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Estimates of positive linear operators in terms of second-order moduli. (English summary)

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Let $\varphi \in C([0, 1])$ be a nonnegative function such that $\varphi > 0$ on $(0, 1)$ and φ^2 is concave. For positive linear operators which preserve linear functions the author presents an estimate by means of the modulus of smoothness $\omega_2^\varphi(f, h)$. This estimate generalizes both a result proved in [R. Păltănea, *Approximation theory using positive linear operators*, Birkhäuser Boston, Boston, MA, 2004; [MR2085239 \(2005f:41061\)](#)] and an estimate for Bernstein operators due to I. Gavrea et al. [East J. Approx. **9** (2003), no. 2, 175–194; [MR1991849 \(2004e:41006\)](#)] for which the weight was $\varphi(x) = \sqrt{x(1-x)}$ ($0 \leq x \leq 1$).

As a by-product, the author also obtains an improvement of Theorem 4.1.2 of [Z. Ditzian and V. Totik, *Moduli of smoothness*, Springer, New York, 1987; [MR0914149 \(89h:41002\)](#)]. Indeed, he shows that, for a weight φ such that φ^2 is concave, if $f \in C([0, 1])$ and $\lambda, t > 0$ such that $\lambda t \in (0, 1/2]$, then

$$\omega_2^\varphi(f, \lambda t) \leq (2 + 3\lambda^2)\omega_2^\varphi(f, t).$$

Reviewed by *Vita Leonessa*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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