Power-Aperture Product Resource Allocation for Radar ISAC

Augusto Aubry
DIETI
University of Naples "Federico II"
Napoli, Italy
augusto.aubry@unina.it

Antonio De Maio
DIETI
University of Naples "Federico II"
Napoli, Italy
ademaio@unina.it

Luca Pallotta
School of Engineering
University of Basilicata
Potenza, Italy
luca.pallotta@unibas.it

***Abstract*—This article deals with the problem of power aperture product (PAP) management in a multifunction phased array radar (MPAR) performing sensing in both line of sight (LOS) and non line of sight (NLOS), and communications. To this end, two different quality metrics are introduced, namely the range where the cumulative detection probability (for sensing) and the channel capacity per bandwidth (for communications) attain a specified value. Then, suitable utility functions are defined to map the quality index relative to the corresponding perceived utility for each task. The resource allocation is hence formulated as a constrained optimization problem whose solution optimizes the global radar quality of service (QoS). The method is finally validated by means of numerical simulations.**

***Index Terms*—integrated sensing and communication (ISAC), multifunction phased array radar (MPAR), radar resource management.**

I. Introduction

In the modern battlefield, the inherent flexibility of the electronically steered array (ESA) can be exploited by the multifunction phased array radar (MPAR) to simultaneously perform many operations, e.g., search, target recognition, communication, electronic counter-counter measure (ECCM) [1], [2]. This is realized thanks to the ESA capabilities of electronically steering a multitude of beams at several different angles with negligible delays. In this context, the radar resource manager (RRM) is devoted to the perform a dynamic scheduling along with the selection and optimization of the parameters, after the assignment of priorities to each function and/or task [3]. Accordingly, to each task a specific amount of the available resources is assigned. However, during the assignment of the resources, the RRM must adhere to physical and technical constraints due to the limited resource budget and performance constraints. Hence, the RRM must decide the optimal allocation of controllable resources to ensure the quality required by the various tasks while respecting their priorities. During the management process, it is necessary to determine an appropriate merit factor for each correlated activity and the relative utility function to achieve an optimized distribution of the degrees of freedom by solving a constrained optimization problem [4].

In this work, an optimized quality-of-service (QoS) solution is derived to properly distribute resources with reference to a MPAR system operating in a challenging integrated sensing and communication (ISAC) scenario [5], [6]. To do this, following the line of reasoning of [4], for each involved task, their respective quality metrics are defined together with their utility functions. As to the specific tasks considered in this paper, they are line of sight (LOS) search, non line of sight (NLOS) search supported by the reflective intelligent surfaces (RIS), and multi-user communication (COM). Therefore, setting the power-aperture product (PAP) as limited resource to be distributed, the resulting resource allocation scheme is formulated as a constrained optimization problem, whose solution allows the maximization of the radar QoS. To demonstrate the effectiveness of the devised procedure, some numerical tests are conducted in realistic radar scenarios.

II. Problem formulation

A multifunction MPAR system equipped with an active ESA antenna is considered. The different tasks to be performed require a specific amount of PAP that is hence assumed as the limited resource to be distributed among them. From a practical point of view the active ESA is composed of many tiles each with a given PAP (see Fig. 1). They are clustered according to the requirements of the system tasks so that each group realizes an overall PAP value.

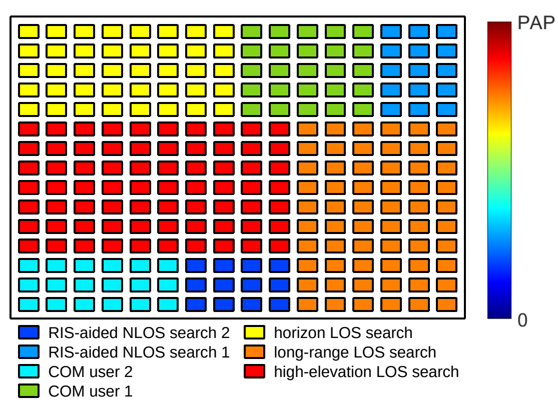


Fig. 1: Pictorial description of the PAP allocation to the different tasks of the active ESA.

