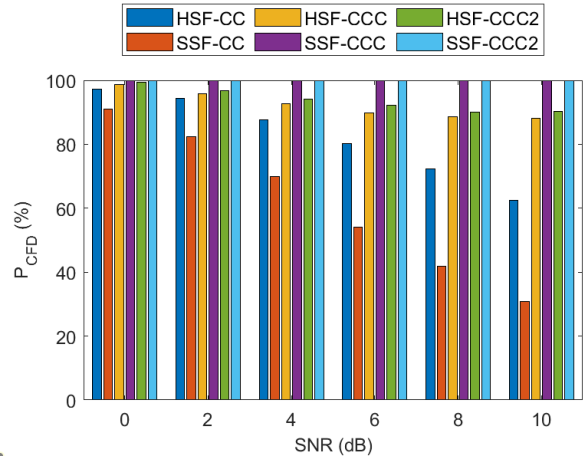


Figure 1: $P_{\text{CFD}} (\%)$ versus SNR for sensor 3 (over a total of $M = 8$ receiving nodes) when it is under failure.

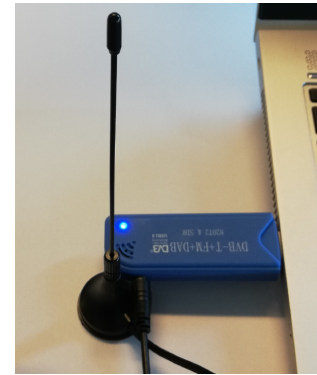


Figure 2: Utilized test-bed consisting of an RTL-SDR device connected through USB to a PC for real data recording.

MHz. Moreover, the received signal is firstly downconverted to an intermediate frequency of 3.57 MHz, and then digitized (using a 8bit ADC) and digitally downconverted to baseband to obtain the I&Q samples. Figure 2 shows the testbed used for signal acquisition consisting of the above RTL-SDR connected to a PC through a USB port.

A FM signal is acquired at central frequency of 103.8 MHz, setting the sample rate equal to 1.024 MHz. The signals have been filtered to select the specific FM channel using a low pass finite impulse response (FIR) filter with a normalized frequency of 0.1. Moreover, a decimation with factor 4 has been applied to reduce the number of samples to 10^3 I&Q samples. Figure 3 shows the power spectral density (PSD) obtained using the method of Welch of the three recorded signals.

By applying a random delay and adding Gaussian noise, we derive replicas for $M = 8$ sensors with specific SNR values. As for the simulated environment, Figure 4 displays the results in terms of P_{CFD} versus SNR. The two subplots show the case in which sensor 2 and sensor 3 experience failure, respectively.

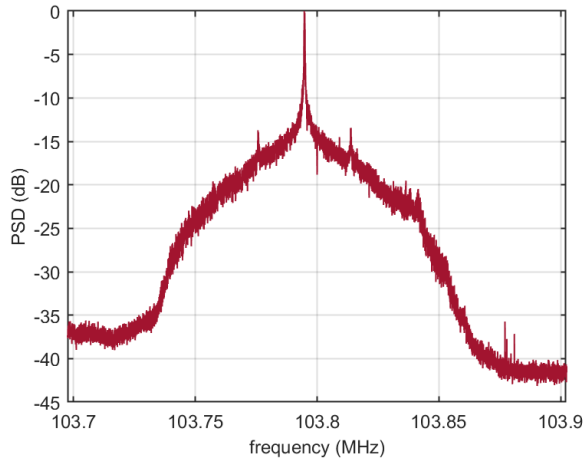


Figure 3: PSD (dB) of the FM signal acquired with the RTL-SDR device centered at 103.8 MHz.

It is interesting to note that the curves shares the same behavior observed over fully simulated data, with the SSF-CCC and SSF-CCC2 having the best performances in both scenarios.

