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Accurate Delay Estimation for Multi-Sensor Passive Locating Systems Exploiting the Cross-Correlation between Signals Cross-Correlations

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***Abstract*—The availability of accurate estimates of the delay or time of arrival (TOA) of the incoming signals is of paramount importance for the position estimation in passive radars with multiple receivers. This correspondence aims at improving estimation of the delays by multiple detectors exploiting the cross-correlation between the cross-correlation estimates (say cross-cross-correlation) of the received signals. The resulting equation system is formulated as a least squares (LS) minimization problem, whose solution is efficiently found computing the pseudo-inverse of the model matrix. In fact, the cross-cross-correlation implicitly performs a filtering operation on the considered signal, approximating the generalized cross-correlator behavior, without using statistical information about the signal spectra. The proposed method is numerically validated in comparison with classic counterparts and theoretical bounds.**

***Index Terms*—Delay estimation, generalized cross-correlation, time difference of arrivals (TDOA), cross-cross-correlation, passive locating systems, passive radar.**

I. Introduction

PASSIVE localization systems exploit the radar/communication signals emitted by a target to localize its position in the surrounding space. Hence, thanks to the absence of a dedicated transmitter, they are able to estimate the target's position without being in turn intercepted while maintaining a low implementation cost. [1]–[8]. In this context, a classic approach is the hyperbolic positioning based on the evaluation of the time difference of arrivals (TDOAs) of the electromagnetic signal emitted by the target/source to localize and recorded by a multitude of non co-located passive receiving sensors. In this respect, several algorithms and methodologies have been developed by the scientific community trying to efficiently solve the localization problem based on the TDOA measurements [9]–[13]. In particular, the above methods start from the availability of accurate estimates of the delay or time of arrival (TOA) associated with the signals acquired by the receiving sensors. Remarkably, this is the starting point also in underwater acoustics [14], [15], indoor/outdoor positioning by acoustic sources [16], and many other challenging applications. Additionally, other recent developments about parameters estimations within the frame of multi-receivers radar systems can be found in [17]–[21].

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A classic way to obtain an estimate of the delay between two replicas of a stationary signal received at two different spatial locations consists in computing their cross-correlation. The delay estimate is the time instant at which the maximum value of the cross-correlation function arises. A possible way to improve the estimation accuracy of the time delay has been developed in the seminal Knapp and Carter's paper [22]. It consists in applying a filter to the incoming signals just before the derivation of the cross-correlation. By doing so, the maximum value of such a filtered cross-correlation, referred to as generalized cross-correlation (GCC), provides the delay estimation between the two involved signals. The performance of GCC has been widely studied and analyzed in several works, e.g. [23]–[26]. However, it is strictly related to the availability of a-priori information about both signal and noise spectral statistics [22]. Therefore, its implementation is difficult in those applications in which the spectral properties of the incoming signals are not fully known or cannot be effectively estimated.

To overcome this drawback, in this correspondence, we devise a novel time delay estimator that, differently from the GCC, does not need any a-priori knowledge about the spectral contents of signal and noise processes. The method is based on the computation of the cross-correlation between each couple of cross-correlations performed on the received signals (called cross-cross-correlations). By doing so, the unknown time delays can be derived from the positions of the maximum values of the cross-cross-correlations through the formalization of a least squares (LS) problem whose solution is obtained through the application of the pseudo-inverse. It would be expected that the novel estimator performs better than the conventional one in the presence of a correlated signal and a relevant level of random noise for two basic reasons. Firstly, the random errors of cross-correlation peak's estimates (due to the additive noise effect) could be reduced by the higher number of equations employed in the pseudo-solution. In addition, if well estimated, each cross-correlation is just a shifted and scaled signal's auto-correlation. As shown in this correspondence, the second cross-correlation corresponds to a signal filtering that acts like the optimum GCC filter for white random noise at low signal-to-noise ratio (SNR) regimes.

Summarizing, the main contributions of the paper are:

- The design of a new time delay estimator based on the computation of the cross-cross-correlation.
- The demonstration that the devised method acts an implicit pre-filtering approximating that of the GCC, but

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without the need of a-priori information about the signal and noise spectra.

- The analyses of the possible advantages of the designed procedure through numerical simulations.

This correspondence is organized as follows. Section II formulates the delay estimation problem, providing the theory of the classic cross-correlation based estimation procedure. Then, the proposed method exploiting the cross-cross-correlations is also derived. Section III demonstrates the validity of the proposed method through numerical simulations also in comparison with classic counterparts. Finally, Section IV concludes the correspondence and provides suggestions for future developments.

A. Notation

We use boldface for vectors $\boldsymbol{a}$ (lower case) with $\boldsymbol{a}(i)$ its i-th entry, and matrices $\boldsymbol{A}$ (upper case). The transpose operator is denoted by the symbols $(\cdot)^T$, whereas $(\cdot)^{-1}$ is the matrix inverse. Then, $\mathbf{0}$ denotes the matrix with all zero entries (its size is determined from the context). $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{R}^N$ are, respectively, the sets of real numbers, complex numbers and N-dimensional vectors of real numbers. The letter j represents the imaginary unit (i.e. $j = \sqrt{-1}$), and for any real (resp. complex) number x, $|x|$ indicates its absolute value (resp. modulus), whereas $(\cdot)^*$ represents the conjugate of its complex-valued argument. Finally, $\mathbb{E}[\cdot]$ denotes statistical expectation.