Let us indicate by $q_i(PAP_i)$, $i = 1, \ldots, L$, the quality metric characterizing the performance of the i-th task, being L the number of tasks. The role of the RRM is to find the optimal assignment of PAP between tasks in order to maximize the weighted sum of their utilities [7, Chap. 3], [4, Chap. 5]. In this context, the task utility function provides the satisfaction level corresponding to the achieved task quality metric value. Moreover, having the tasks different importance ruled by different priorities, these utilities are suitably weighted when the overall RRM utility metric is evaluated. Specifically, denoting by $\boldsymbol{PAP} = [PAP_1, PAP_2, \ldots, PAP_L]^T \in \mathbb{R}^L$ the vector containing as i-th entry the PAP attributed to the i-th task, $i = 1, \ldots, L$, the PAP distribution is obtained as the optimal solution to the following constrained optimization problem [4, Chap. 5]

$$\begin{cases} & \max_{\boldsymbol{PAP}} u(\boldsymbol{PAP}) \\ \text{s.t.} & \sum_{i=1}^{L} PAP_i \leq PAP_{tot} \\ & PAP_i \geq 0, i = 1, \ldots, L \end{cases}, \tag{1}$$

where

$$u(\boldsymbol{PAP}) = \sum_{i=1}^{L} w_i u_i(q_i(PAP_i)), \tag{2}$$

PAP_{tot} is the total amount of PAP available at the MPAR, $u_i(\cdot)$ and w_i, $i = 1, \ldots, L$, are the utility functions and the weights ruling the priorities of the L tasks.

III. Task quality and utility

A figure of merit is now introduced for each specific task. In particular, for all surveillance functions it is provided by the cumulative detection range, say R, defined as the range where the cumulative probability of detection (P_d) overcomes a desired value [4], [7]–[9]. Similarly, for the COM function, the quality metric can be defined as the communication range, indicated as R_{com}, corresponding to the maximum distance at which a minimum bit-rate can be conveyed reliably.

A. Search task quality metric

Let us indicate with $P_d(R')$ the single-look P_d at range R', and assume that S is the number of scans the target needs to reach the range R from the pop-up range R_m. The cumulative P_d at range R is the probability that a target is detected at least once in a given number of dwells [4], [8], i.e.,

$$P_c(R|R_m) = 1 - \prod_{n=0}^{S-1} \left[1 - P_d\left(R_m - n v_r t_f - \Delta\right)\right], \tag{3}$$

with v_r the target radial speed, t_f the frame time, and Δ a sample of a uniform random variable in the interval $[0, v_r t_f]$, with $v_r t_f$ the distance traveled by the target in a single scan, modeling the initial target position in the corresponding radar cell. The single-look P_d can be evaluated once the desired false alarm probability, say P_{fa}, is set. More specifically, assuming a Swerling 0 (SW0) model for the target amplitude and a coherent integration of the pulses in a dwell, the single-look detection probability at range R' can be obtained as [10]

$$P_d(R') = Q_M\left(\sqrt{2\text{SNR}}, \sqrt{-2\log P_{fa}}\right) \tag{4}$$

where $Q_M(\cdot,\cdot)$ the Marcum Q-function. Note that, the functional dependence on the variable R' of the P_d is embedded in the expression of the coherent signal to noise ratio (SNR).

Assuming a monostatic radar configuration using the same beam in transmission and reception, the search-form of the radar range equation (RRE) can be derived as [10]

$$\text{SNR}^{\text{LOS}} = PAP \frac{\sigma}{4\pi k T_s R_{\text{LOS}}^4 L_s^{\text{LOS}} L_{\text{steer}}^{\text{LOS}}} \frac{t_f}{\Omega} \tag{5}$$

where R_{LOS} is the range radar-target, T_s is the system noise temperature, L_s^{LOS} is the combined two-way system operational loss [10], σ is the target radar cross section (RCS), and k is the Boltzmann's constant. Moreover, $L_{\text{steer}}^{\text{LOS}}$ is the term accounting for the scanning gain loss of the steered antenna in the LOS scenario, in the pointing direction (θ_0, ϕ_0), which implicitly embeds the spatial selectivity in the antenna gain.

