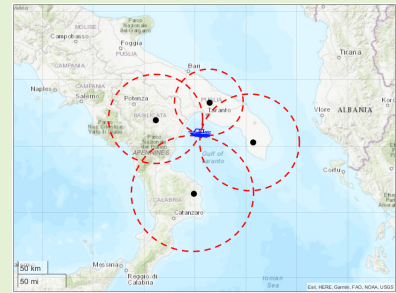


Passive TOA Localization for Maritime Surveillance Using AIS Signals

Vincenzo Carotenuto[✉], Senior Member, IEEE, Antonio De Maio[✉], Fellow, IEEE, Massimo Marrazzo, and Luca Pallotta[✉], Senior Member, IEEE

Abstract—Maritime localization traditionally relies on cooperative systems such as the Automatic Identification System (AIS), where vessels actively transmit their positions. However, AIS spoofing (i.e., the intentional falsification of transmitted coordinates) undermines the reliability of these approaches. This paper investigates Time of Arrival (TOA)-based passive localization as an independent, physical-layer solution for vessel position verification. A key challenge in this scenario is the presence of an unknown timing offset that is common to all receiving base stations (BSs) for each AIS transmission, resulting from transmitter delays, protocol timing references, and receiver processing latencies. To address this issue, the localization problem is formulated by explicitly incorporating the unknown temporal bias into the measurement model. Several estimation strategies (either iterative or not) are then investigated/designed, including Maximum Likelihood (ML), Least-Squares (LS), and constrained formulations, to solve the resulting nonlinear problem. Experimental validation using real AIS datasets shows that the considered approaches enable accurate passive vessel localization using existing AIS infrastructures. In particular, all the considered methods provide a favorable trade-off between estimation accuracy and computational complexity, while satisfying requirements for real-time applicability.



Index Terms—automatic identification system (AIS), maritime surveillance, passive localization, time of arrival (TOA).

I. INTRODUCTION

Maritime localization systems have traditionally relied on cooperative techniques based on the Automatic Identification System (AIS). AIS-equipped vessels broadcast messages containing navigational and identification information over Very High Frequency (VHF) channels, which are received by neighboring vessels and shore-based AIS Base Stations (BSs), enabling large-scale monitoring and situational awareness of maritime traffic. However, the open broadcast nature of AIS and the lack of authentication mechanisms make it vulnerable to spoofing and manipulation attacks. Malicious users may transmit AIS signals with falsified vessel identities or positions, or even generate messages corresponding to non-existent vessels, thereby creating “ghost targets” [1]. These vulnerabilities raise significant concerns regarding the reliability of AIS-

based information, particularly in safety-critical and security-sensitive maritime scenarios. Since such attacks typically alter the message content but not the physical propagation of the transmitted signal, physical-layer measurements can provide an independent mechanism for verifying vessel positions.

In this context, passive localization techniques [2], [3] offer a complementary approach that does not rely on AIS message content. Instead, vessel positions can be estimated from the Time of Arrival (TOA) of intercepted AIS signals received by multiple spatially distributed nodes. By exploiting the signal propagation time between transmitter and receivers, TOA measurements enable range estimation and subsequent multilateration across multiple receivers. A key motivation for considering TOA is its robustness to AIS spoofing. While the position information contained in AIS messages can be deliberately manipulated, TOA is determined by the physical propagation time of the transmitted signal between the vessel and the receiving BSs and is therefore not directly affected by modifications to the message content. Hence, TOA provides a complementary physical-layer observable for localization in the presence of spoofed AIS data. Moreover, AIS employs a Time Division Multiple Access (TDMA)-based access mechanism with well-defined time slots, making timing measurements particularly suitable for localization and naturally compatible with a TOA-based formulation.

A fundamental challenge in TOA-based passive localization is the knowledge of the signal transmission time, which is often unknown in scenarios such as AIS-based maritime mon-

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V. Carotenuto, A. De Maio (corresponding author), and M. Marrazzo are with the Department of Electrical Engineering and Information Technology (DIETI), Università degli Studi di Napoli “Federico II”, via Claudio 21, I-80125 Napoli, Italy (e-mail: vincenzo.carotenuto@unina.it, ademai@unina.it, massimo.marrazzo@unina.it).

M. Marrazzo is also with the Italian Coast Guard Headquarters, Rome, Italy.

L. Pallotta is with the Department of Engineering (DiING), University of Basilicata, via dell’Ateneo Lucano 10, 85100 Potenza, Italy (e-mail: luca.pallotta@unibas.it).

itoring [4]–[11]. This unknown transmission instant introduces a common timing offset across all the receiving stations, which must be properly accounted for to obtain accurate position estimates. Time Difference of Arrival (TDOA) techniques avoid the need for transmitter-network synchronization by relying on differences in arrival times between pairs of receivers [12]–[15]. However, this comes at the cost of increased estimation uncertainty. As a matter of fact, under comparable geometry and noise conditions, TDOA-based position estimates can exhibit approximately twice the variance of TOA-based estimates. Consequently, while TDOA can simplify synchronization requirements, TOA remains preferable when suitable mechanisms or compensation strategies are available to handle the unknown transmission time, offering improved localization accuracy [16].

A substantial body of research has developed TOA-based localization algorithms across various domains, including wireless sensor networks [17]–[19], ultra-wideband systems [20], and underwater acoustic networks [21]–[26]. From a technical perspective, TOA localization frameworks rely on a variety of estimation techniques to accurately determine source positions while maintaining robustness against measurement noise and environmental uncertainties. Among them

- Maximum Likelihood (ML) estimation [6], [27]: maximizes the likelihood of the observed TOA measurements, yielding statistically efficient estimates when the noise model assumptions hold.
- Least-Squares (LS) estimation [28]–[30]: minimizes the sum of squared residuals between measured and predicted arrival times, offering a lower computational complexity. Notably, under Gaussian noise, ML simplifies to a (possibly weighted) LS estimator.
- Constrained estimators [16], [31], [32]: leverage prior knowledge (such as geographical boundaries) to improve estimation performance.

These fundamental estimation principles not only form the basis of classical multilateration, but also underpin more advanced passive localization algorithms in practical maritime scenarios. In fact, in the context of maritime situational awareness, several passive localization algorithms have been developed and evaluated using AIS signals intercepted by BSs [33]–[36]. Although most of these approaches mainly focus on TDOA-based formulations, their underlying estimation principles are closely related to TOA techniques. In particular, [33], [34] demonstrated AIS signal localization using Extended Kalman Filters (EKF), improving tracking performance over conventional multilateration. These studies provide a relevant precedent for exploiting AIS receiving infrastructure for localization purposes. Moreover, [35] combined TDOA and Frequency Difference of Arrival (FDOA) within an Interactive Multiple Model (IMM) framework to better capture vessel maneuvers. Moreover, [36] proposed an asynchronous TDOA method for unsynchronized receiver networks, enhancing deployment flexibility. Recent contributions have also addressed related problems in different application scenarios, including simultaneous localization and synchronization in distributed Multiple Input Multiple Output-Orthogonal Frequency Di-

vision Multiplexing (MIMO-OFDM) systems [37], hybrid TOA/Frequency Of Arrival (FOA) localization with clock bias and drift [38], joint localization and synchronization of moving agents using one-way TOA measurements in asynchronous networks [39], and sequential one-way TOA/Doppler-shift-based joint localization and synchronization [40], [41]. Notice also that passive localization can exploit other observables, such as Received Signal Strength (RSS), Angle Of Arrival (AOA), FDOA, or their combinations [42]. While these approaches are relevant alternatives, they may require additional sensing capabilities, or algorithmic complexity. In contrast, TOA is directly available from the AIS receiving infrastructure considered in this work, making it a natural choice given the AIS signal characteristics, the considered spoofing scenario, and the existing measurement capabilities.

Building on these developments, TOA-based localization provides an effective physical-layer mechanism for validating AIS-reported vessel positions and detecting anomalous behavior. As already pointed out, a fundamental issue is the presence of an unknown temporal offset common to all receiving BSs for a given AIS transmission, caused by the uncertainty in the signal transmission time. This offset introduces systematic errors in the measured distances and must therefore be explicitly accounted for in the localization model. Several approaches have been proposed in the literature to address this issue. For instance, asynchronous TOA methods iteratively correct the timing offset to enable synchronous localization [43], while joint estimation schemes simultaneously estimate both the transmitter position and the relative clock offsets using multi-antenna receivers and LS formulations [44]. Statistical models, instead, treat the bias as an unknown parameter in one-way TOA measurements, which can also support spoofing detection [45].

In this paper, several TOA-based passive localization techniques are investigated/designed and compared for vessel positioning using AIS signals intercepted by multiple shore-based BSs. The localization problem is first addressed under a ML framework, which is solved using an iterative Fixed-Point (FP) method to efficiently handle the nonlinear estimation. Building on this, a constrained LS formulation, referred to as CLS-TOA, is introduced to explicitly account for the unknown temporal offset arising from transmission-time uncertainty. Due to potential ill-conditioning in the model matrix, particularly in scenarios with a limited number of receiving stations or unfavorable geometric configurations, CLS-TOA solutions may experience performance degradation; to mitigate the impact, a modified version (MCLS-TOA) is proposed, which replaces estimates associated with poorly conditioned configurations with their corresponding ML solutions, providing a mostly non-iterative robust approach. In addition, two iterative LS schemes are derived to enable computationally efficient alternatives that maintain satisfactory accuracy.

Experimental evaluation has been conducted using real AIS data representative of dynamic vessel trajectories, collected over a large-scale monitoring area from BSs belonging to the Italian AIS network. The results show that the proposed methods achieve accurate localization performance under realistic operating conditions. In particular, the CLS-TOA approach

ensures satisfactory accuracy on average; however, it may suffer from localized performance degradation at specific points along the vessel trajectory due to ill-conditioning possibly induced by unfavorable sensor geometries. The MCLS-TOA method effectively mitigates this issue, leading to improved robustness under challenging measurement conditions. Iterative schemes, on the other hand, achieve numerical performance close to that of the ML estimator under the considered experimental conditions, with a comparable computational burden. Overall, all these methods are suitable for real-time implementation, given the AIS data acquisition rate. Finally, TOA localization demonstrates higher performance than TDOA-based algorithms. Although TDOA theoretically cancels the unknown temporal offset, its position estimates exhibit roughly twice the measurement variance of TOA under the same noise conditions. This can significantly reduce the position estimation accuracy especially when the number of receiving sensors is limited.

The main contributions of this paper can be summarized as follows.