II. System Model and Proposed Solution

A passive locating system composed by M sensors, whose physical displacements in the area of interest are not known, is herein considered. Figure 1 depicts a schematic of a possible displacement of the receiving sensors aimed at intercepting the signals emitted by the radar in order to localize it properly elaborating their respective TOAs at the central processing unit. It is also worth pointing out that the receivers are also assumed to not be co-located, otherwise the hyperbolic localization is not yet possible [27].

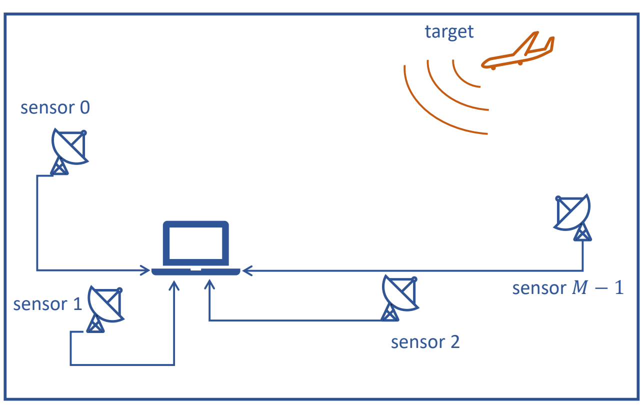

Figure 1. Pictorial representation of the considered passive receiving radar system.

More precisely, each receiving sensor acquires a delayed copy of the signal transmitted by the source (or target) to be localized. Then, the reference node elaborates all received signals to provide an estimate of the target position starting from the delay estimates. Therefore, indicating with $s(t)$ the random signal transmitted by the object to be identified, the signal received at the i-th sensing node can be described by means of the following equation [28]–[30]

$$r_i(t) = \alpha_i s(t - t_i) e^{j2\pi\nu_i t} + w_i(t), \quad i = 0, \dots, M-1, \tag{1}$$

where $\alpha_i \in \mathbb{C}$, $i = 0 \dots, M-1$, is a complex unknown scaling factor accounting for the channel propagation effects, as well as the distance between the transmitter and the i-th sensing node, ν_i, $i = 0 \dots, M-1$, is the Doppler frequency[1] at the i-th receiving node. Finally, $w_i(t)$, $i = 0 \dots, M-1$, is the thermal noise contribution at each receiving sensor modeled as a zero-mean complex Gaussian random variable with unknown variance σ_i^2, i.e. $w_i \sim \mathcal{CN}\left(0, \sigma_i^2\right)$, and assumed to be uncorrelated with the signal. Additionally t_i, $i = 0 \dots, M-1$, indicates the time delay or TDOA at each receiving node to be estimated, evaluated with respect to the delay of the first sensor, say t_0, assumed in the following, without loss of generality, equal to 0 s. Moreover, due to the random displacement of sensors, there is no a-priori known functional dependence between time instants t_i.

Indicating with τ the delay variable[2] representing the lag between the two signals (i.e. their respective delay), the cross-correlation $R(\tau)$ between two signals can be estimated, when the ergodicity property is verified, as

$$R_{ij}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} r_i(t) r_j^*(t - \tau) dt, \tag{2}$$

where T is the observation time. Before proceeding further, an important remark now occurs. Precisely, in this correspondence, the delay estimation problem and its solution are formalized by assuming a continuous-time processing chain. Nevertheless, the theoretic cross-correlation expressed by the integral in (2) can be numerically computed as sum of the two sequences sampled with a fixed rate to reduce the overall computational burden but set to ensure a negligible loss of information [31], [32]. Additionally, an interpolation can be applied on the final cross-correlations to reduce the estimation error while searching for their peaks.

The classic way to obtain an estimate of the delay difference consists in evaluating the peak position of cross-correlation magnitude (in the ideal noise-free case) derived by (2), that is

$$\hat{\tau} = \arg\max_{\tau} \left\{ |R_{ij}(\tau)| \right\}. \tag{3}$$

Now, considering all couples of sensors (i, j), $i, j = 0, \dots, M-1$, all the cross-correlation maxima can be properly

[1]The explicit dependence on Doppler frequency is no longer considered in the following algebraic derivations. In fact, in practical applications, the Doppler shift component is negligible for wideband radar sensors typically used to detect short signals with unknown bandwidth (the analysis bandwidth is more than some tens of MHz, even up to 1 GHz). Conversely, narrowband radars may need a preliminary procedure for Doppler compensation.

[2]It is also worth to underline that, even if τ depends on the considered couple, the subscripts ij on τ are omitted to simplify the notation.