As to the NLOS scenario, encompassing a RIS that aids the detection *over the corner* [11], let us indicate with $R_{1,\text{NLOS}}$ the distance radar-RIS and $R_{2,\text{NLOS}}$ the range RIS-target. Therefore, the search-form of the RIS-aided RRE is

$$\text{SNR}^{\text{NLOS}} = \frac{PAP\, G_{\text{RIS}}^2 A_{\text{RIS}}^2 \eta_{\text{RIS}}^2 \sigma}{R_{1,\text{NLOS}}^4 R_{2,\text{NLOS}}^4 (4\pi)^3 k T_s L_s^{\text{NLOS}} L_{\text{steer}}^{\text{NLOS}}} \frac{t_f}{\Omega}. \tag{6}$$

with L_s^{NLOS} the combined system operational loss [10] and $L_{\text{steer}}^{\text{NLOS}}$ the total scanning loss in the NLOS scenario. Moreover, A_{RIS} is the RIS area, that for a uniform rectangular geometry can be expressed as $\delta_x \delta_y N_1 N_2$, with $\delta_x = \delta_y = \lambda_0/2$ the patch size along x- and y-direction, respectively, and N_1, N_2 the respective number of patches, with λ_0 the operating wavelength. Additionally, η_{RIS} is the RIS efficiency, whereas G_{RIS} is the RIS peak gain.

It is now worth observing that a commonly reference value for the objective P_c is 0.9. For this reason, the corresponding cumulative detection R is usually denoted by R_{90}, as in the remainder of this paper.

B. COM task quality metric

The focus is on the transmission of a signal composed by the superposition of U orthogonal waveforms to U COM receiving users. At receiver, assuming an additive white Gaussian noise (AWGN) channel and performing a matched filter operation on the incoming signal, the channel capacity per bandwidth (expressed in bit/s/Hz) for the h-th user can be defined as [12]–[14]

$$C = \log_2\left(1 + \text{SNR}_h^{\text{COM}}\right), \tag{7}$$

where $\text{SNR}_h^{\text{COM}}$ is the SNR at the h-th COM user receiver, that can be expressed in terms of PAP as

$$\mathrm{SNR}_h = PAP_h \frac{A_e^{\mathrm{rx},h}}{\lambda_0^2 R_{h,\mathrm{COM}}^2 L_s^{\mathrm{COM}} L_{\mathrm{steer}}^{\mathrm{COM}} k T_s^{\mathrm{COM}} B^{\mathrm{COM}}}. \tag{8}$$

where $R_{h,\mathrm{COM}}$ is distance between transmitter and receiver, $A_e^{\mathrm{rx},h}$ is the effective area of the h-th user receiving antenna, $L_{\mathrm{steer}}^{\mathrm{COM}}$ is the total scanning loss in the COM scenario, and L_s^{COM} is the combined system operational loss. Moreover, T_s^{COM} and B^{COM} are the noise system temperature and effective bandwidth of the receiver.

Finally, denoting by C_{desired} the reference value for the objective channel capacity, its corresponding range R_{com} is directly derived from (7)-(8).

C. Task utility

The utility provides a description of the degree of satisfaction reached when each task is completed. A possible way to define the utility for the i-th considered task is [9]

$$u_i(R_c) = \begin{cases} 0, & R_c < R_{t_i} \\ \frac{R_c - R_{t_i}}{R_{o_i} - R_{t_i}}, & R_{t_i} \le R_c \le R_{o_i} \\ 1, & R_c > R_{o_i} \end{cases} \tag{9}$$

where R_{t_i} and R_{o_i} are the threshold and objective ranges of the i-th task, respectively.

D. Optimization algorithm

To obtain a solution to the non-convex resource allocation problem in (1), the iterative optimization algorithm in [15] is exploited. Therein, the interior-point approach to constrained optimization is employed, which amounts to solve a sequence of approximate minimization problems which include non-negative constrained slack variables and equality constraints. The solution algorithm is based on the availability of an oracle that provides the values for the objective function for each choice of the parameters and with the desired accuracy. This is possible thanks to the analytic expressions which in implicit form rule the relationships among the objective and the different design parameters.

IV. Numerical results

This section is devoted to assess the functioning of the MPAR focusing on a scenario comprising seven tasks: three refer to search in LOS scenarios (shortly referred to as Horizon, Long-range, and High-elevation, respectively), two COMs with different users, and two to search in a NLOS surveillance using two different RISs. Problem (1) is solved using the Mathworks Matlab® *Quality-of-Service Optimization for Radar Resource Management* [16] which performs a constrained minimization of a given objective function.