- 1) A TOA-based passive localization framework tailored to AIS signals is developed, explicitly accounting for the presence of a common unknown timing offset affecting all receiving sensors.
- 2) Several estimation strategies (both iterative and non-iterative) for joint position and offset estimation are formulated or designed, including ML, constrained LS, and modified constrained LS formulations, as well as iterative hybrid LS/ML estimation schemes, highlighting their practical advantages and disadvantages.
- 3) An extensive experimental validation of the considered positioning strategies is conducted using real AIS datasets. The comparative analysis aims to highlight the complementary characteristics, strengths, and limitations of the different approaches under realistic maritime conditions. Hence, the results demonstrate the effectiveness of the proposed methods and show the performance advantage of TOA over TDOA approaches under comparable conditions.

The remainder of this paper is organized as follows. Section II provides a brief overview of the AIS transmission characteristics, along with the motivations and challenges of applying passive TOA-based localization techniques to AIS signals. Section III describes the adopted measurement model and presents five algorithms for estimating vessel positions from TOA measurements. Section IV reports the experimental assessment and performance evaluation of the studied approaches using data collected from the Italian AIS network. Finally, Section V draws conclusions and outlines possible directions for future research.

NOTATION

Boldface letters represent vectors $\mathbf{a}$ (lowercase) and matrices $\mathbf{A}$ (uppercase). The transpose operator is written as $(\cdot)^T$, while $(\cdot)^{-1}$ indicates the inverse. The sets $\mathbb{R}^N$ and $\mathbb{R}^{N \times M}$ refer to the space of N -dimensional real vectors and $N \times M$ real matrices, respectively. Moreover, $\|\cdot\|$ denotes

the Euclidean norm, and $\mathbf{0}$ is the zero vector (or matrix) of appropriate dimension. The gradient of a scalar function f with respect to $\mathbf{a}$ is written as $\nabla_{\mathbf{a}} f$, and the Hessian as $\nabla_{\mathbf{a}}^2 f$. $\mathbf{A} \succeq \mathbf{0}$ indicates that the matrix $\mathbf{A}$ is positive semidefinite. Finally, $\inf\{\cdot\}$ and $\sup\{\cdot\}$ denote the infimum (greatest lower bound) and the supremum (least upper bound) of a set, respectively.

II. TOA-BASED LOCALIZATION APPLIED TO AIS SIGNALS

AIS is a maritime communication system designed for vessel identification, tracking, and collision avoidance, in which AIS-equipped vessels broadcast messages containing navigational and identification information over two dedicated VHF channels centered at 161.975 MHz (AIS 1) and 162.025 MHz (AIS 2). These messages are received by both neighboring vessels and shore-based BSs, enabling situational awareness and large-scale monitoring of vessel traffic.

AIS operates according to a TDMA scheme, in which time is divided into frames and slots synchronized to Coordinated Universal Time (UTC) [46]. Each frame has a duration of 60 s and is aligned with the beginning of a UTC minute. A frame is further divided into 2250 time slots of duration $T_s \approx 26.67$ ms, with AIS transmissions occurring in one or more consecutive slots depending on the specific AIS message type, as illustrated in Figure 1. This structured access scheme requires accurate time synchronization among AIS-equipped units, which is typically achieved through Global Navigation Satellite System (GNSS) receivers providing high-precision timing references.

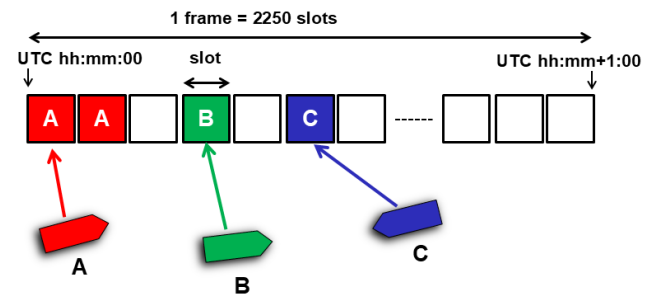


Fig. 1: AIS channel TDMA scheme and signal structure: each frame is divided into 2250 slots of 26.67 ms.

Within the AIS network, vessels are equipped with either Class A or Class B transponders, which differ in terms of access mechanism, transmission power, and operational requirements [46]. Class A transponders are mandated for large commercial vessels and ships subject to international regulations. They operate under a Self-Organized TDMA (SO-TDMA) protocol, whereby transmissions are scheduled according to a dynamic slot map that is autonomously maintained and shared among nearby vessels. As a result, Class A transmissions are generally well aligned with the nominal TDMA slot structure. Class B transponders are typically used by smaller vessels and recreational craft. They generally employ a Carrier-Sense TDMA (CS-TDMA) access mechanism, in which transmissions are initiated based on channel sensing rather than strict

slot reservation. Consequently, Class B transmissions may not be perfectly aligned with the nominal slot boundaries, leading to increased uncertainty in the effective transmission time with respect to the TDMA frame structure.

Figures 2(a) and 2(b) illustrate the RF power-versus-time envelope of AIS transmissions within a single TDMA slot for Class A and Class B transponders, respectively. In both cases, due to practical hardware constraints, the RF signal does not instantaneously reach its nominal transmit power. Instead, a ramp-up interval is required to satisfy spectral mask requirements and limit out-of-band emissions, resulting in a gradual increase of the transmitted power at the beginning of the slot. According to the AIS standard, this ramp-up period corresponds to a maximum time interval of 8 bits (i.e., ≈ 0.833 ms, given the AIS bit time of ≈ 104.17 μ s). Clearly, the effective value of this transient time depends on the specific characteristics of the transponder.

The figures also highlight that, for Class A transponders, which operate according to the SO-TDMA protocol, AIS signal transmissions are nominally aligned with the beginning of the assigned time slot. In contrast, for Class B transponders operating under a CS-TDMA access scheme, transmissions are not strictly aligned with the nominal slot boundary due to the additional time required to perform carrier sensing and verify channel availability.

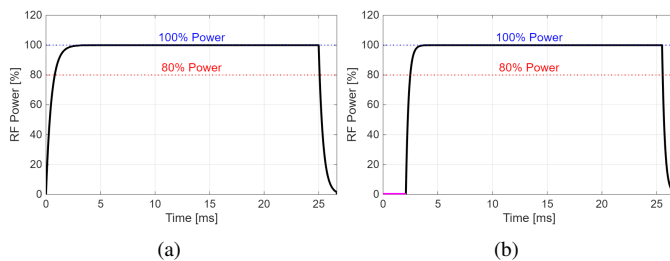


Fig. 2: RF power-versus-time envelope within a single TDMA time slot for (a) Class A transponders and (b) Class B transponders operating under CS-TDMA.

Although AIS is widely used for maritime traffic monitoring and situational awareness, its open broadcast nature and lack of authentication mechanisms make it vulnerable to spoofing and manipulation attacks, whereby AIS signals with falsified vessel identities or positions can be transmitted over the AIS channel. These vulnerabilities have raised increasing concerns regarding the reliability of AIS-based information, especially in safety-critical and security-sensitive maritime scenarios.

In this context, passive localization techniques based on physical-layer measurements, such as TOA, provide an attractive means to independently verify reported vessel positions and to support the detection and mitigation of spoofing behaviors, without requiring modifications to existing AIS transponders. Indeed, the inherent time synchronization of the AIS network enables the use of multiple shore-based AIS BSs as nodes in a passive localization system, without requiring additional dedicated hardware. By equipping AIS BSs with the capability to measure the arrival time of received AIS signals, the existing infrastructure can therefore be leveraged

to perform passive localization of AIS-equipped vessels. This enables the detection of some anomalous situations by correlating the estimated position with the coordinates reported within the AIS messages. Notice that, the AIS BSs used in the experimental campaign are equipped with reception time stamping capabilities, meaning that they provide the time stamp associated with the reception of each AIS signal. These time stamps are used as TOA measurements by the proposed localization algorithms. Hence, the proposed approach exploits the existing AIS receiving infrastructure and does not require additional dedicated localization hardware, provided that the participating BSs can provide reception time stamps.

However, the application of TOA-based localization techniques to AIS signals is not straightforward and entails several challenges. In particular, TOA-based methods rely on precise knowledge of the signal transmission start time and arrival time at the receivers, which represent the core timing information for localization. In AIS systems, these quantities are affected by protocol-level mechanisms and hardware-induced delays and are therefore not directly available for conventional TOA processing.

With respect to the transmission time, the nominal start of the TDMA slot represents a key timing reference. However, for Class A transponders, practical hardware effects and residual synchronization errors introduce an inherent jitter relative to this ideal boundary. In Class B transponders, this uncertainty is further increased by the variable delay of the CS mechanism, which makes the actual transmission start time less predictable.

Additional challenges arise in the measurement of the arrival time. In AIS systems, arrival-time measurements are not referenced to the physical onset of RF transmission, but rather to a protocol-defined point within the received packet, typically corresponding to the start of the payload. Consequently, the measured arrival time includes not only the propagation delay, but also additional transmitter-dependent delays, such as signal ramp-up effects which are generally unknown and vary across transponders.

All these effects introduce an unknown timing offset that is common across all receiving BSs for a given AIS transmission and cannot be assumed known or constant across different vessels. Accounting for this common unknown offset is therefore essential when applying TOA-based localization techniques to AIS signals. A suitable modeling of this effect enables the use of TOA measurements collected by existing AIS infrastructures for passive vessel localization, without requiring explicit calibration of transmitter- or receiver-specific delays, and motivates the measurement model and solution approaches introduced in the following section.

It is worth noting that the accuracy of arrival-time estimates for AIS signals is inherently limited by several factors. The narrowband nature of AIS transmissions (approximately 25 kHz) constrains the achievable timing resolution, while the protocol-driven detection of the packet start makes the estimates sensitive to synchronization, receiver processing, and Signal-to-Noise Ratio (SNR). As a result, even after compensating for deterministic offsets, arrival-time measurements are typically affected by random errors on the order of tens of microseconds. This corresponds to ranging uncertainties in

the order of a couple kilometers, highlighting the intrinsic limitations of TOA-based localization using AIS signals.

Before concluding, a further aspect to point out is the inherently limited number of receiving sensors in AIS-based TOA localization. Indeed, AIS BSs are deployed to ensure coverage rather than redundancy, so only a limited number of BSs typically receive the same vessel transmission. As a result, localization performance is directly dependent on the number and geometry of the available sensors. In particular, a reduced number of BSs may lead to ill-conditioned problems and increased sensitivity to noise, thus motivating the development of estimation strategies that remain reliable with few sensors.