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used to estimate the relative signals' delays acquired by the M sensors, viz. $\tau = t_i - t_j$. We can observe that $R_{ij}(\tau)$ produces some redundant estimates due to its intrinsic symmetric definition as well as reduces to the auto-correlation for $i = j$. Therefore, (2) is evaluated only for $j > i$ to eliminate all the above-mentioned redundant information. By doing so, the total number of admitted combinations of M sensors is

$$Q = \binom{M}{2} = \frac{1}{2}\left(M^2 - M\right). \tag{4}$$

To simplify the notation, in the following, each cross-correlation considered in (2) is numbered by a single subscript $q = 0, \dots, Q-1$, that is

$$\begin{aligned} R_q(\tau) = R_{ij}(\tau), \quad & q = 0, \dots, Q-1, \\ & i,j = 0, \dots, M-1 \,(\text{with } j > i). \end{aligned} \tag{5}$$

Then, it is possible to estimate the $M-1$ delays in the minimum mean square error (MMSE) sense by computing the apex of each cross-correlation magnitude and writing the corresponding equation as a linear combination of $M-1$ unknowns (i.e. the signal delays). More precisely, we can solve the overdetermined system made by the Q equations consisting of a linear combination of the $M-1$ unknowns equal to the index of the maximum of the cross-correlations considered in (2), that is

$$t_i - t_j = \hat{\tau}_{ij}, \quad i,j = 0, \dots, M-1 \,(j > i), \tag{6}$$

where

$$\hat{\tau}_{ij} = \arg\max_{\tau} \left\{ |R_{ij}(\tau)| \right\}. \tag{7}$$

Resorting to a more compact matrix form, (6) can be rewritten as

$$\boldsymbol{A}\boldsymbol{t} = \boldsymbol{\tau}, \tag{8}$$

with

$$\boldsymbol{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_{M-1} \end{bmatrix}, \qquad \boldsymbol{\tau} = \begin{bmatrix} \hat{\tau}_{01} \\ \vdots \\ \hat{\tau}_{(M-2)(M-1)} \end{bmatrix}$$

The model matrix $\boldsymbol{A}$ of size $Q \times (M-1)$ can be built as follows. Firstly, all possible cross-correlations R_{ij} are listed and sequentially re-numbered by the index q with $1 \leq q \leq Q$. The two corresponding vectors of size Q, $\boldsymbol{l}_1$ and $\boldsymbol{l}_2$, containing the values assumed by the indices i and j associated with the q-th cross-correlation, are introduced[3]. Then, the (q,k)-th element of $\boldsymbol{A}$ is

$$\boldsymbol{A}(q,k) = \begin{cases} 1 & \text{if } k = \boldsymbol{l}_1(q), \\ -1 & \text{if } k = \boldsymbol{l}_2(q), \\ 0 & \text{otherwise} \end{cases}$$

[3]As an example, for the case of $M = 3$ sensors (i.e., $Q = 3$), the cross-correlations are $\{R_{01}, R_{02}, R_{12}\}$, and consequently the indices are $\boldsymbol{l}_1 = [0,0,1]^T$ and $\boldsymbol{l}_2 = [1,2,2]$.

As a consequence, the solution to (8) is given by the pseudo-inverse of $\boldsymbol{A}$, that is

$$\hat{\boldsymbol{t}} = \left(\boldsymbol{A}^T\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T\boldsymbol{\tau}. \tag{9}$$

It is also worth observing that, if the SNR at a number F of sensors is not sufficient for a reliable cross-correlation computation, a sensor failure procedure could be applied. It would consist in identifying the sensors under failure and then discarding all the equations associated with them. In such a case the overall locating system would estimate the target position from $M - F > 2$ sensors in place of M with a consequent performance degradation. However, this topic is out for the focus of the proposed paper, and will be treated in future works.

A. Proposed method

This section describes the proposed solution to improve the delay estimate in the presence of $M > 2$ receiving sensors with unknown positions in the area of interest. In particular, the idea behind the proposed method lies in the fact that it may approximate the ideal GCC scheme with no a-priori knowledge about the signal spectra. Moreover, since the devised overdetermined system is capable to self correct estimation errors, it is expected that it is capable of improving the delay estimation with respect to the classic CC. This will be better clarified in the following and proved in the analyses section. Thus, let us first introduce the cross-cross-correlation estimate, i.e. the cross-correlation between two cross-correlations, that is

$$\begin{aligned} C_{ijlm}(\delta) =& \frac{1}{2T}\int_{-T}^{T} R_{ij}(\tau)R_{lm}^*(\tau-\delta)d\tau \\ =& \frac{1}{2T}\int_{-T}^{T} R_q(\tau)R_p^*(\tau-\delta)d\tau, \\ & q,p = 0,\dots,Q-1 \,(\text{with } p > q). \end{aligned} \tag{10}$$

Once again, the choice $p > q$ is performed to avoid redundant equations.

In addition, to increase the number of available equations, the flipped cross-cross-correlation estimate (that can be also seen as the convolution between the two cross-correlations) can be also considered, namely

$$\begin{aligned} F_{ijlm}(\delta) =& \frac{1}{2T}\int_{-T}^{T} R_{ij}(\tau)R_{lm}(\delta-\tau)d\tau \\ =& \frac{1}{2T}\int_{-T}^{T} R_q(\tau)R_p(\delta-\tau)d\tau, \\ & q,p = 0,\dots,Q-1 \,(\text{with } p > q), \end{aligned} \tag{11}$$

where the second cross-correlation is time-reversed with respect to (10).