Further confirmation of the previous test results can be seen in Figure 5, which reports the P_{CFD} for two faulty sensors as the SNR varies for the above described partially-simulated environment. The test is conducted using 10^2 Monte Carlo trials, and a challenging scenario featuring $M = 10$ sensors was considered with both sensors number 2 and 3 set in a faulty condition at the same time. Both SSF-CCC and SSF-CCC2 performed satisfactorily, while all other detectors struggled to identify a couple of failures. This is due to the fact that the other detectors often only reveal the presence of one failure among the two. As a result, these detectors, in which the threshold is a priori set, are unable to detect both types of failures simultaneously.

To finally demonstrate the advantages of the proposed algorithms for TDOA estimation under sensor failure condition, we estimate the corresponding RMSE by removing the delay associated with each sensor declared in failure. Furthermore, we report the Cramér-Rao lower bound (CRLB) for delay estimation to show a benchmark in the performance. Specifically, it is based on the CRLB derived for two sensors in the complex case [13] generalized to multiple sensors [14], [15]. The results are presented in Figure 6 and show the RMSE as a function of the number of sensors having set the SNR to 0 dB. For comparison purposes, we have also considered the CC, CCC, and CCC2 not performing sensor failure detection as summarized in Section II. In the given plot, subplot a) displays the performance of all sensors when they are working correctly. Classic methods such as CC, CCC, and CCC2 work better than their counterparts performing failure detection. However, the respective losses are still limited, especially for SSF-CCC and SSF-CCC2. In contrast, subplot b) shows that the classic CC/CCC/CCC2 methods cannot provide satisfactory estimation performance

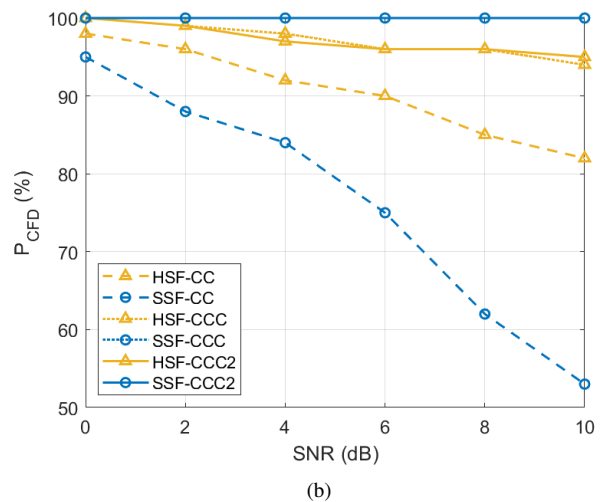
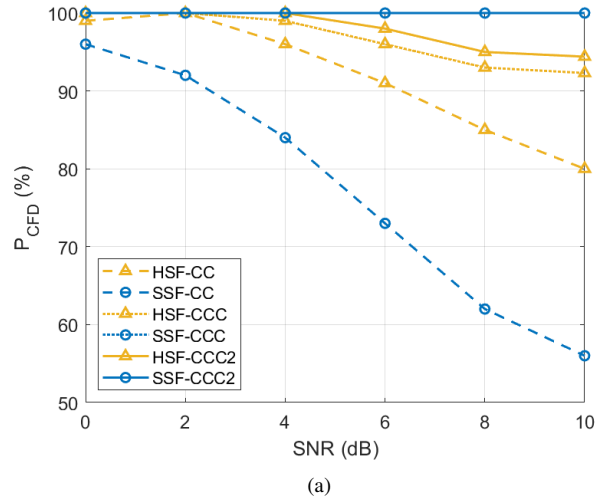


Figure 4: P_{CFD} (%) for faulty sensor versus SNR for real-recorded data; $M = 8$ sensors and sensor number a) 2 and b) 3 in failure.

if even one sensor is in failure. Both SSF-CCC and SSF-CCC2 show good performances with a significant gain over their heuristic counterparts. The graphs demonstrate that the RMSE decreases as more sensors are used, approaching the benchmark. The SSF-CCC2 method performs the best as it utilizes all the equations from the accurate sensors, resulting in improved delay estimations.