III. PROBLEM FORMULATION AND TOA-BASED LOCALIZATION ALGORITHMS

TOA measurements form the basis of many passive localization techniques, as they enable the localization of a signal source by exploiting the propagation time delays between the signal transmission and its reception at multiple sensors. To formalize the problem in a two-dimensional setting, consider a target located at an unknown position $\mathbf{p} = [x_p, y_p]^T \in \mathbb{R}^2$ that transmits a signal at a time τ_0 , which is received by N passive sensors forming a localization network at times τ_i , $i = 1, \dots, N$. In the absence of measurement errors, the distance r_i between the target and the i th receiving sensor can be expressed as [11]

$$r_i = c(\tau_i - \tau_0) = \|\mathbf{s}_i - \mathbf{p}\|, \quad i = 1, \dots, N, \quad (1)$$

where $c = 3 \times 10^8$ m/s denotes the signal propagation speed and $\mathbf{s}_i = [x_i, y_i]^T \in \mathbb{R}^2$ the known position of the i th sensor. In practice, the measured arrival times τ_i , $i = 1, \dots, N$, are affected by noise and other system imperfections, and the transmission time τ_0 is generally unknown. Consequently, passive TOA-based localization must account for both measurement noise and the unknown transmission time.

Nonetheless, in some practical scenarios, partial information about τ_0 may be available. For instance, when dealing with the localization of emitters operating under TDMA schemes, preliminary information about the transmission time can be obtained by exploiting its inherent time synchronization. Specifically, TDMA schedules transmissions within predefined time slots theoretically aligned to a known absolute time reference.

Assuming that the receivers forming the passive localization network are synchronized to the same reference, a nominal transmission time $\tilde{\tau}_0$ can be inferred as the start time of the slot in which the signal is detected. However, the actual emission time τ_0 may deviate from this nominal value due to transmitter-dependent delays (i.e., the transmission does not occur exactly at the beginning of the time slot) and/or imperfect slot alignment of the transmitter (i.e., small discrepancies between the scheduled slot timing and the true reference), leading to the following model for the transmission time

$$\tau_0 = \tilde{\tau}_0 + \delta_{tx}, \quad (2)$$

where δ_{tx} is an unknown transmitter-dependent timing offset.

As for the arrival times τ_i , $i = 1, \dots, N$, in practice they are affected both by noise and by specific signal- and receiver-dependent delays. Indeed, the estimation of the arrival time at

a sensor can only be performed after the transmitted signal is detected. The detection process requires the received signal to reach a sufficient power level, and additional processing time is needed for the receiver to accurately determine the arrival instant. As a consequence, the measured arrival time is typically not taken at the very beginning of the signal, but rather at the point where the signal is deemed detectable. Additionally, different emitters may exhibit different rise times or signal transitions. This implies that the effective detection delay can vary across emitters, introducing further signal-dependent contributions to the measured arrival time. Combined with receiver-specific processing delays, these effects lead to deviations between the ideal nominal arrival time and the actual measured time.

Assuming that the deterministic part of the combined effects are approximately identical across all measuring AIS BSs, it can be accounted for via a common offset in the localization model¹.

Consequently, the actual measured arrival time at the i th sensor can be expressed as

$$\tilde{\tau}_i = \tau_i + \delta_{rx} + \tilde{n}_i, \quad i = 1, \dots, N, \quad (3)$$

where τ_i is the nominal arrival time, δ_{rx} represents an unknown delay common to all receivers, accounting for the combined signal- and receiver-dependent effects, whereas $\tilde{n}_i$ represents the measurement noise at the i th sensor.

Accounting for the models in (2) and (3), the TOA-based localization model in (1) can be expressed in terms of the available pseudo-range measurements as

$$\begin{aligned} \tilde{r}_i &= c(\tilde{\tau}_i - \tilde{\tau}_0) \\ &= c(\tau_i + \delta_{rx} + \tilde{n}_i - \tau_0 + \delta_{tx}) \\ &= c(\tau_i - \tau_0) + c(\delta_{rx} + \delta_{tx}) + c\tilde{n}_i \\ &= \|\mathbf{s}_i - \mathbf{p}\| + \Delta + n_i, \end{aligned} \quad (4)$$

where $\Delta = c(\delta_{rx} + \delta_{tx})$ is an unknown offset common to all receiving sensors, and $n_i = c\tilde{n}_i$ denotes additive measurement noise. Leveraging this measurement model, the following subsections present ML and linearized LS solution techniques for the joint estimation of the target position and the common timing offset.

A. ML Solution Techniques

The ML estimation relies on the maximization of the likelihood (or equivalently the log-likelihood) function of the observations with respect to the unknown parameters. In the considered scenario, assuming that the noise samples n_i , $i = 1, \dots, N$, are statistically independent and identically distributed zero-mean Gaussian random variables with variance σ^2 , the likelihood function of the observation vector $\tilde{\mathbf{r}} = [\tilde{r}_1, \dots, \tilde{r}_N]^T$, given the unknown target position $\mathbf{p}$ and

¹Note that, in the adopted model, any potential receiver-specific timing bias arising, for example, from residual synchronization errors or receiver-processing latencies is treated as a residual model mismatch.

the common offset Δ , can be written as

$$f(\tilde{\mathbf{r}} | \mathbf{p}, \Delta) = (2\pi\sigma^2)^{-\frac{N}{2}} \prod_{i=1}^N \exp\left(-\frac{(\tilde{r}_i - \|\mathbf{s}_i - \mathbf{p}\| - \Delta)^2}{2\sigma^2}\right). \quad (5)$$

In practical coastal VHF environments, TOA errors may exhibit heteroscedastic behavior, and their statistics may depend on the received-signal SNR, which is also influenced by the vessel-to-BS distance, as well as on multipath, propagation conditions, and BS-specific effects. Nevertheless, the available measurements provide preliminary empirical support for the equal-variance approximation across BSs adopted in (5), as discussed in Section IV.

Taking the logarithm of (5) yields

$$\log f(\tilde{\mathbf{r}} | \mathbf{p}, \Delta) = -\frac{N}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^N (\tilde{r}_i - \|\mathbf{s}_i - \mathbf{p}\| - \Delta)^2. \quad (6)$$

Since the first term does not depend on the unknown parameters, maximizing the log-likelihood is equivalent to minimizing the quadratic term in the summation. Therefore, the ML-TOA estimator of $\mathbf{p}$ and Δ is obtained as the solution to the following non-convex optimization problem

$$(\hat{\mathbf{p}}_{\text{ML}}, \hat{\Delta}_{\text{ML}}) = \arg \min_{\mathbf{p}, \Delta} J(\mathbf{p}, \Delta), \quad (7)$$

where

$$J(\mathbf{p}, \Delta) = \sum_{i=1}^N (\tilde{r}_i - \|\mathbf{s}_i - \mathbf{p}\| - \Delta)^2. \quad (8)$$

The cost function in (8) is clearly nonlinear with respect to the target position $\mathbf{p}$ due to the presence of Euclidean distance terms. As a consequence, the joint minimization problem in (7) does not admit a closed-form solution in general.

To handle (7), let us first observe that, for a given target position $\mathbf{p}$, the ML estimate of the offset Δ can be obtained by minimizing (8) with respect to Δ . This leads to the following closed-form expression

$$\hat{\Delta}_{\text{ML}}(\mathbf{p}) = \frac{1}{N} \sum_{i=1}^N (\tilde{r}_i - \|\mathbf{s}_i - \mathbf{p}\|). \quad (9)$$

Conversely, for a given value of Δ , the ML estimate of the target position $\mathbf{p}$ is obtained by minimizing (8) with respect to $\mathbf{p}$. Setting the gradient of $J(\mathbf{p}, \Delta)$ with respect to $\mathbf{p}$ equal to zero yields the stationarity condition

$$\nabla_{\mathbf{p}} J(\mathbf{p}, \Delta) = \sum_{i=1}^N (\tilde{r}_i - \|\mathbf{s}_i - \mathbf{p}\| - \Delta) \frac{\mathbf{s}_i - \mathbf{p}}{\|\mathbf{s}_i - \mathbf{p}\|} = 0. \quad (10)$$

After straightforward algebraic manipulations, (10) can be equivalently rewritten as

$$\mathbf{p} = \alpha - \frac{1}{N} \sum_{i=1}^N (\tilde{r}_i - \Delta) \frac{\mathbf{s}_i - \mathbf{p}}{\|\mathbf{s}_i - \mathbf{p}\|}, \quad (11)$$

Algorithm 1 Iterative FP Solution for the ML Estimation of $\mathbf{p}$ and Δ

Require: Sensor positions $\mathbf{s}_i$, $i = 1, \dots, N$, pseudo-range measurements $\tilde{r}_i$, $i = 1, \dots, N$, initial position guess $\hat{\mathbf{p}}$.

1: **Initialization:** Set the iteration index $k = 0$, and initialize $\hat{\mathbf{p}}_{\text{ML}}^{(0)} = \hat{\mathbf{p}}$.

2: **repeat**

3: **Offset update:** Update the offset estimate as

$$\hat{\Delta}_{\text{ML}}^{(k+1)} = \frac{1}{N} \sum_{i=1}^N (\tilde{r}_i - \|\mathbf{s}_i - \hat{\mathbf{p}}_{\text{ML}}^{(k)}\|).$$

4: **Position update:** Given the current position estimate $\hat{\mathbf{p}}_{\text{ML}}^{(k)}$ and the current offset estimate $\hat{\Delta}_{\text{ML}}^{(k+1)}$, compute

$$\hat{\mathbf{p}}_{\text{ML}}^{(k+1)} = \alpha - \frac{1}{N} \sum_{i=1}^N (\tilde{r}_i - \hat{\Delta}_{\text{ML}}^{(k+1)}) \frac{\mathbf{s}_i - \hat{\mathbf{p}}_{\text{ML}}^{(k)}}{\|\mathbf{s}_i - \hat{\mathbf{p}}_{\text{ML}}^{(k)}\|}.$$

5: **Iteration update:** Set $k \leftarrow k + 1$.

6: **until** a stopping criterion is satisfied or a maximum number of iterations is reached

7: **Output:** $\hat{\mathbf{p}}_{\text{ML}} = \hat{\mathbf{p}}_{\text{ML}}^{(k)}$, $\hat{\Delta}_{\text{ML}} = \hat{\Delta}_{\text{ML}}^{(k)}$.

where $\alpha = \frac{1}{N} \sum_{i=1}^N \mathbf{s}_i$ denotes the centroid of the sensor positions. Notice that, an alternative fixed point method to handle the ML solution of (7) has been proposed in [47].