A. Parameter setting

Tests conducted in this paper refer to a MPAR operating in X-band with its central frequency $f_0 = 10$ GHz and the nominal P_{fa} is set to 10^{-6} for search operations. For each considered task, the antenna coverage sector is specified in terms of angle and range limits, as specified in the following, in the order azimuth, elevation, and range:

- $[-45°, 45°]$, $[0°, 4°]$, 40 km (Horizon);
- $[-30°, 30°]$, $[0°, 30°]$, 70 km (Long-range);
- $[-45°, 45°]$, $[30°, 45°]$, 50 km (High-elevation);
- $[-45°, 45°]$, $[0°, 45°]$, 45 km (COM user 1);
- $[-45°, 45°]$, $[0°, 45°]$, 55 km (COM user 2);
- $[15°, 20°]$, $[28°, 32°]$, 4 km (RIS 1);
- $[15°, 20°]$, $[16°, 20°]$, 3.5 km (RIS 2);

Other parameters for the three search tasks are $T_s = 913$ K, $v_r = 250$ m/s, $\sigma = 1$ m^2, and $t_f = \{0.5, 6, 2\}$ s, $L_s^{\mathrm{LOS}} = \{22, 19, 24\}$ dB, $L_{\mathrm{steer}}^{\mathrm{LOS}} = \{0.01, 0.13, 2.31\}$ dB, for Horizon, Long-range, and High-elevation, respectively. Whereas, for COM tasks, $T_s^{\mathrm{COM}} = 916$ K, $B^{\mathrm{COM}} = 40$ MHz, $A_e^{\mathrm{rx},h} = 0.7 \times 10^{-3}$ m^2, $L_s^{\mathrm{COM}} = 27$ dB, and $L_{\mathrm{steer}}^{\mathrm{COM}} = \{0.15, 0.62\}$ dB, for the two users respectively. Finally, for RIS the parameters are $t_f = 2$ s, $T_s = 913$ K, $v_r = 50$ m/s, $\sigma = 0.02$ m^2, $L_s^{\mathrm{NLOS}} = 19$ dB, $G_{\mathrm{patch}} = 4$ dB, $N_1 = N_2 = 101$, $\eta_{\mathrm{RIS}} = 0.8$, $R_1^{\mathrm{NLOS}} = 1$ km, and $L_{\mathrm{steer}}^{\mathrm{NLOS}} = \{1.25, 0.44\}$ dB, respectively for the two RISs.

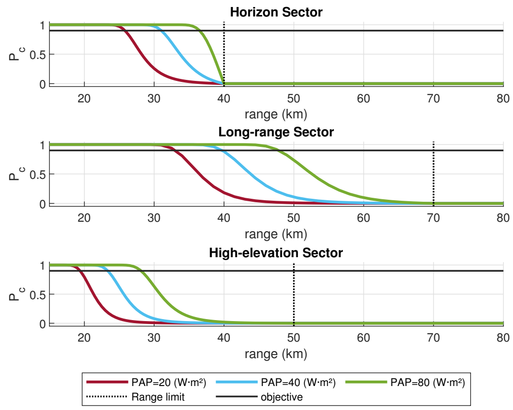
B. Results

The analyses assume a SW0 fluctuating target model for both the high-speed targets considered in three LOS search functions and for the small unmanned aerial vehicle (UAV)s to be detected via RIS-aided surveillance. The cumulative P_d (3) and channel capacity per bandwidth (7) are shown in Fig. 2 versus range for three different values of the PAP assigned to each task, viz. $\{20, 40, 80\}$ W·m^2.

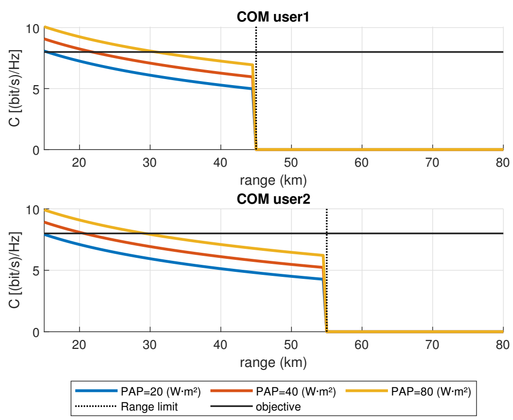
For each subfigure in Fig. 2, the corresponding range limit is also reported as a dotted line. Hence, QoS values overcoming these limits are not of interest and set to zero. Moreover, the desired value for the cumulative P_d (i.e., $P_{c_{\mathrm{desired}}} = 0.9$), and for the channel capacity per bandwidth (i.e., $C_{\mathrm{desired}} = 8$ bit/s/Hz) are highlighted in the same graph using a continuous black line. Hence, the corresponding ranges R_{90} and R_{com} are computed for each PAP numerically solving the equations $P_c(R) - P_{c_{\mathrm{desired}}} = 0$ and $C(R) - C_{\mathrm{desired}} = 0$ with respect to the variable R. The obtained results are shown in Fig. 3, with the task quality depicted as a function of the allocated resource for each task. Evidently, the task quality increases until its limit as the utilized PAP grows.