Now, representing all the combinations of (q,p) as row and column indices of a $Q \times Q$ square matrix, the combinations so that $p > q$ in (10)-(11) are located under the main diagonal of the matrix. Then, the total number of admitted combinations of all (both direct and flipped) cross-cross-correlations is

$$L = 2\binom{Q}{2} = \frac{1}{4}M^4 - \frac{1}{2}M^3 - \frac{1}{4}M^2 + \frac{1}{2}M. \tag{12}$$

As observed before, in the noise-free case the apex of the magnitude of the cross-cross-correlation, $|C_{ijlm}(\delta)|$, should be at the index $t_i - t_j - t_l + t_m$, while that of $|F_{ijlm}(\delta)| = |C_{ijml}(\delta)|$ should be at the index $t_i - t_j + t_l - t_m$. Hence, we are now able to estimate the $M-1$ delays in the MMSE sense solving the overdetermined system made by the L equations, consisting of the linear combination of the $M-1$ unknowns equal to the index of the maximum of the standard and flipped cross-cross-correlations considered in (10) and (11), that is

$$t_i - t_j - t_l + t_m = \bar{\delta}_{ijlm}, \quad i,j,l,m = 0,\ldots,M-1\,(j>i \text{ and } m>l), \tag{13}$$

and

$$t_i - t_j + t_l - t_m = \breve{\delta}_{ijlm}, \quad i,j,l,m = 0,\ldots,M-1\,(j>i \text{ and } m>l), \tag{14}$$

where

$$\bar{\delta}_{ijlm} = \arg\max_{\delta}\left\{|C_{ijlm}(\delta)|\right\}, \tag{15}$$

and

$$\breve{\delta}_{ijlm} = \arg\max_{\delta}\left\{|F_{ijlm}(\delta)|\right\}. \tag{16}$$

Again, resorting to a compact matrix form, (13)-(14) can be rewritten as

$$\boldsymbol{B}\boldsymbol{t} = \boldsymbol{\delta}, \tag{17}$$

with

$$\boldsymbol{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_{M-1} \end{bmatrix}, \qquad \boldsymbol{\delta} = \begin{bmatrix} \bar{\delta}_{0102} \\ \vdots \\ \bar{\delta}_{(M-3)(M-1)(M-2)(M-1)} \\ \breve{\delta}_{0102} \\ \vdots \\ \breve{\delta}_{(M-3)(M-1)(M-2)(M-1)} \end{bmatrix}$$

The model matrix $\boldsymbol{B}$ of size $L \times (M-1)$ can be constructed as herein described. Firstly, all possible cross-cross-correlations C_{ijlm} are listed and sequentially re-numbered by the index r with $1 \leq r \leq L/2$. The four corresponding vectors of size $L/2$, $\boldsymbol{l}_1$, $\boldsymbol{l}_2$, $\boldsymbol{l}_3$, and $\boldsymbol{l}_4$ containing respectively the values assumed by the indices i, j, l, m associated with the r-th cross-cross-correlation, are hence introduced. Then, starting from $\boldsymbol{B} = \boldsymbol{0}$, the (r,k)-th element of $\boldsymbol{B}$ is obtained adding 1 if $k = \boldsymbol{l}_1(r)$, -1 if $k = \boldsymbol{l}_2(r)$, -1 if $k = \boldsymbol{l}_3(r)$, 1 if $k = \boldsymbol{l}_4(r)$. The same four vectors of indices are used for the flipped cross-cross-correlations $F_{ijlm} = C_{ijml}$ to fill the elements $L/2+1 \leq r \leq L$ of $\boldsymbol{B}$, after switching the rule by adding 1 if $k = \boldsymbol{l}_1(r)$, -1 if $k = \boldsymbol{l}_2(r)$, 1 if $k = \boldsymbol{l}_3(r)$, -1 if $k = \boldsymbol{l}_4(r)$. As a consequence, $\boldsymbol{B}$ results in a matrix made of several null elements and some non-zero elements equal to ± 1 and ± 2.

Therefore, the solution to (17) is obtained through the pseudo-inverse of $\boldsymbol{B}$, that is

$$\hat{\boldsymbol{t}} = \left(\boldsymbol{B}^T\boldsymbol{B}\right)^{-1}\boldsymbol{B}^T\boldsymbol{\delta}. \tag{18}$$

It is important to note that $\boldsymbol{B}$ depends only on the number of sensors M, therefore it can be computed and a-priori stored. Indeed, its pseudo-inverse can be computed off-line, with the matrix $\boldsymbol{B}^T\boldsymbol{B}$ of size $(M-1)\times(M-1)$ that can be easily inverted due to its reduced dimension that is the same as $\boldsymbol{A}^T\boldsymbol{A}$ in the classic cross-correlation. Moreover, indicating with D the number of available data, each equation for the complete correlation has a cost approximately equal to $C_1 = 3(2D)\log_2(2D)$ if computed through FFT. Each complete cross-cross-correlation has a cost of $C_2 = 3(4D)\log_2(4D)$. Being Q and L the number of equations in the two cases, the costs are QC_1 for the conventional and $QC_1 + LC_2$ for the new method, respectively. However, this latter can be severely reduced if it is computed from the cross-correlations windowed around their maxima. Hence, the new method approaches the cost of the conventional one if the cross-cross-correlations are computed after windowing with a neglecting additional cost. These considerations allow to frame the proposed method as a fast algorithm, very useful in practical applications.