Equation (11) is nonlinear in $\mathbf{p}$ and cannot be solved explicitly, since the unknown position appears on both sides of the equation. Moreover, it implicitly assumes that the offset Δ is known, which is generally not the case in practical scenarios.

These observations naturally motivate an iterative solution strategy. Starting from an initial position estimate, the offset Δ is first estimated using (9). The resulting estimate is then used to update the target position according to (11), where the right-hand side is evaluated using the current position estimate. This alternating update of Δ and $\mathbf{p}$ gives rise to the FP iterative algorithm, summarized in Algorithm 1.

The iterative procedure is stopped when the change between two consecutive estimates falls below a prescribed threshold or when a maximum number of iterations is reached. This criterion indicates that the solution has become numerically stable and that further iterations do not produce appreciable changes in the estimated parameters.

As a final remark, it is worth noting that the considered FP procedure belongs to the class of iterative nonlinear optimization algorithms. As such, its performance may depend on the quality of the initial position estimate, and convergence to local minima cannot be excluded in general.

B. Linearized LS Solution Techniques

Linearized LS methods are well-established approaches in passive localization, providing computationally efficient alternatives to nonlinear optimization techniques [11], [48]. These methods rely on suitably linearizing the original nonlinear measurement model in the form

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b}, \quad (12)$$

where $\theta \in \mathbb{R}^p$ denotes the vector of unknown parameters, $\mathbf{A} \in \mathbb{R}^{N \times p}$ with $N \geq p$ is the measurement matrix, and $\mathbf{b} \in \mathbb{R}^N$ is the data vector. In the presence of measurement errors, θ can be estimated using the standard LS criterion as

$$\hat{\theta} = \arg \min_{\theta} \|\mathbf{A}\theta - \mathbf{b}\|^2. \quad (13)$$

Provided that $\mathbf{A}$ has full column rank, this LS problem admits the well-known closed-form solution

$$\hat{\theta} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}. \quad (14)$$

In some scenarios, additional functional relationships among the unknown parameters may be known a priori [4], [5], [49], leading to constrained optimization problems of the form

$$\begin{cases} \text{minimize}_{\theta} & f(\theta) = \|\mathbf{A}\theta - \mathbf{b}\|^2, \\ \text{subject to} & g(\theta) = 0, \end{cases} \quad (15)$$

where $g(\theta)$ is a bespoke quadratic constraint. Such class of quadratically constrained quadratic problems can be solved by introducing the Lagrangian

$$\mathcal{L}(\theta, \lambda) = f(\theta) + \lambda g(\theta), \quad (16)$$

where $\lambda \in \mathbb{R}$ is the Lagrange multiplier. Specifically, according to [50, Theorem 3.2], assuming that the Hessian of the quadratic constraint is not the zero matrix, i.e., $\nabla_{\theta}^2 g(\theta) \neq 0$, and

$$\inf\{g(\theta) : \theta \in \mathbb{R}^p\} < 0 < \sup\{g(\theta) : \theta \in \mathbb{R}^p\}, \quad (17)$$

a vector θ^* is a global minimizer of problem (15) if and only if $g(\theta^*) = 0$ and there is a multiplier λ^* such that the Kuhn-Tucker condition

$$\nabla_{\theta} \mathcal{L}(\theta^*, \lambda^*) = 0 \quad (18)$$

is satisfied with the Hessian $\nabla_{\theta}^2 \mathcal{L}(\theta^*, \lambda^*)$ positive semidefinite, i.e., $\nabla_{\theta}^2 \mathcal{L}(\theta^*, \lambda^*) \succeq 0$.

The general linearized LS formulation, both unconstrained and constrained, provides the foundation for the specific one-shot and iterative methods considered in this work for the estimation of the target position and the common timing offset.

1) Joint Estimation of Target Position and Common Offset:

Neglecting the noise contribution in (4), the TOA-based measurement model can be written as

$$\tilde{r}_i - \Delta = \|\mathbf{s}_i - \mathbf{p}\|, \quad i = 1, \dots, N. \quad (19)$$

Squaring both sides of (19) yields

$$\tilde{r}_i^2 + \Delta^2 - 2\tilde{r}_i \Delta = \|\mathbf{s}_i\|^2 + \|\mathbf{p}\|^2 - 2\mathbf{s}_i^T \mathbf{p}, \quad i = 1, \dots, N. \quad (20)$$

Defining the augmented unknown parameter vector as [9]

$$\theta_{\text{LS}} = \begin{bmatrix} \mathbf{p} \\ \Delta \\ \|\mathbf{p}\|^2 - \Delta^2 \end{bmatrix}, \quad (21)$$

the system of equations resulting from (20) can be written in matrix form as

$$\mathbf{A}_{\text{LS}} \theta_{\text{LS}} = \mathbf{b}_{\text{LS}}, \quad (22)$$

where

$$\mathbf{A}_{\text{LS}} = \begin{bmatrix} -2\mathbf{s}_1^T & 2\tilde{r}_1 & 1 \\ -2\mathbf{s}_2^T & 2\tilde{r}_2 & 1 \\ \vdots & \vdots & \vdots \\ -2\mathbf{s}_N^T & 2\tilde{r}_N & 1 \end{bmatrix}, \quad \mathbf{b}_{\text{LS}} = \begin{bmatrix} \tilde{r}_1^2 - \|\mathbf{s}_1\|^2 \\ \tilde{r}_2^2 - \|\mathbf{s}_2\|^2 \\ \vdots \\ \tilde{r}_N^2 - \|\mathbf{s}_N\|^2 \end{bmatrix}. \quad (23)$$

The model in (22) is linear in the augmented unknown parameter vector θ_{LS} . Thus, provided that $\mathbf{A}_{\text{LS}}$ has full column rank, the LS estimate of the unknown parameters is given by

$$\hat{\theta}_{\text{LS}} = (\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}})^{-1} \mathbf{A}_{\text{LS}}^T \mathbf{b}_{\text{LS}}. \quad (24)$$

This formulation treats the entries of the augmented parameter vector as unconstrained. However, by construction, the components of θ_{LS} are not independent. Indeed, the auxiliary parameter $\|\mathbf{p}\|^2 - \Delta^2$ is functionally related to the position vector $\mathbf{p}$ and the offset Δ . To explicitly enforce this dependency, the following constrained formulation can be considered:

$$\begin{cases} \text{minimize}_{\theta_{\text{LS}}} & \|\mathbf{A}_{\text{LS}} \theta_{\text{LS}} - \mathbf{b}_{\text{LS}}\|^2, \\ \text{subject to} & \theta_{\text{LS}}^T \mathbf{D} \theta_{\text{LS}} + \mathbf{h}^T \theta_{\text{LS}} = 0, \end{cases} \quad (25)$$

where $\mathbf{D}$ and $\mathbf{h}$ are defined as

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}. \quad (26)$$

The constrained optimization problem in (25) can be solved in closed form as

$$\hat{\theta}_{\text{CLS}}(\lambda) = (\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}} + \lambda \mathbf{D})^{-1} \left(\mathbf{A}_{\text{LS}}^T \mathbf{b}_{\text{LS}} - \frac{\lambda}{2} \mathbf{h} \right), \quad (27)$$

where λ is determined by solving the polynomial equation

$$\hat{\theta}_{\text{CLS}}(\lambda)^T \mathbf{D} \hat{\theta}_{\text{CLS}}(\lambda) + \mathbf{h}^T \hat{\theta}_{\text{CLS}}(\lambda) = 0. \quad (28)$$

This is a seventh-degree polynomial whose real roots correspond to candidate solutions of the constrained problem. Among these candidates, only those corresponding to a positive semidefinite Hessian of the Lagrangian represent a valid minimum. For the problem in (25), the Lagrangian is

$$\mathcal{L}(\theta_{\text{LS}}, \lambda) = \|\mathbf{A}_{\text{LS}} \theta_{\text{LS}} - \mathbf{b}_{\text{LS}}\|^2 + \lambda (\theta_{\text{LS}}^T \mathbf{D} \theta_{\text{LS}} + \mathbf{h}^T \theta_{\text{LS}}), \quad (29)$$

with Hessian

$$\nabla_{\theta_{\text{LS}}}^2 \mathcal{L} = 2(\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}} + \lambda \mathbf{D}). \quad (30)$$

Requiring this Hessian to be positive semidefinite, namely

$$\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}} + \lambda \mathbf{D} \succeq \mathbf{0}, \quad (31)$$

implies that the valid Lagrange multiplier λ must satisfy

$$\lambda \in \left[-\frac{1}{\lambda_{\max}}, -\frac{1}{\lambda_{\min}} \right], \quad (32)$$

where $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum eigenvalues, respectively, of

$$(\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}})^{-1/2} \mathbf{D} (\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}})^{-1/2}. \quad (33)$$

Within this interval, the polynomial equation in (28) has a unique real root [50] corresponding to the valid CLS solution².

Before proceeding further, it is worth noting that both the solution to the unconstrained LS problem in (22) and the CLS problem in (25) rely on the matrix $\mathbf{A}_{\text{LS}}$, which depends on the measurements $\tilde{r}_i, i = 1, \dots, N$. As a result, for certain sensor-target geometries, both the matrices $\mathbf{A}_{\text{LS}}$ and $(\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}} + \lambda \mathbf{D})$ can become ill-conditioned, potentially affecting the reliability of the LS and CLS solutions, respectively. This problem is particularly relevant in scenarios with a small number of sensors where poor geometry can further degrade conditioning.

A first solution to mitigate this phenomenon, as proposed in this paper, consists in developing a modified version of the CLS-TOA (hereafter referred to as MCLS-TOA), as described below. Specifically, the CLS-TOA estimator is first computed, and the condition number of the matrix $(\mathbf{A}_{\text{LS}}^T \mathbf{A}_{\text{LS}} + \lambda \mathbf{D})$ is evaluated. If this value exceeds a predetermined threshold (indicative of ill-conditioning of the matrix) the ML-TOA estimation algorithm is subsequently applied. In this way, the procedure retains the non-iterative nature of the CLS-TOA under well-conditioned scenarios, while ensuring improved numerical stability and estimation reliability when ill-conditioning occurs.

As a second solution to the above problems, relies on the use of alternative LS formulations in which the measurement matrix does not depend on the available measurements. This prevents scenario-dependent ill-conditioning and enhances numerical robustness, particularly in configurations characterized by a limited number of sensors.