V. CONCLUSIONS

The paper suggested a new design for estimating delays in passive radars based on the processing of the signal emitted by a non-cooperative target, even when there might be sensor failures. The overall pipeline consists of two stages: in the first stage, some equations marked as outliers are canceled, and in the second one, a failure identification procedure based on the statistical behavior of warning scores is used to identify and discard measurements from one or more

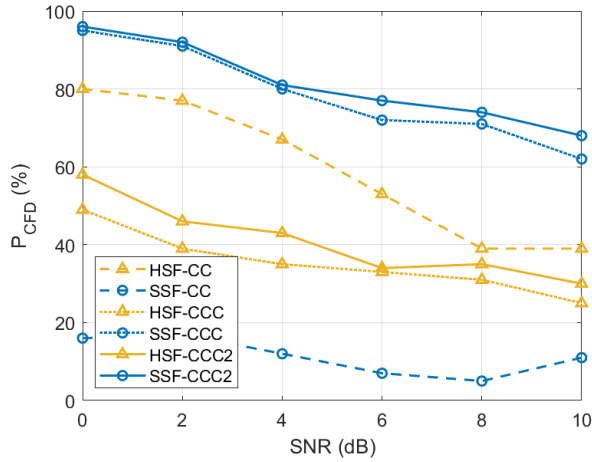


Figure 5: P_{CFD} (%) for faulty sensor versus SNR for real-recorded data; $M = 10$ sensors and sensors number 2 and 3 both in failure.

faulty sensors. Numerical simulations (also conducted starting from real measurements) have confirmed that the algorithm is successful in recognizing and dismissing a faulty sensor while accurately calculating the delays of the incoming signals. Additionally, it has been shown that the proposed methods are robust even in situations where no sensors are failing, with only minor deviations from the standard method. As possible future researches, it would be interesting to extend the analyses to sets of sensors under different operating principles.

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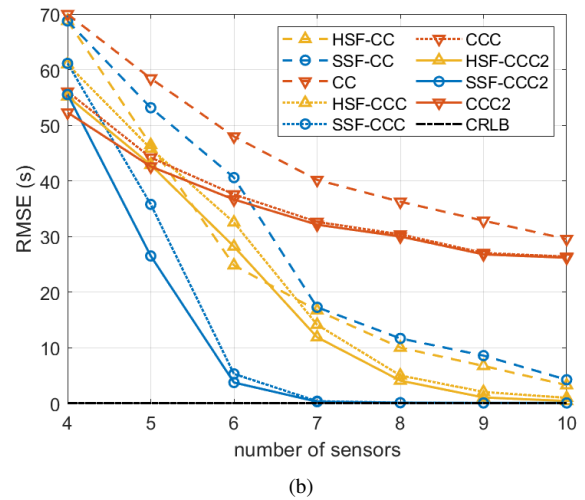
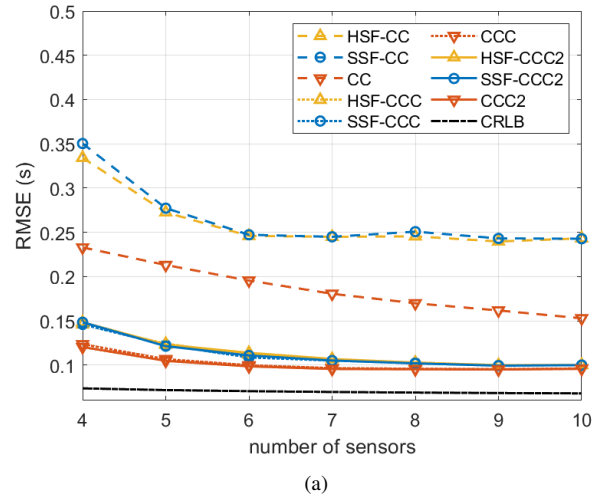


Figure 6: RMSE (s) of the delay estimate with respect to the number of passive receiving sensors M under a) $H_{F,0}$ and b) $H_{F,1}$ hypotheses for the different considered algorithms.

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