Beyond the procedure described above, an additional version of the proposed algorithm is also considered. It exploits all the linear equations of both the systems defined in (8) and (17)

$$\boldsymbol{C}\boldsymbol{t} = \boldsymbol{\xi}, \tag{19}$$

where

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{B} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\tau} \\ \boldsymbol{\delta} \end{bmatrix},$$

whose solution is obtained with the pseudo-inverse of $\boldsymbol{C}$, that is

$$\hat{\boldsymbol{t}} = \left(\boldsymbol{C}^T\boldsymbol{C}\right)^{-1}\boldsymbol{C}^T\boldsymbol{\xi}. \tag{20}$$

Algorithm 1 summarizes the steps involved in the procedure for the proposed methods.

A final remark now occurs. Since the devised approach is based on the LS solution, it tends to find the best result accounting for all the involved measurements, without considering for the presence of possible outliers. In such a situation, possibly wrong delays could be detected in order to apply a sensor failure procedure, discarding completely erroneous measurements from the available data by deleting the corresponding equations from the (overdetermined) linear system set. This situation will be investigated in future developments.

III. Performance Assessment

In this section the performance of the proposed methodology for estimating the TOA of signals received at M passive locating sensors is assessed. To do this, the considered figure

Algorithm 1 Procedure for the proposed methods

Input: Received signals at each sensor r_i, $i = 0, \dots, M-1$;
Output: Estimated time delays for each sensor $\hat{t}_i$.

1) **Model Matrix Definition**
   - Compute the model matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of size $Q \times (M-1)$ and $L \times (M-1)$, respectively;
   - Build-up the model matrix $\boldsymbol{C} = \left[\boldsymbol{A}^T, \boldsymbol{B}^T\right]^T$;
2) **Measurements Acquisition**
   - Perform the measurements of the peaks' positions of the cross-correlations through (8) and store them in the vector $\boldsymbol{\tau}$;
   - Perform the measurements of the peaks' positions of the cross-cross-correlations and its flipped version through (16) and (17) and store them in the vector $\boldsymbol{\delta}$;
   - Construct the measurements vector $\boldsymbol{\xi} = \left[\boldsymbol{\tau}^T, \boldsymbol{\delta}^T\right]^T$.
3) **Solutions Computation**
   - Compute the solution with the pseudo-inverse of $\boldsymbol{B}$ through (19) if the first method is chosen.
   - Compute the solution with the pseudo-inverse of $\boldsymbol{C}$ through (21) if the second method is chosen.

of merit is the root mean square error (RMSE) of the estimated TOAs, which is theoretically given by

$$\text{RMSE} = \sqrt{\mathbb{E}\left[\left|\hat{t} - t\right|^2\right]}. \tag{21}$$

Since a closed-form expression for the RMSE in (21) is not available, it has been extensively studied by performing Monte Carlo simulations made of 10^3 independent trials. In our tests, the transmitted signal $s(t)$ is assumed to be a zero-mean stationary complex Gaussian random process with unit variance and having a Gaussian-shaped auto-correlation function given by [25]

$$\rho_s(\tau) = \exp\left(-\tau^2/\sigma_a^2\right),$$

where σ_a^2 is the variance of the auto-correlation function. The incoming signal is also corrupted by white circularly symmetric complex Gaussian noise sharing the same variance $\sigma_i^2 = \sigma^2$ for all the M sensors. Setting the number of available data to $D = 10^3$ (that is a reasonable value in the context of fast detection of unknown signals), an observation period of $T = 10^3$ s is used for the numerical tests, having assumed a unitary sampling frequency[4]. It is also worth to highlight here that, during the cross- and cross-cross-correlations evaluation from discrete signals, they are nearly ideally interpolated to emulate a continuous domain for delays, by a zero padding by a factor 1000 in the Discrete Fourier Transform (DFT) domain. The study cases considered herein comprise M sensors, and the $M-1$ time of arrivals have been picked-up at each Monte Carlo run as a realization of a uniform random variable within the interval $[0, 1]$ s, namely $t_i = \mathcal{U}[0, 1]$, $i = 1, \dots, M-1$. The trials have been parametrically run to investigate the performance versus the number M of sensors, the autocorrelation width σ_a, and the SNR level $1/\sigma^2$.

The analyses reported in this section are conducted comparing the proposed technique of (18) based on the use of the cross-cross-correlation of the received signals (dubbed CCC for short) with the classic method of (9) that exploits the cross-correlation only (referred to as CC), the classical GCC [22], and the GCC with phase transform (GCC-PHAT) [13], [33]. In addition, the algorithm of (20) that utilizes both the systems of equations given by the two methods above (called CCC2) is also considered.

Moreover, the Cramér-Rao lower bound (CRLB), devised within the context of passive time delay estimation when no a-priori information on the unknowns is available, is used as performance benchmark of the new estimators introduced in this paper. Precisely, referring to the expression provided by Schultheiss in [23], [24] for the case of multiple sensors, the CRLB, derived in [34] for two sensors in the complex case, modifies as:

$$\text{CRLB} = \frac{\pi}{T}\left\{\int_0^\infty \frac{M\omega^2 |G_{ss}(\omega)|^2/|G_{nn}(\omega)|^2}{1 + MG_{ss}(\omega)/G_{nn}(\omega)} d\omega\right\}^{-1}, \tag{22}$$

where $G_{ss}(\omega)$ and $G_{nn}(\omega)$ are the signal and noise power spectral density also indicated as autospectra, respectively.