2) Iterative Mixed LS/ML Estimation of Target Position and Common Offset: To eliminate the dependency of the measurement matrix on the observations $\tilde{r}_i, i = 1, \dots, N$, this section focuses on the development of an Iterative LS (ILS) estimation strategy that exploits preliminary knowledge of the common offset Δ . The key idea is to decouple the estimation of the target position and the offset by alternating between the two optimizations. Precisely, starting from an initial offset estimate, the measurements are suitably corrected and the target position is computed through a formulation that does not explicitly include Δ among the unknown parameters. The resulting position estimate is then used to refine the offset estimate according to a ML criterion. This alternating procedure is repeated until a specific condition is met, progressively improving both estimates.

To formalize the problem, let us observe that for a given offset Δ and neglecting the noise contribution, the TOA-based measurement model in (4) can be written as

$$\bar{r}_i = \tilde{r}_i - \Delta = \|\mathbf{s}_i - \mathbf{p}\|, \quad i = 1, \dots, N, \quad (34)$$

where $\bar{r}_i, i = 1, \dots, N$, denotes the offset-compensated pseudo-range measurement. Squaring both sides of (34) yields

$$\bar{r}_i^2 = \|\mathbf{s}_i\|^2 - 2\mathbf{s}_i^T \mathbf{p} + \|\mathbf{p}\|^2, \quad i = 1, \dots, N. \quad (35)$$

²The case in which λ coincides with one of the boundaries in (32) requires special treatment. However, since this situation is unlikely, it is not considered in the present work.

Defining the augmented parameter vector

$$\boldsymbol{\theta}_{\text{ILS}} = \begin{bmatrix} \mathbf{p} \\ \|\mathbf{p}\|^2 \end{bmatrix} \in \mathbb{R}^{d+1}, \quad (36)$$

the squared-range equations in (35) can be expressed in linear form as

$$\mathbf{A}_{\text{ILS}} \boldsymbol{\theta}_{\text{ILS}} = \mathbf{b}_{\text{ILS}}, \quad (37)$$

where

$$\mathbf{A}_{\text{ILS}} = \begin{bmatrix} -2\mathbf{s}_1^T & 1 \\ -2\mathbf{s}_2^T & 1 \\ \vdots & \vdots \\ -2\mathbf{s}_N^T & 1 \end{bmatrix}, \quad \mathbf{b}_{\text{ILS}} = \begin{bmatrix} \bar{r}_1^2 - \|\mathbf{s}_1\|^2 \\ \bar{r}_2^2 - \|\mathbf{s}_2\|^2 \\ \vdots \\ \bar{r}_N^2 - \|\mathbf{s}_N\|^2 \end{bmatrix}. \quad (38)$$

It is worth noting that, in this formulation, the measurement matrix $\mathbf{A}_{\text{ILS}}$ does not depend on the observations, and therefore its rank and conditioning depend solely on the sensor geometry. Provided that $\mathbf{A}_{\text{ILS}}$ has full column rank, the LS estimate of the augmented parameter vector is given by

$$\hat{\boldsymbol{\theta}}_{\text{ILS}} = (\mathbf{A}_{\text{ILS}}^T \mathbf{A}_{\text{ILS}})^{-1} \mathbf{A}_{\text{ILS}}^T \mathbf{b}_{\text{ILS}}. \quad (39)$$

Using the position estimate $\hat{\mathbf{p}}_{\text{ILS}}$, obtained from the first 2 entries of $\hat{\boldsymbol{\theta}}_{\text{ILS}}$, the offset Δ can be updated according to the ML estimator in (9) as

$$\hat{\Delta}_{\text{ML}} = \frac{1}{N} \sum_{i=1}^N (\tilde{r}_i - \|\mathbf{s}_i - \hat{\mathbf{p}}_{\text{ILS}}\|). \quad (40)$$

The updated offset estimate is then used to further correct the pseudo-range measurements, and the procedure is repeated until a prescribed stopping criterion is met or a maximum number of iterations is reached. Algorithm 2 summarizes the proposed iterative mixed unconstrained LS/ML estimation of target position and common offset, hereafter referred to as the ILS-TOA approach. The algorithm is initialized with a preliminary protocol-based offset estimate $\bar{\Delta}$. However, in the absence of prior information, the initialization $\bar{\Delta} = 0$ can also be adopted.

The iterative formulation in Algorithm 2 treats the entries of $\boldsymbol{\theta}_{\text{ILS}}$ as unconstrained, thereby neglecting the inherent dependency between the auxiliary parameter $\|\mathbf{p}\|^2$ and the target position. However, improved estimation performance can be achieved by explicitly enforcing this dependency through an Iterative Constrained LS (ICLS) approach. Specifically, the unconstrained estimation in Step 5 of Algorithm 2 can be replaced with the solution of the following constrained formulation:

$$\begin{cases} \text{minimize} & \|\mathbf{A}_{\text{ILS}} \boldsymbol{\theta}_{\text{ICLS}} - \mathbf{b}_{\text{ILS}}\|^2, \\ \boldsymbol{\theta}_{\text{ICLS}} & \\ \text{subject to} & \boldsymbol{\theta}_{\text{ICLS}}^T \mathbf{P} \boldsymbol{\theta}_{\text{ICLS}} + \mathbf{q}^T \boldsymbol{\theta}_{\text{ICLS}} = 0, \end{cases} \quad (41)$$

where the matrices $\mathbf{P}$ and $\mathbf{q}$ are defined as

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}. \quad (42)$$

The constrained problem in (41) has been handled in [4], [5]. Here, the corresponding solution approach is adopted in

Algorithm 2 Iterative Mixed Unconstrained LS/ML Estimation of Target Position and Common Offset

Require: Sensor positions $\mathbf{s}_i$, $i = 1, \dots, N$, pseudo-range measurements $\tilde{r}_i$, $i = 1, \dots, N$, initial offset estimate $\bar{\Delta}$.

1: **Initialization:** Set $\hat{\Delta}_{\text{ML}}^{(0)} = \bar{\Delta}$ and initialize the iteration index $k = 0$.

2: **repeat**

3: **Position estimation:** Given the current offset estimate $\hat{\Delta}_{\text{ML}}^{(k)}$, form the corrected pseudo-ranges

$$\bar{r}_i^{(k)} = \tilde{r}_i - \hat{\Delta}_{\text{ML}}^{(k)}, \quad i = 1, \dots, N.$$

4: **Compute**

$$\beta_i^{(k)} = \left(\bar{r}_i^{(k)}\right)^2 - \|\mathbf{s}_i\|^2, \quad i = 1, \dots, N,$$

and form the data vector

$$\mathbf{b}_{\text{ILS}}^{(k)} = \left[\beta_1^{(k)}, \beta_2^{(k)}, \dots, \beta_N^{(k)}\right]^T.$$

5: **Estimate the augmented parameter vector**

$$\hat{\boldsymbol{\theta}}_{\text{ILS}}^{(k)} = (\mathbf{A}_{\text{ILS}}^T \mathbf{A}_{\text{ILS}})^{-1} \mathbf{A}_{\text{ILS}}^T \mathbf{b}_{\text{ILS}}^{(k)},$$

and extract the target position estimate $\hat{\mathbf{p}}_{\text{ILS}}^{(k)}$.

6: **Offset estimation:** Update the offset estimate as

$$\hat{\Delta}_{\text{ML}}^{(k+1)} = \frac{1}{N} \sum_{i=1}^N (\tilde{r}_i - \|\mathbf{s}_i - \hat{\mathbf{p}}_{\text{ILS}}^{(k)}\|).$$

7: **Iteration update:** Set $k \leftarrow k + 1$.

8: **until** a stopping criterion is satisfied or a maximum number of iterations is reached

9: **Output:** $\hat{\mathbf{p}}_{\text{ILS}} = \hat{\mathbf{p}}_{\text{ILS}}^{(k)}$, $\hat{\Delta}_{\text{ML}} = \hat{\Delta}_{\text{ML}}^{(k)}$.

the context of AIS-based localization. Specifically, problem (41) admits the closed-form solution

$$\hat{\boldsymbol{\theta}}_{\text{ICLS}}(\lambda) = \left(\mathbf{A}_{\text{ILS}}^T \mathbf{A}_{\text{ILS}} + \lambda \mathbf{P}\right)^{-1} \left(\mathbf{A}_{\text{ILS}}^T \mathbf{b}_{\text{ILS}} - \frac{\lambda}{2} \mathbf{q}\right), \quad (43)$$

where λ is determined by solving

$$\hat{\boldsymbol{\theta}}_{\text{ICLS}}(\lambda)^T \mathbf{P} \hat{\boldsymbol{\theta}}_{\text{ICLS}}(\lambda) + \mathbf{q}^T \hat{\boldsymbol{\theta}}_{\text{ICLS}}(\lambda) = 0, \quad (44)$$

which is a fifth-degree polynomial equation. The real roots of this polynomial correspond to feasible points of the constrained problem. Among these candidates, the optimum solution is selected as the one that satisfies the positive semidefiniteness of the Hessian and minimizes the original cost function.

It is worth noting that the iterative procedure for estimating the target position and common offset remains the same as in the unconstrained case. The only difference is that, at each iteration, the unconstrained LS step for updating the position is replaced with the constrained LS update in (43).

IV. PERFORMANCE ASSESSMENT

The performance of the proposed TOA-based localization methods is evaluated in a real-world scenario considering

an AIS-based passive localization system composed of four BSs. The BSs are deployed in southern Italy at the following locations: Monte Redentore (40.6561° N, 17.2889° E), Monte San Michele (39.2861° N, 16.9794° E), Salento Turrise (40.0621° N, 18.1424° E), and Stigliano (40.3955° N, 16.2321° E). The geometric configuration of the BS network is illustrated in Figure 3, where $\mathbf{s}_i$, $i = 1, \dots, 4$, denotes the geographic coordinates of the i -th BS³. The experimental analysis is carried out on six distinct datasets (provided by the Italian Coast Guard Headquarters), all sharing the same BS configuration, while differing in terms of the vessel positions or trajectories, as illustrated in the six subplots of Figure 3. The number of available measurements varies across the different datasets, namely $M = \{1427, 541, 1804, 863, 182, 943\}$.

It is worth pointing out that the Italian AIS BSs are equipped with TOA measurement capabilities featuring a temporal granularity of 1 μ s. The measured TOA is not computed at the exact beginning of the AIS signal, but after a delay corresponding to the first 32 bits, which constitutes the preamble used for time synchronization with the AIS data field, in agreement with the AIS standard. This choice is also consistent with practical receiver design considerations, since TOA estimation is typically performed after the synchronization phase, when transient effects in the signal, which are accounted for by the ramp-up interval, are no longer present. However, since this delay is common to all AIS transmitters and receivers, it is absorbed into the temporal offset term of the localization model; therefore, it does not introduce any bias in the position estimation and does not require explicit compensation within the proposed framework.

