Uplink Sum-Rate Optimization in RIS-Assisted NOMA Systems with Spatial Correlation

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***Abstract*—This paper addresses the uplink sum-rate maximization in RIS-assisted power-domain NOMA systems. The aim is to optimize RIS phase shifts under unit-modulus constraints, resulting in a non-convex problem with coupled variables. To tackle this, an alternating optimization (AO) method is proposed, in which the RIS configuration is updated for one user at a time, keeping others fixed. Each step uses a phase-only conjugate gradient method (CGM), inspired by adaptive array processing. Various models for spatial correlation among the RIS elements are explicitly considered. Simulations show that the proposed AO-CGM approach improves both the sum-rate and the user fairness compared to conventional methods.**

***Index Terms*—6G networks, alternating optimization (AO), non-orthogonal multiple access (NOMA), reconfigurable intelligent surface (RIS).**

I. INTRODUCTION

WIRELESS communications for sixth-generation (6G) are becoming increasingly complex, driven by the need to simultaneously support communication and sensing functions [1], [2]. In this context, non-orthogonal multiple access (NOMA) has emerged as a key multiple access technique, offering improved quality of service (QoS), higher spectral efficiency, and massive connectivity [3], [4]. Specifically, power-domain NOMA (P-NOMA) allows multiple users to share the same time-frequency resources by leveraging power-domain multiplexing. In the uplink, users with stronger channel gains are decoded first and removed via successive interference cancellation (SIC), allowing the decoding of weaker users' signals. Another key enabler for 6G networks is the reconfigurable intelligent surface (RIS) [5], [6], a surface composed of many passive, reconfigurable elements that can reflect incident signals in a controlled way. By intelligently tuning the phase shifts of these elements, RIS can dynamically create virtual line-of-sight (LOS) paths even in obstructed scenarios [7]. Several approaches have been proposed to integrate RIS into NOMA-based communication systems effectively [8]–[10]. However, most existing methods rely on semidefinite relaxation (SDR) techniques, which involve significant computational complexity.

This work focuses on a RIS-assisted uplink communication scenario based on P-NOMA, assuming no LOS path between the base station (BS) and the users. The problem is formulated as a constrained optimization task, where the RIS phase shift vector is optimized to maximize the uplink sum-rate at the BS under unit-modulus constraints. The resulting problem is non-convex due to both the nature of the objective function and the unit-modulus constraint on each RIS element. Additionally, since the RIS configuration is shared across all users, the objective function includes non-separable terms dependent on a common RIS parameter set. As a result, joint optimization across users becomes computationally intractable and lacks a closed-form solution. To solve the problem, a cost-effective approach based on alternating optimization (AO) is proposed in our previous work [11] in the case of spatially uncorrelated scenarios. This paper builds on the same setup, but it offers a novel contribution by considering scenarios involving phase-correlation among RIS elements. The RIS phase shifts are optimized iteratively one user at a time, while the configurations for the remaining users are kept fixed. This decomposition transforms the original coupled and non-convex problem into a sequence of more tractable subproblems. Each subproblem corresponds to a phase-only design task, which is addressed by adapting the core principles of the conjugate gradient descent method (CGM) introduced in [12]. Hence, the same methodology as in [12], [13] is applied to solve the RIS phase optimization problem individually for each user. The effectiveness of the proposed AO-based RIS phase optimization algorithm is demonstrated in the analysis section in challenging scenarios characterized by RIS phase correlations.[1]

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[1]*Notation:* We use boldface for vectors $\boldsymbol{a}$ (lower case) and matrices $\boldsymbol{A}$ (upper case). The transpose and the conjugate transpose operators are denoted by the symbols $(\cdot)^T$ and $(\cdot)^\dagger$ respectively. $\mathbf{Diag}\,(\boldsymbol{a})$ is the diagonal matrix whose i-th diagonal element is the i-th entry of $\boldsymbol{a}$, while $\mathbf{diag}(\boldsymbol{A})$ is the vector containing the diagonal entries of $\boldsymbol{A}$. $\boldsymbol{I}$ refers to the identity matrix (its size is determined from the context). $\mathbb{R}^N$ and $\mathbb{T}^N$ are the sets of N-dimensional vectors of real numbers and elements of the phase-only torus, respectively. The symbol $\odot$ indicates the element-wise or Hadamard product. The letter $j = \sqrt{-1}$ represents the imaginary unit. For any complex number, $|\cdot|$ indicates its modulus, whereas $\|\cdot\|$ denotes the Frobenius norm.