Let us note that the GCC is based on the ideal assumption of a perfect a-priori knowledge of the signal and noise spectra to derive the prefilter. Hence, the CRLB in (22) is not a fair benchmark for the GCC estimator, although widely used in the literature [23]–[26].

The analytic expression of the GCC prefilter to be applied to the cross-correlation for the case at hand is [22], [26]

$$|H(\omega)|^2 = \frac{G_{ss}(\omega)}{2G_{ss}(\omega)G_{nn}(\omega) + |G_{nn}(\omega)|^2}. \tag{23}$$

Let us note that the expression in (23) tends to be proportional to $G_{ss}(\omega)$ (that is a scaled version of the true signal cross-spectrum magnitude) for large white noise. Therefore, before proceeding with the discussion of the simulation results, it is somehow useful to observe that the second cross-correlation utilized by the proposed algorithm implicitly performs a filtering operation on the considered signal. More precisely, its shape approximates that of the GCC pre-filter [22]–[26] in the presence of white noise and low SNRs. This is demonstrated by observing the filter responses of the proposed method and that of the GCC. They are reported in Figure 2 for some SNRs (viz., -3, 0, and 3 dB) and for $\sigma_a = 2$. Clearly, the filter response of the proposed method matches better and better with the optimum one as the SNR reduces.

In Figure 3 the RMSE is plotted versus the number of sensors for the above illustrated simulating scenario setting the auto-correlation width to $\sigma_a = 2$ and SNR $= 0$ dB.

The curves clearly show how a higher number of receiving sensors permits to reduce the delay estimation error. This trend is observed for both the new algorithms (CCC and CCC2) that gain over their classic counterpart being closer to the CRLB.

[4]For sake of simplicity of processing routines and without loss of generality, we have normalized all the involved quantity (i.e., observation time, sampling frequency, ...) to a unitary sampling period.

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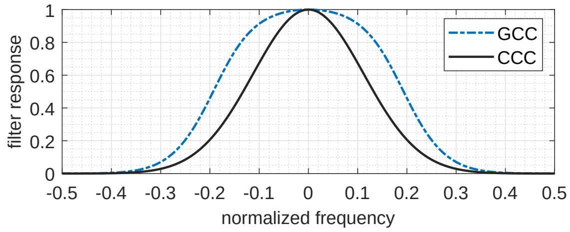
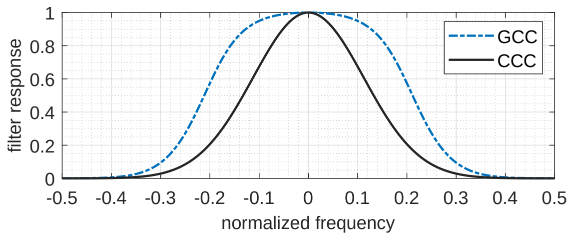
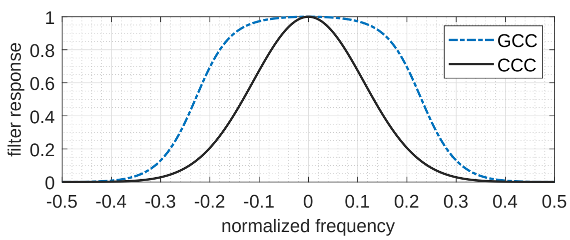


(a) SNR $= -3$ dB



(b) SNR $= 0$ dB



(c) SNR $= 3$ dB

Figure 2. Normalized magnitude of filter responses for the GCC and the proposed CCC for $\sigma_a = 2$. Subplots refer to a) SNR $= -3$ dB, b) SNR $= 0$ dB, c) SNR $= 3$ dB.

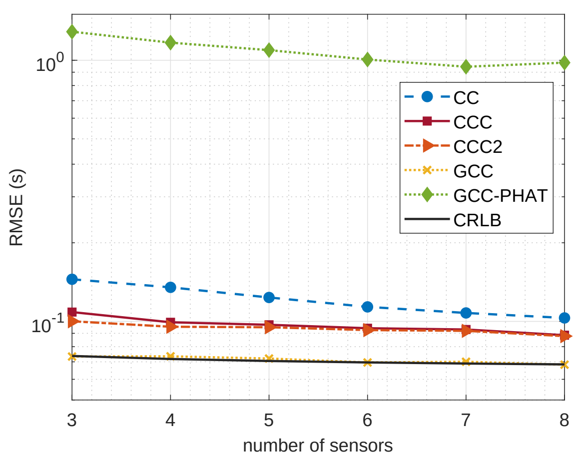


Figure 3. RMSE (s) of the delay estimate versus number of passive receiving sensors M.

This behaviour can be easily explained observing that as the number of sensors increases, the number of available equations in the LS problem grows.

Similarly, Figure 4 shows the RMSE as a function of the signal's auto-correlation width for a given number of sensors ($M = 4$) and for SNR $= 0$ dB.