In the following, the proposed TOA-based algorithms are validated on the above-described measured data. In addition, a performance comparison is carried out with respect to the CLS and ML algorithms operating in the TDOA domain, denoted as CLS-TDOA and ML-TDOA [3], respectively. This comparison is included for methodological completeness, since TOA and TDOA formulations rely on different statistical models. In ideal conditions, TDOA measurements are generally obtained as differences of independent TOA measurements, which results in error terms with doubled variance and in the loss of one independent measurement. Consequently, the TOA formulation is herein expected to provide superior estimation performance.

For a fair comparison, the considered iterative algorithms are instantiated as follows. Both the ILS-TOA and ICLS-TOA algorithms are initialized with $\bar{\Delta} = 0$ m. The ML-TOA solution is computed using the FP formulation of Section III initialized with the CLS-TOA output. The threshold on the matrix condition number for the MCLS-TOA is set to 4×10^{16} . Notice that, this threshold is empirically selected so as to identify severely ill-conditioned CLS-TOA solutions. In fact, results (not reported here for brevity) indicate that, if

³Since the localization model is formulated in a two-dimensional Cartesian coordinate system, all geodetic coordinates are first converted into a local Cartesian system before applying the localization methods. Specifically, the conversion is performed using the WGS84 ellipsoid as the Earth model, with $\mathbf{s}_1$ selected as the reference point and zero altitude assumed for all points for simplicity.

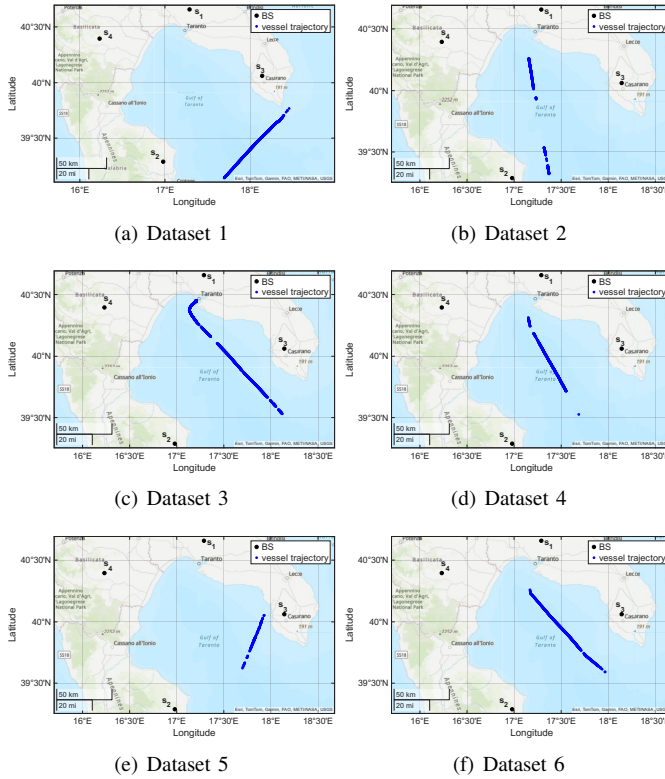


Fig. 3: Geographic distribution of the AIS datasets considered in the experimental analysis.

5 km is taken as the maximum tolerable RMSE, threshold values up to 5×10^{16} remain acceptable. For all iterative methods, the maximum number of iterations is set to 100. Finally, the ML-TDOA estimate is evaluated by iteratively solving the nonlinear LS problem stemming from the ML formulation under Gaussian noise, initialized with the CLS-TDOA solution.

The accuracy of the localization algorithms is quantified in terms of the root mean square error (RMSE), defined as

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{m=1}^M \|\hat{\mathbf{p}}_m - \mathbf{p}_m\|^2}, \quad (45)$$

where $\mathbf{p}_m$ and $\hat{\mathbf{p}}_m$, $m = 1, \dots, M$, denote the true and estimated vessel positions, respectively. In this respect, it is worth noting that the data provided by the Italian Coast Guard Headquarters were collected under normal operating conditions and that no known AIS-spoofing events were reported during the considered data-acquisition period. On this basis, the vessel positions $\mathbf{p}_m$ extracted from the processed AIS messages are adopted as the assumed ground truth for the performance assessment. Accordingly, the reported RMSE values should be interpreted relative to these reference positions.

The availability of these reference positions also enables a preliminary trajectory-level characterization of the TOA errors. In particular, within each dataset, the marginal TOA-error standard deviations exhibit comparable values across the available BSs, thereby providing empirical support, at the aggregate dataset level, for the BS-wise homoscedastic error

model adopted in (5). A more detailed position-dependent statistical characterization of the TOA errors, aimed at further assessing both the equal-variance and Gaussian assumptions, would require multiple measurements acquired at each vessel position under comparable operating conditions. Such repeated measurements are not available in the considered datasets, and the corresponding analysis is therefore left for future investigation.

Table I reports the RMSE values obtained by the considered TOA- and TDOA-based localization algorithms over the six AIS datasets.

In particular, from Table I, it can be observed that TOA-based approaches generally outperform their TDOA-based counterparts across all datasets, highlighting the benefit of accounting for the unknown temporal offset at the algorithm design stage. Among the reference methods, ML-TOA mostly achieves the lowest RMSE values, confirming its optimality under ideal modeling assumptions. The proposed MCLS-TOA method exhibits competitive performance across all datasets and achieves the lowest RMSE in two out of six cases (datasets 4 and 6), while maintaining good performance in the remaining scenarios as well as being non-iterative for most of the estimation instances. In particular, compared with the standard CLS-TOA approach, MCLS-TOA provides a substantial RMSE reduction by mitigating the numerical instability and the associated performance degradation observed for CLS-TOA in Datasets 1, 3, 5, and 6. However, since RMSE is particularly sensitive to extreme localization errors, the observed reduction alone does not establish whether MCLS-TOA improves localization accuracy over the whole vessel trajectory or primarily mitigates a subset of extreme localization errors. A more detailed examination of this behavior is provided later in this section.

As for ILS-TOA and ICLS-TOA, both methods exhibit stable performance across the considered datasets, and outperform the MCLS-TOA method for some of the analyzed datasets. Interestingly, the ICLS-TOA algorithm shows higher performance than the ILS-TOA in all cases except Dataset 4, where the two methods achieve comparable results. This result confirms the effectiveness of incorporating constraints into the LS formulation. Overall, the RMSE values show that the proposed ICLS-TOA approach achieves an accuracy close to that of ML-TOA while retaining comparable computational complexity. It is worth underlining that the RMSE values reported in Table I are consistent with the intrinsic limitations of AIS-based TOA measurements. In fact, as discussed in Section II, the narrowband nature of AIS signals, together with SNR-related effects, leads to effective TOA uncertainties on the order of tens of microseconds, corresponding to kilometer-level range-equivalent errors. The reported values should be interpreted as the accuracy achievable by a passive verification layer based on the existing AIS infrastructure. Moreover, in the context of maritime surveillance and AIS position verification through physical-layer information, kilometer-level RMSE values can still be useful for detecting inconsistencies between the AIS-reported position and the position inferred from signal-propagation measurements.

To further corroborate the results reported in Table I,

TABLE I: RMSE (in km) obtained by the considered localization algorithms over the six datasets

Dataset	CLS-TDOA	ML-TDOA	ML-TOA	ILS-TOA	ICLS-TOA	CLS-TOA	MCLS-TOA
Dataset 1	8.35	5.94	3.95	5.13	4.67	58.75	5.67
Dataset 2	4.16	2.55	2.05	2.29	2.24	2.83	2.83
Dataset 3	3.55	2.84	2.15	2.48	2.36	26.86	2.72
Dataset 4	2.80	2.46	1.77	1.97	1.97	1.96	1.73
Dataset 5	4.69	2.95	1.85	2.76	2.48	24.13	2.54
Dataset 6	3.24	2.71	1.79	2.21	2.16	36.50	1.76

Figure 4 shows the boxplot diagrams of the position error for datasets 2 and 3, including the ML-TOA, ILS-TOA, ICLS-TOA, CLS-TOA, and MCLS-TOA algorithms. In agreement with the previous analysis, these plots indicate that, for dataset 2, the different methods exhibit comparable error behavior, whereas for dataset 3 the CLS-TOA method produces a significant number of large outliers. This behavior is strongly mitigated by the MCLS-TOA, confirming the effectiveness of the proposed correction strategy in ill-conditioned cases.

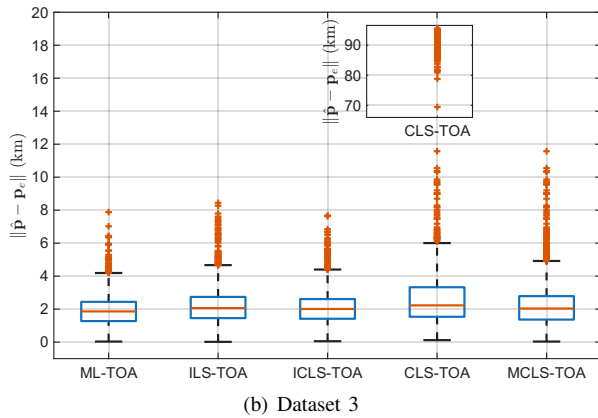
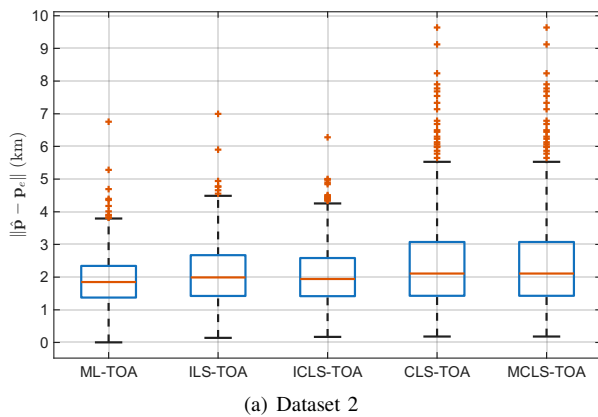


Fig. 4: Boxplots of the root square error for datasets 2 and 3 using ML-TOA, ILS-TOA, ICLS-TOA, CLS-TOA, and MCLS-TOA.