In Fig. 4 the utility functions for the above considered tasks are reported, particularizing the general form given by (9) setting the objective ranges to $R_o = \{38, 65, 45, 35, 45, 2, 2\}$ km and the threshold ranges to $R_t = \{25, 45, 30, 5, 15, 0.153, 0.153\}$ km for the three search, two COM and two RIS-aided tasks, respectively.

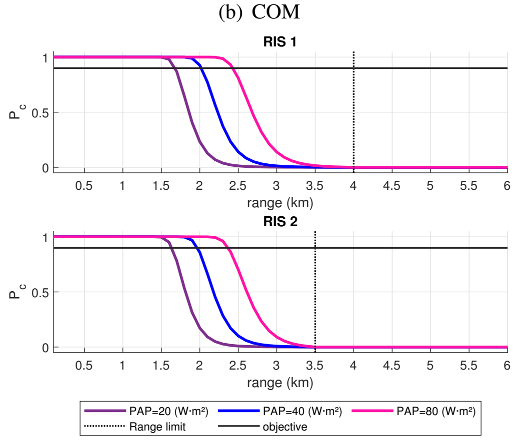
Moreover, the PAP can be mapped to the utility space as shown in Fig. 5. Interestingly, the Long-range, High-elevation, Horizon, and COM user 2 need to exploit non negligible PAP values to reach non zero utilities, viz., 65, 102, 18, and 22 W·m^2, respectively. Conversely, the other tasks can reach nonzero utilities with very low values of assigned PAP. Additionally, the Long-range and High-elevation search functions demand high PAP values to obtain the maximum



(a) LOS search



(b) COM



(c) RIS

Fig. 2: Cumulative P_d (subfigures a and c) for the LOS and NLOS search tasks, and channel capacity per bandwidth (subfigure b) for the COM tasks.

utility, i.e., 300 and 504 W·m^2. Interestingly, the operation that requires the minimum PAP value to attain the maximum utility is the RIS-aided search 1 with PAP = 39 W·m^2.

Now, the simulation analyzes the case where the resource allocation is performed under normal operational conditions (i.e., when no optimization is performed) where the maximum utility is reached for each operating task. Hence, they exploit all the necessary resource to fulfil its demanded nominal objective, viz. cumulative P_d and/or channel capacity per bandwidth. To highlight this distribution, Fig. 6 shows a graphical representation of the antenna coverage sectors and the objective value R_{90} (resp. R_{com}). Additionally, on the right side of this diagram a bar chart indicating the PAP allocated to each task is also reported (see the bar on the right side). Specifically, the maximum utility values are obtained with the allocation $\boldsymbol{PAP} = [103, 300, 503, 103, 190, 39, 42]^T$ W·m^2, corresponding to a total PAP of about 1280 W·m^2.

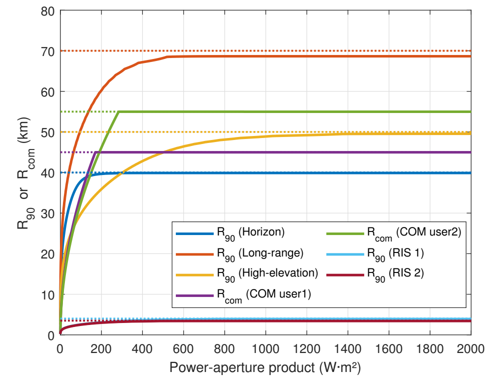
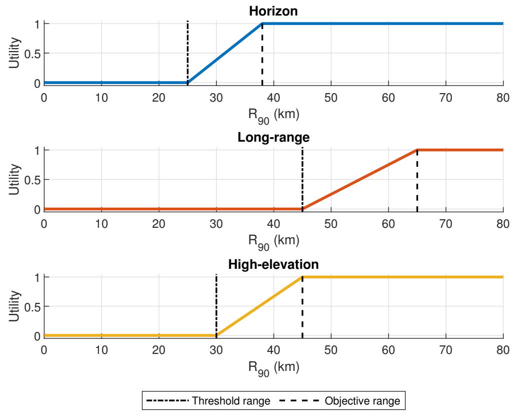
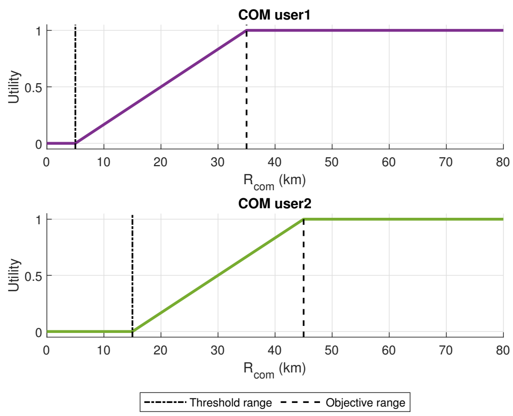