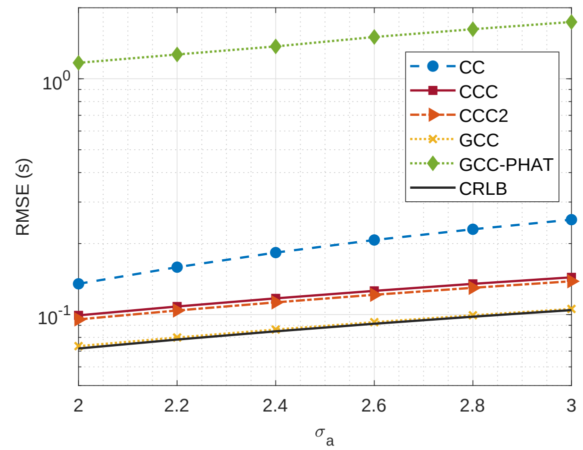


Figure 4. RMSE (s) of the delay estimate versus signal's auto-correlation standard deviation σ_a.

Interestingly, in this situation all estimators tend to experience almost a linear increment in the estimation error, but with a lower slope for the proposed techniques with respect to the classic one. Moreover, both the proposed version of the algorithm ensure almost the same performance.

The next analysis, whose results are depicted in Figure 5, shows the behavior of the quoted estimation methods as a function of SNR for $M = 4$ sensors and $\sigma_a = 2$. As expected, the evidence is that all estimators tend to reduce their errors getting closer and closer to the CLRBs as the SNR increases. Moreover, as observed in the previous analysis, the proposed algorithms share better performance than their classic competitor. These results suggests that the proposed approach can be successfully applied when a low number of sensor is available especially in low SNR regimes.

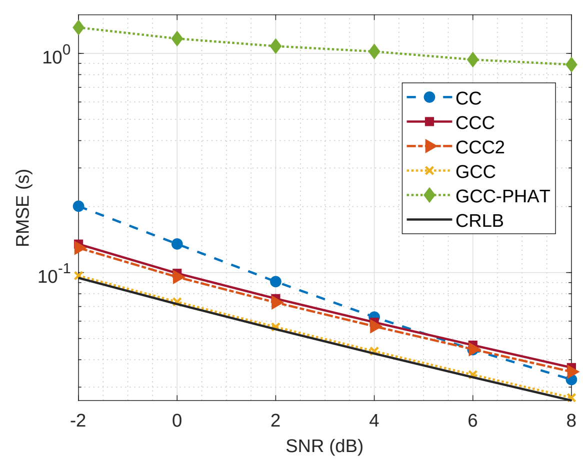


Figure 5. RMSE (s) of the delay estimate versus SNR.

Summarizing the above results, it could be claimed that the CCC has better performance than CC for a low SNR since it performs a filtering operation on the received signal which approximates the optimum pre-filter of the GCC, but without

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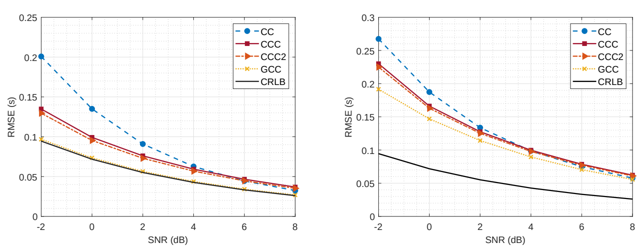
requiring a-priori information about the signal's bandwidth. Moreover, it has been observed that CCC and CCC2 allow to reach better estimation because they add further equations with respect to CC. Similarly, adding further sensors also increases the number of equations to both CC and CCC/CCC2 systems, that enhance robustness with respect to estimation errors. As a consequence, the performance of CC and CCC/CCC2 becomes closer for higher number of sensors. It can be explained observing that since the CCC and CCC2 start from a better point (i.e. lower RMSE), they improve slower than the CC.

The next analysis is devoted to study of the impact of multipath on the performance of the proposed method and its robustness in this challenging situation. As a matter of fact, both the proposed methods and their counterparts do not account for more than one receiving path at each node in their signal model. Therefore, the presence of multipath produces a model mismatch that in turn will have a deleterious effect on the performances of the estimators. To this aim, we evaluate the RMSE versus SNR in the presence of two replicas of the useful signal, i.e., the line of sight (LOS) signal and its reflected version. In particular, the second path is modeled as having one tenth of power with respect to the LOS and a uniform random phase in $[0, 2\pi]$. Results are given in Figure 6, where the evidence is that all the considered estimators experience a performance degradation due to the effect of the second path. Nevertheless, the trend between different curves observed in the LOS scenario is maintained also in this case.



(a) LOS (b) multipath

Figure 6. RMSE (s) of the delay estimate versus SNR in a multipath scenario. Subplots refer to a) LOS and b) multipath.

Another useful analysis regards the evaluation of the performance when a different signal is transmitted by the target. This is required to emphasize the effectiveness of the proposed method independently on shape and characteristics of the transmitted waveform. Therefore, Figure 7 reports the RMSE versus SNR, when a chirp signal having a normalized bandwidth equal to 0.4 is considered at the transmitting side. From the curves it is observed that even if performance degrades with respect to the case of random signals with Gaussian autocorrelation, the trend of the curves is the same. Moreover, the GCC shows a degradation performance such that now the proposed methods overcome it.