The quantitative results reported in Table I are further supported by a qualitative analysis based on the spatial distribution of the estimated positions. In particular, Figures 5, and 6 provide a visual comparison of the position estimates obtained for Dataset 2 and 3, respectively, by superimposing the estimated vessel positions on a topographic map of the area of interest.

The visual inspection of the results is in agreement with

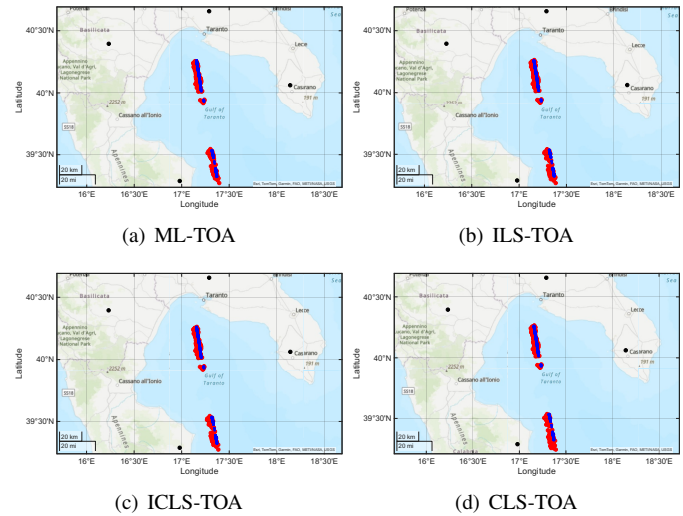


Fig. 5: Localization results for Dataset 2. Estimated vessel positions obtained using the ML-TOA, ILS-TOA, ICLS-TOA, and CLS-TOA algorithms are overlaid on a topographic map of the area of interest. Black dots indicate the BS locations, blue dots denote the true vessel positions, and red dots represent the estimated positions.

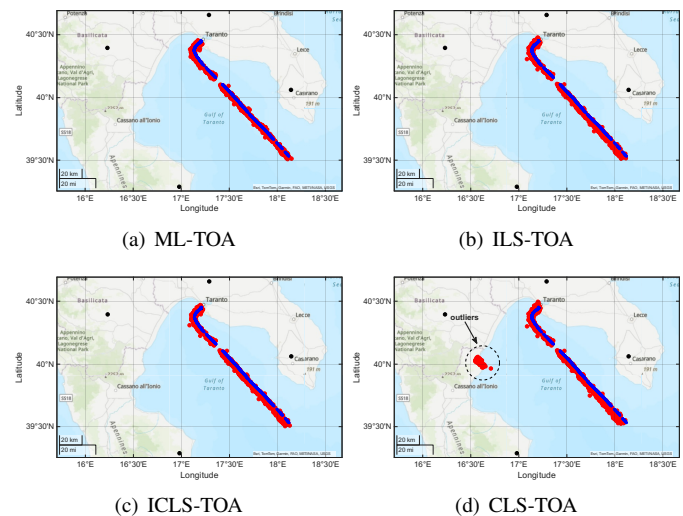


Fig. 6: Localization results for Dataset 3. Estimated vessel positions obtained using the ML-TOA, ILS-TOA, ICLS-TOA, and CLS-TOA algorithms are overlaid on a topographic map of the area of interest. Black dots indicate the BS locations, blue dots denote the true vessel positions, and red dots represent the estimated positions.

the RMSE values reported in Table I. Specifically, algorithms yielding lower RMSE values produce position estimates that are more tightly clustered around the true vessel trajectory, whereas methods characterized by higher RMSE exhibit greater dispersion and, in some cases, a bias with respect to the ground-truth positions.

Notably, the CLS-TOA algorithm occasionally produces incorrect estimates, which are effectively mitigated by its modified version, MCLS-TOA. As discussed in Section III, this behavior can be attributed to the ill-conditioning of the Hessian matrix in (30), which in turn is associated with unfavorable target-sensor geometries, particularly when only a small number of sensors is available (as in the considered scenario). This effect is further illustrated in Figure 7, where the condition number of the CLS-TOA model matrix is shown together with the localization maps obtained using both CLS-TOA and MCLS-TOA for two different datasets. The upper subplots refer to Dataset 2, while the lower subplots correspond to Dataset 3.

For Dataset 2, the condition number remains nearly constant across all measurements, and the CLS-TOA localization map shows no outliers, indicating a well-conditioned estimation problem. As a result, the MCLS-TOA map closely matches that obtained with CLS-TOA.

In contrast, for Dataset 3, the condition number increases by approximately two orders of magnitude for several measurements. The estimates associated with these large condition numbers are highlighted in green in the CLS-TOA map, where it is evident that most of them correspond to large deviations. As expected, in the MCLS-TOA map these points are effectively corrected, thereby resolving the instability caused by the ill-conditioned model matrix.

It is worth underlining that, the large RMSE values obtained by CLS-TOA for some datasets are associated with occasional extreme localization errors, reflecting the numerical instability of the squared-range LS formulation. Therefore, in these cases, the RMSE is particularly sensitive to outliers and should not be interpreted alone. Instead, its behavior should be considered jointly with the histograms of localization error, which allow the typical localization performance to be distinguished from the effect of occasional extreme failures.

Beyond the accuracy assessment, it is important to examine the convergence properties of the iterative algorithms. The RMSE values reported previously describe the final performance, but do not provide insight into the evolution of the estimates across iterations. In this respect, the convergence behavior of ILS-TOA, ICLS-TOA, and ML-TOA in terms of both position RMSE and offset estimate is analyzed. Specifically, with reference to Dataset 2, Figures 8 and 9 illustrate the evolution of the RMSE of the estimated position and temporal offset versus the iteration index for the ILS-TOA, ICLS-TOA, and ML-TOA algorithms.

The ML-TOA algorithm reaches its minimum RMSE and a stable offset estimate almost at the first iteration, thus exhibiting immediate convergence. This behavior is expected, as the ML approach is initialized with the CLS solution, which provides a reliable initial estimate for both position and offset. Hence, only minor updates are required in subsequent

iterations, leading to fast convergence.

For the ILS-TOA and ICLS-TOA approaches, two different initializations of the temporal offset are considered, namely $\Delta_t^{(0)} = 0$ ms and $\Delta_t^{(0)} = 3.3$ ms, where $\hat{\Delta}_t = \bar{\Delta}/c$. The latter value is derived from a priori information on the time instant within the AIS signal at which the BSs measure the arrival time, i.e., 32 bits after the ramp-up interval. This corresponds to a delay of $32 \times 104.17 \mu\text{s}$, i.e., approximately 3.3 ms. Regardless of the temporal offset initialization, both methods exhibit a monotonic decrease of the RMSE and convergence toward a stable value of the offset estimate within the first few iterations. In particular, when initialized with $\Delta_t^{(0)} = 0$ ms, the ILS-TOA and ICLS-TOA approaches require approximately 15 (resp. 12) and 5 (resp. 3) instances to reach convergence in terms of RMSE (offset estimate), respectively. When the initialization is set to $\Delta_t^{(0)} = 3.3$ ms, the convergence of the ILS-TOA method becomes faster, requiring approximately 10 iterations for the position estimate (instead of 15) and 7 for the offset estimate (instead of 12). This behavior is expected, as the initial offset guess is closer to the actual value. On the other hand, the ICLS-TOA algorithm shows reduced sensitivity to the initialization, which improves estimation accuracy across iterations. It is worth noting that, after a limited number of iterations (fewer than 15), the estimates obtained with both ILS-TOA and ICLS-TOA stabilize, and the two initialization cases become practically indistinguishable, also in comparison with ML-TOA. This behavior confirms that the proposed iterative schemes are not overly sensitive to the initial offset estimate. Moreover, it highlights the benefit of a more accurate initialization, derived from prior knowledge of the AIS timing structure, in terms of convergence speed.

To conclude the analysis, boxplots of the temporal offset estimates are also considered. Figure 10 reports the results for datasets 2 and 3, including the ML-TOA, ILS-TOA, ICLS-TOA, CLS-TOA, and MCLS-TOA algorithms. The boxplot analysis provides additional insight into the distribution of the estimates, complementing the evaluation based on the average offset values. Overall, all the considered algorithms (except for the CLS-TOA which suffers ill-conditioning of the model matrix for some target-sensor geometries) exhibit similar behavior, with boxplots approximately centered around the same median values and showing comparable dispersions. Moreover, for Dataset 2, all methods present a limited number of outliers (less than 2% of the estimates are identified as outliers). An interesting difference emerges for Dataset 3 instead. In fact, due to the ill-conditioning of the CLS-TOA Hessian matrix, this algorithm produces a significant number of outliers (namely, 35.49% of the estimates are classified as outliers for CLS-TOA, while only 3.88% for ML-TOA), that are also significantly farther from the concentration box of the estimates. These extreme values are effectively eliminated by the MCLS-TOA algorithm, which mitigates the instability caused by the poor conditioning of the model matrix and restores a distribution comparable to that produced by the other methods (in fact, it has almost the same percentage of outliers as the ML-TOA).

As a remark, it is worth noting that AIS transmissions occur at relatively low rates, typically ranging from a few seconds to