Fig. 3: Task quality versus assigned resource. Dashed curves indicate the range limit for each specific task.

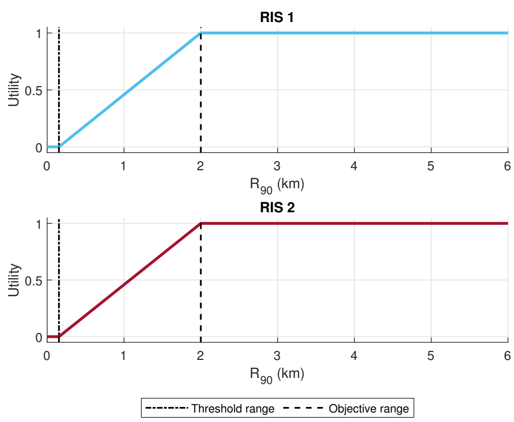
Comparing the bar chart of Fig. 6 with the diagram representing the utility versus resource of Fig. 5, it is evident that in the case of normal operational conditions, all tasks are capable of obtaining the maximum utility. However, in some operating conditions, the total amount of resources available at the MPAR cannot allow to assign the ideally required PAP to each task. This can be also explained observing that, a non negligible part of the available resources should be reserved to other tasks (e.g., tracking) [17]. Hence, the RRM should compute the optimal PAP allocation, once its maximum available value is set. In this study, the maximum PAP is set to the 50% of that under normal operational conditions, that is 640 W·m^2. Moreover, the following set of priority weights is enforced, $\boldsymbol{w} = [0.36, 0.10, 0.10, 0.07, 0.07, 0.15, 0.15]^T$, providing low priorities to COM tasks with respect to search ones. Solving Problem (1) results in the resource distribution reported in left bar chart of Fig. 6, viz. $\boldsymbol{PAP} = [103, 152, 122, 96, 87, 38, 42]^T$ W·m^2. To give insights into the obtained results, Fig. 7 shows for each task the optimal resource allocation in terms of PAP versus the R_{90} (resp. R_{com}) together with the corresponding utility. As expected, the RRM allocates PAP so that the maximum utility is reached for the Horizon search function, being the task with highest priority, with a corresponding $R_{90} = 38$ km. Analogously, also the two RIS-aided search tasks experience an allocation of PAP that allows to reach the maximum utility with $R_{90} = 2$ km. This is because they have a medium priority (i.e., a weight 0.15) together with the fact that they have low requirements in terms of resource. The worst case is the High-elevation where the PAP allocation only



(a) LOS search



(b) COM



(c) RIS

Fig. 4: Utility functions for each considered task.

ensures a utility of 0.10, being its priority weight low.

V. Conclusions

This paper has addressed the problem of optimal PAP allocation in a MPAR system performing ISAC operations, i.e., search in LOS and NLOS together with COM functionality. To maximize the QoS, a constrained optimization problem is formulated whose objective function is the weighted sum of the utilities achieved with the assigned PAP to each task and numerically solved. The validity of the designed allocation strategy has been proved by simulations in a challenging operative scenario. As expected, the results have emphasized that the MPAR tends to mostly allocate the available resources to the high priority tasks at the expense of the others. By doing so, it is ensured that the utilities for the most important tasks attain values close to their objectives, whereas for the remainder tasks a lower level of satisfaction is obtained.