Finally, to show the impact of different SNRs on the estimation performance, a 2D localization scenario with 4 receiving sensor nodes is considered. Specifically, as shown in Figure 8, 3 radars are located at the vertices of an equilateral triangle with side equal to 500 m, and the reference sensor is

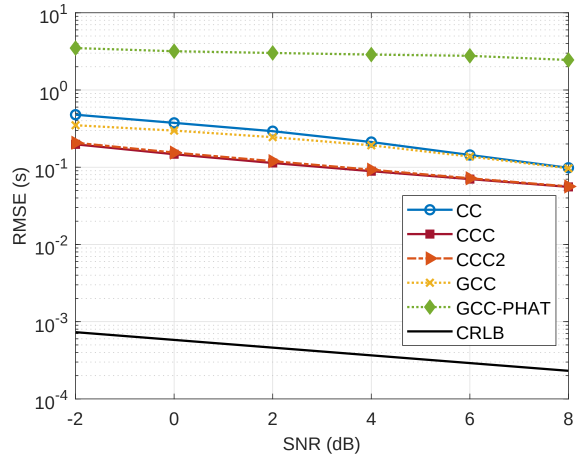


Figure 7. RMSE (s) of the delay estimate versus SNR for a transmitted chirp.

located at the origin of the system.

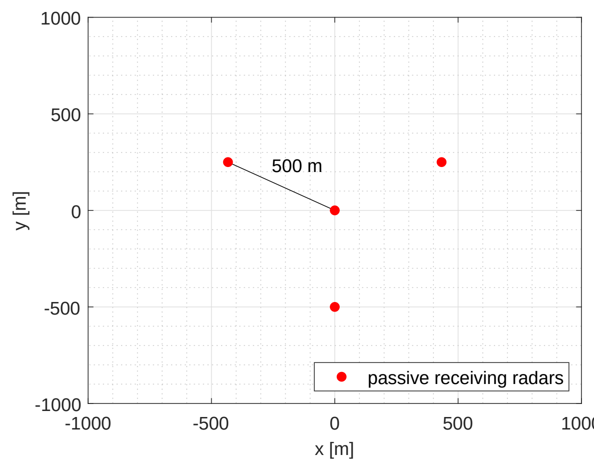
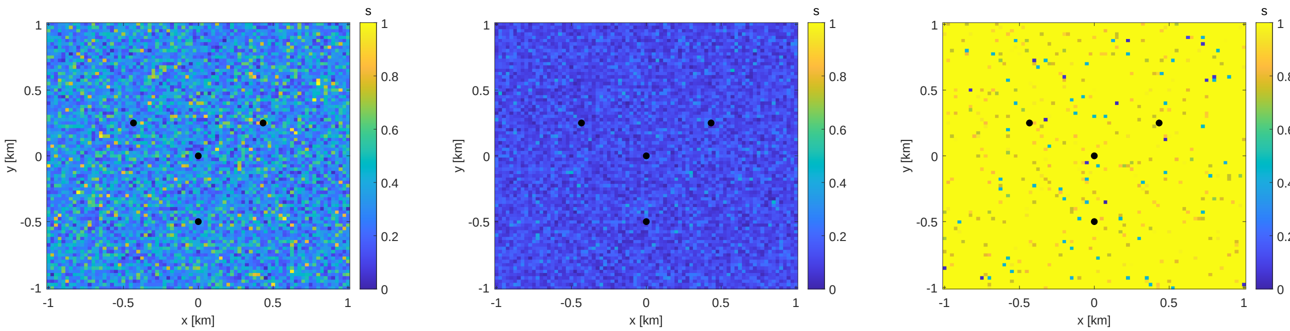


Figure 8. Geometric configuration of the sensor nodes.

Results are given in terms of RMSE maps in Figure 9 for CC, CCC, and GCC-PHAT. The values are computed setting the SNR at the reference sensor equal to $\text{SNR}_0 = 0$ dB and evaluating it at the other sensors position by means of the following re-scaling:

$$\text{SNR}_i = \text{SNR}_0 \frac{d_0^2}{d_i^2}, \quad i = 0, \ldots, M-1,$$

with d_i, $i = 0, \ldots, M-1$, the distance between target position and the i-th sensor. A grid of 4 square km (with a resolution of 25 m) centered at the reference sensor is considered and, for each point, the overall delays RMSE is computed. The radars positions are indicated with black dots in the maps. The results highlight that the overall error in estimating the involved delays is lower when the CCC algorithm is used, coherently with the results observed in previous tests.



(a) CC (b) CCC (c) GCC-PHAT

Figure 9. RMSE (s) map of the delay estimate considering $\text{SNR}_0 = 0$ dB.

IV. Conclusions

In this correspondence, a new algorithm capable of accurately estimating the time delays from multiple passive receiving sensors whose location is not a-priori known has been designed. The proposed procedure is based on the computation of the cross-correlation between each couple of cross-correlations performed on the replicas of the transmitted signal acquired at each sensing node. The resulting system of equations has been formalized as a LS problem whose solution can be found through a pseudo-inverse derived off-line. The main advantages of the proposed method are its fast computation and its effectiveness without any (a-priori or estimated) knowledge about signal and noise statistical spectra. Finally, numerical simulations have demonstrated the efficiency and effectiveness of the proposed method, approaching the performance of the generalized cross-correlator, also in comparison with classic counterparts.

Possible future researches might concern the optimization of the LS solution possibly accounting for the presence of outliers. Additionally, the proposed method could be applied to more sophisticated signal models, e.g. accounting for the presence of multipath (for instance increasing the number of unknowns in the system of equations), as well as to perform tests on real-recorded radar data. Finally, the extension of the devised model when a phased array (beamforming) at each receiving node is available would be also of interest.

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