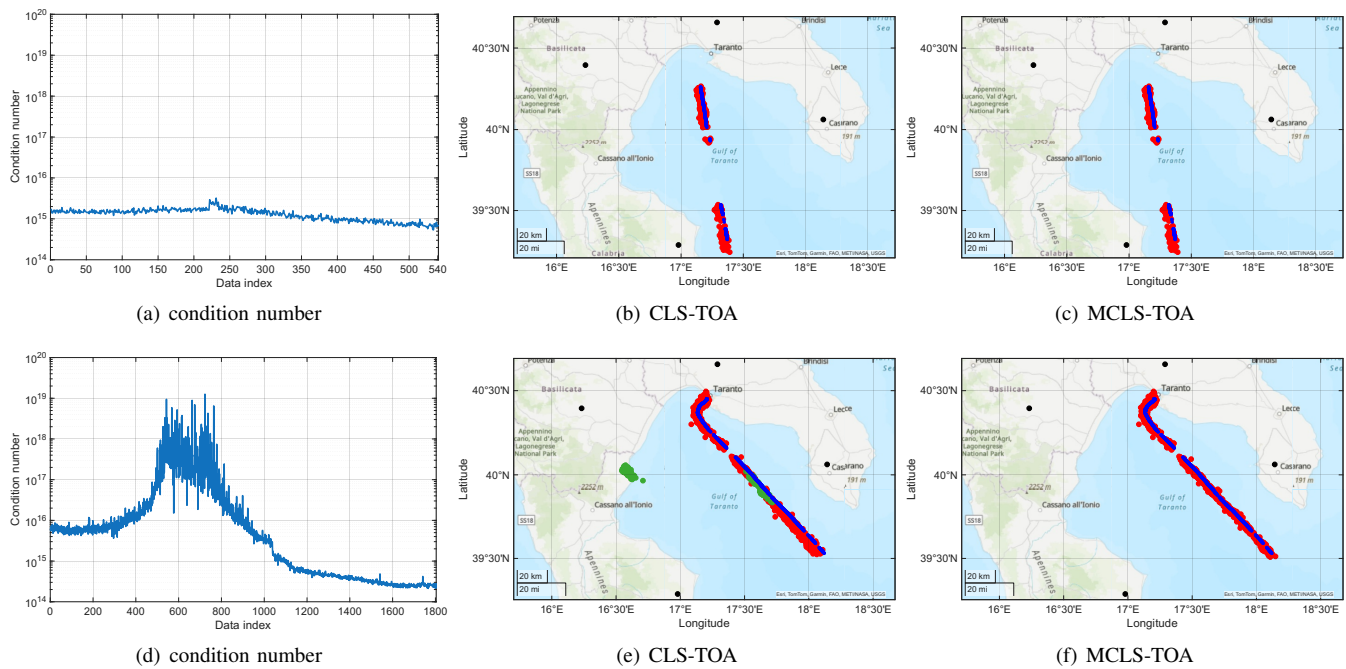


Fig. 7: Stability analysis. From left to right, the subplots show: the condition number of the CLS-TOA model matrix versus the measurement index, the estimated vessel positions obtained with CLS-TOA, and the estimated vessel positions obtained with MCLS-TOA. The top row refers to Dataset 2, while the bottom row corresponds to Dataset 3.

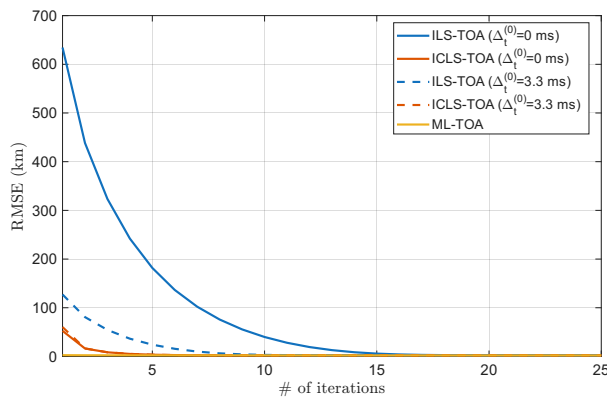


Fig. 8: Convergence behavior of the ILS-TOA, ICLS-TOA, and ML-TOA algorithms on Dataset 2. The plot shows the RMSE of the estimated position versus the number of iterations. The iterative algorithms are initialized with $\Delta_t^{(0)} = 0$ ms and $\Delta_t^{(0)} = 3.3$ ms.

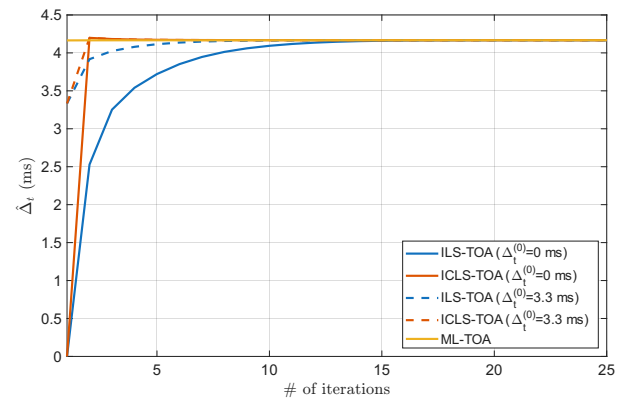


Fig. 9: Convergence behavior of the ILS-TOA, ICLS-TOA, and ML-TOA algorithms on Dataset 2. The plot shows the evolution of the estimated temporal offset versus the number of iterations. The iterative algorithms are initialized with $\Delta_t^{(0)} = 0$ ms and $\Delta_t^{(0)} = 3.3$ ms.

some minutes depending on the message type and the vessel dynamics. As a consequence, the time interval between consecutive received signals is sufficiently large to relax real-time processing constraints. Therefore, the computational complexity of the considered localization algorithms does not represent a limiting factor, and iterative or computationally demanding solutions can be effectively employed without impacting the practical deployment. Moreover, since the proposed approach relies solely on physical-layer measurements, it enables the estimation of vessel positions even for AIS messages that do not explicitly include position information, thereby extending

localization capabilities beyond standard AIS position reports.

A limitation of the present experimental evaluation is that all the considered datasets were collected using the same four-BS configuration. Although the datasets include different vessel trajectories and numbers of measurements, the available data do not permit a systematic assessment of localization performance under different BS geometries or with different numbers of receiving sensors. Since both the number and spatial configuration of the BSs affect the observability, measurement redundancy, and geometric diversity of the localization problem, further experiments involving additional network configurations would provide a more comprehensive

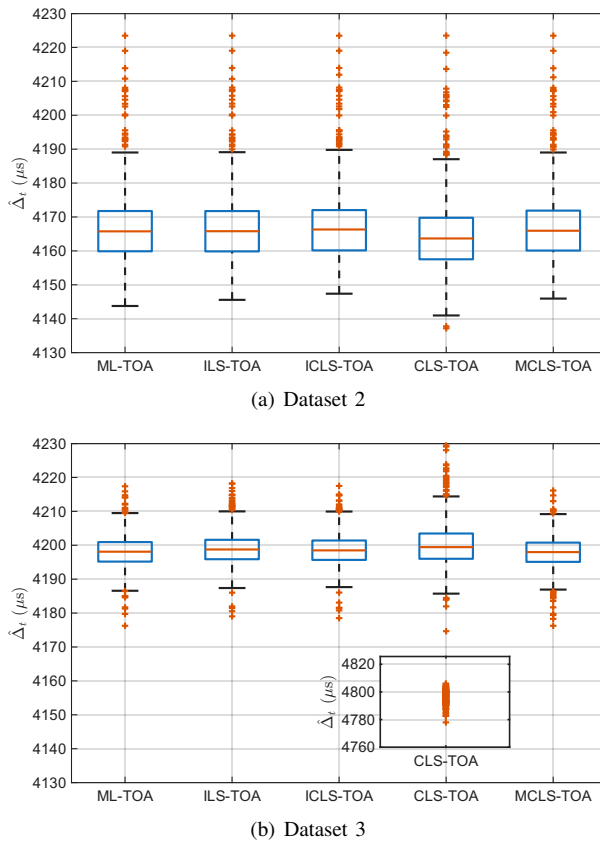


Fig. 10: Boxplots of the offset estimates for datasets 2 and 3 using ML-TOA, ILS-TOA, ICLS-TOA, CLS-TOA, and MCLS-TOA.

assessment of the robustness and scalability of the proposed approaches. It is however worth noting that, for this specific problem, it is unlikely that more than four BSs would follow the same vessel. Such an investigation is left for future work.

A final consideration concerns non-cooperative scenarios where a vessel deliberately turns off its AIS transponder [51]. In such conditions, AIS-based localization methods can no longer operate directly because no cooperative signal is available during the outage period. A possible mitigation strategy is represented by dead reckoning techniques based on the last available AIS information, including position, speed, and course over ground [52]. However, the prediction accuracy rapidly degrades over time due to vessel maneuvers, environmental perturbations (e.g., wind, currents, waves), and navigation uncertainties. Consequently, dead reckoning may provide only short-term trajectory continuity, while its error typically grows unbounded with the AIS outage duration. For prolonged non-cooperative scenarios, additional sensing modalities are required, such as radar, Synthetic Aperture Radar (SAR) imagery, Electro-Optical/Infrared (EO/IR) sensors, or multi-sensor tracking approaches. The extension of the proposed framework toward hybrid non-cooperative tracking architectures could be the object of future work.

V. CONCLUSIONS

This work investigated the application of TOA-based passive localization techniques for maritime situational awareness using AIS signals. Unlike traditional cooperative AIS system, which rely on transmitted positional information and may therefore be vulnerable to spoofing or manipulation, TOA-based approaches provide an independent physical-layer mechanism for verifying vessel positions through the propagation time of AIS signals across multiple spatially distributed BSs.

A key challenge in this context is the presence of an unknown timing offset common to all receiving BSs for a given AIS transmission. To address this issue, the localization problem has been formulated accounting for this unknown temporal bias explicitly in the measurement model. Several estimation approaches have been considered, including ML, LS, and constrained methods. In particular, CLS-TOA is a non-iterative approach that explicitly accounts for the unknown timing offset. However, it may suffer from ill-conditioning of the measurement model under poor target-sensor geometry. To mitigate this effect, the MCLS-TOA method, mostly non-iterative, has been also proposed. Moreover, iterative schemes such as ILS-TOA and ICLS-TOA have been designed to provide additional practical solutions to the resulting nonlinear estimation problem. In particular, replacing the nuisance timing offset with its closed-form ML estimate, the dimensionality of the original joint optimization problem is reduced, making the solution less sensitive to a limited number of sensors.

Comparative evaluation using real AIS data showed that the considered localization schemes provide an effective balance between estimation accuracy and computational complexity. In particular, they achieve positioning performance close to the ML estimator while keeping real-time operation requirements compatible with the data rate of AIS transmissions. The results also indicated that the iterative approaches can achieve up to a 20% reduction in position RMSE as compared with conventional ML-TDOA localization on some datasets, while MCLS-TOA achieved reductions of up to 30%. These findings highlight the potential of TOA-based techniques as an effective alternative to TDOA-based localization, enabling accurate and real-time passive vessel positioning using existing AIS infrastructures.

As next research avenues, bespoke a posteriori corrections ensuring feasibility of the estimated position could be incorporated into the proposed localization framework in order to satisfy additional geographical and/or timing constraints. More generally, such constraints could also be directly integrated into a new constrained optimization formulation. While potentially improving localization robustness and physical consistency, these approaches would require careful analysis, since the resulting estimation problem may no longer admit closed-form or polynomial-based solutions.

Future research may also address the development and validation of calibration-based or joint multitransmission estimation strategies to account for potential receiver-specific timing biases arising, for instance, from residual synchronization errors or differences in receiver-processing latency. Finally, the proposed TOA-based localization framework may also be

extended to support the detection and classification of some spoofing attacks (such as falsified positions, ghost vessels, trajectory manipulation, fake collision alert, and/or search and rescue deception), thereby further enhancing maritime situational awareness.

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