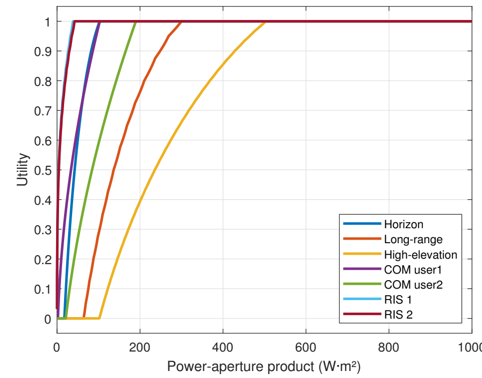


Fig. 5: Utility versus resource for the different radar operations.

Acknowledgments

The work of Augusto Aubry and Antonio De Maio was supported by the European Union under the Italian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, partnership on “Telecommunications of the Future” (CUP J33C22002880001, PE00000001 - Program “RESTART”).

References

[1] P. Moo and Z. Ding, *Adaptive Radar Resource Management*. Academic Press, 2015.

[2] A. Farina, A. De Maio, and S. Haykin, Eds., *The Impact of Cognition on Radar Technology*, ser. Radar, Sonar and Navigation. Institution of Engineering and Technology, 2017. [Online]. Available: https://digital-library.theiet.org/content/books/ra/sbra520e

[3] D. Friedman, *The Double Auction Market: Institutions, Theories, and Evidence*. Routledge, 2018.

[4] A. Charlish and F. Hoffmann, “Cognitive Radar Management,” *Novel Radar Techniques and Applications: Waveform Diversity and Cognitive Radar, and Target Tracking and Data Fusion*, vol. 2, pp. 157–193, 2017.

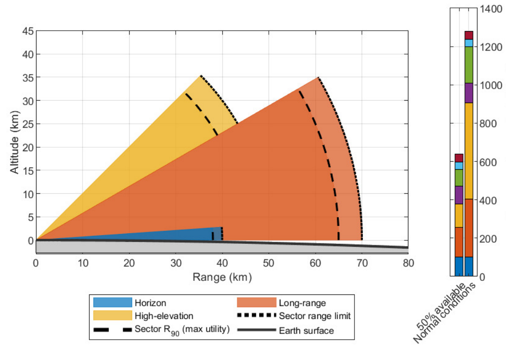
[5] D. Gaglione, C. Clemente, C. V. Ilioudis, A. R. Persico, I. K. Proudler, J. J. Soraghan, and A. Farina, “Waveform Design for Communicating Radar Systems using Fractional Fourier Transform,” *Digital Signal Processing*, vol. 80, pp. 57–69, 2018. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1051200418302070

[6] F. Liu, Y. Cui, C. Masouros, J. Xu, T. X. Han, Y. C. Eldar, and S. Buzzi, “Integrated Sensing and Communications: Towards Dual-Functional Wireless Networks for 6G and Beyond,” *IEEE journal on selected areas in communications*, 2022.

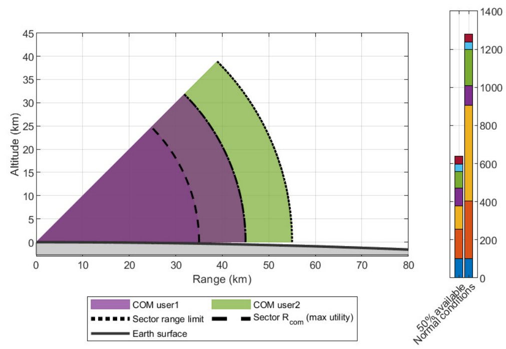
[7] A. Charlish and F. Katsilieris, “Array Radar Resource Management,” *Novel Radar Techniques and Applications: Real Aperture Array Radar, Imaging Radar, and Passive and Multistatic Radar*, vol. 1, pp. 135–171, 2017.

[8] J. D. Mallett and L. E. Brennan, “Cumulative Probability of Detection for Targets Approaching a Uniformly Scanning Search Radar,” *Proceedings of the IEEE*, vol. 51, no. 4, pp. 596–601, 1963.

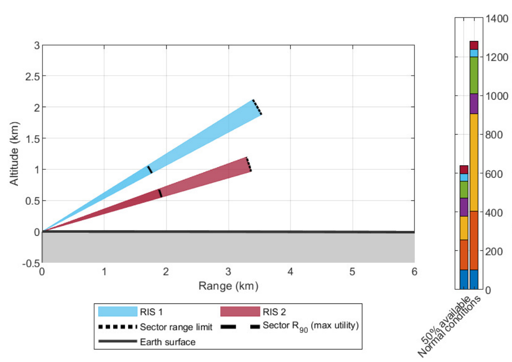
[9] F. Hoffmann and A. Charlish, “A Resource Allocation Model for the Radar Search Function,” in *2014 International Radar Conference*. IEEE, 2014, pp. 1–6.



(a) LOS search

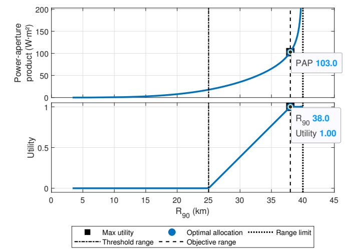


(b) COM

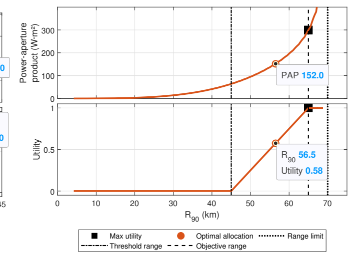


(c) RIS

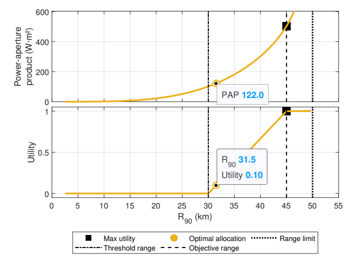
Fig. 6: Normal and optimized resource allocation of MPAR search, COM, and RIS-aided tasks.



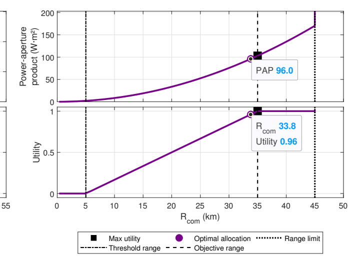
(a) Horizon



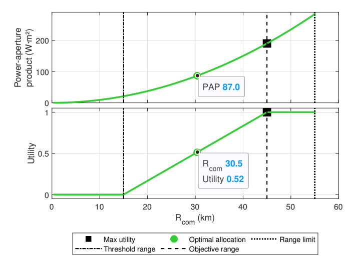
(b) Long-range



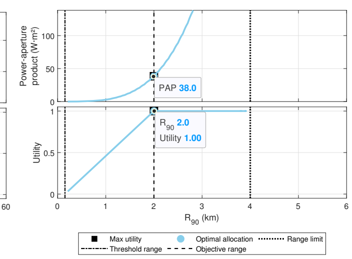
(c) High-elevation



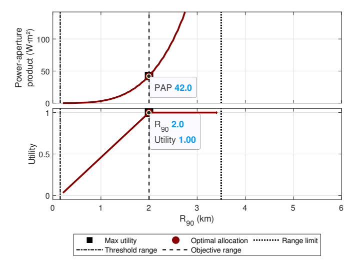
(d) COM user 1



(e) COM user 2



(f) RIS 1



(g) RIS 2

Fig. 7: Optimized resource allocation of MPAR search, COM, and RIS-aided tasks.

[10] M. A. Richards, J. Scheer, W. A. H., and W. L. Melvin, *Principles of Modern Radar*. Citeseer, 2010, vol. 1.

[11] A. Aubry, A. De Maio, and M. Rosamilia, "Reconfigurable Intelligent Surfaces for N-LOS Radar Surveillance," *IEEE Transactions on Vehicular Technology*, vol. 70, no. 10, pp. 10 735–10 749, 2021.

[12] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic Beamforming using Dumb Antennas," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, 2002.

[13] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[14] M. Kountouris, "Multiuser Multi-Antenna Systems with Limited Feedback," Ph.D. dissertation, Télécom ParisTech, 2008.

[15] "Find minimum of constrained nonlinear multivariable function," https://it.mathworks.com/help/optim/ug/fmincon.html?s_tid=doc_ta, Mathworks Matlab.

[16] "Quality-of-Service Optimization for Radar Resource Management," https://it.mathworks.com/help/radar/ug/quality-of-service-optimization-for-resource-management-in-multifunction-phased-array-radar.html, Mathworks Matlab.

[17] D. K. Barton, *Radar Equations for Modern Radar*. Artech House, 2013.