II. System model

The involved system consists of a single-antenna BS, a RIS comprising N reflecting elements, and K single-antenna NOMA users. Due to the presence of obstacles, direct communication between the users and the BS is obstructed, making the RIS-reflected paths the sole means of connectivity. Through the RIS, each user establishes a virtual LOS link to the BS. The goal is to optimize the phase shifts of the RIS elements in this system to enhance the achievable data rates for the NOMA users. Being the RIS positioned to maintain LOS between the BS and the users, the channels BS-RIS and users-RIS are all modeled as independent, frequency-flat Rayleigh fading channels. Consequently, the channel BS-RIS is expressed as

$$\boldsymbol{h} = d_1^{-\alpha_1/2}\boldsymbol{h}_0, \tag{1}$$

where $\boldsymbol{h}_0 \sim \mathcal{CN}(\boldsymbol{0}, \eta_{h_0}\boldsymbol{C}_{h_0})$ represents the small-scale fading component, modeled as an independent and identically distributed (i.i.d.) complex Gaussian vector with zero mean and covariance matrix $\eta_{h_0}\boldsymbol{C}_{h_0}$.

Similarly, the channel RIS-U_k is modeled as

$$\boldsymbol{g}_k = d_{k,2}^{-\alpha_{k,2}/2}\boldsymbol{g}_{k,0}, \tag{2}$$

where $\boldsymbol{g}_{k,0} \sim \mathcal{CN}(\boldsymbol{0}, \eta_{g_k}\boldsymbol{C}_g)$ denotes the corresponding small-scale fading term. Notice that, the matrix $\boldsymbol{C}_g$ accounts for phase correlations between RIS elements. Here, d_1 and $d_{k,2}$ denote the distances between the BS and the RIS, and between the RIS and user U_k, respectively; α_1 and $\alpha_{k,2}$ are the path loss exponents reflecting the propagation environment. It is assumed that the channels $\boldsymbol{h}$ and $\boldsymbol{g}_k$ are statistically independent, according to the spatial separation of the BS, RIS, and users. Furthermore, the block fading model is assumed, whereby channel realizations remain constant over each transmission block and vary independently across blocks.

In the uplink system, each user U_k, $k = 1, \dots, K$, encodes its message into an information symbol s_k with unit power $\mathbb{E}\{|s_k|^2\} = 1$ and transmits $\sqrt{p_k}s_k$, where p_k represents the user's transmit power. The transmission is assisted by a RIS, characterized by a diagonal phase-shift matrix $\boldsymbol{\Theta} = \mathbf{Diag}\left(e^{j\theta_1}, \dots, e^{j\theta_N}\right)$, which adjusts the phase of the reflected signals to enhance the communication link between the users and the BS. Hence, the received signal at the BS is given by

$$y = \sum_{i=1}^{K}\sqrt{p_k}(\boldsymbol{h}^H\boldsymbol{\Theta}\boldsymbol{g}_k)s_k + n_0, \tag{3}$$

with n_0 a zero-mean circularly symmetric complex Gaussian random variable with variance σ^2 characterizing the additive white Gaussian noise (AWGN) channel.

From (3), it is evident that each user's signal is affected by interference from the signals of all other users transmitted simultaneously. In accordance with the uplink NOMA principle, SIC is performed at the BS to mitigate this interference. Without loss of generality, the users are indexed based on their effective channel gains in descending order, such that

$$\left|\boldsymbol{h}^H\boldsymbol{\Theta}\boldsymbol{g}_K\right| \leq \cdots \leq \left|\boldsymbol{h}^H\boldsymbol{\Theta}\boldsymbol{g}_1\right|. \tag{4}$$

At the BS, decoding starts with user U_1, whose signal is detected while treating the others as interference. After successfully decoding s_1, its effect is removed from the received signal through SIC. The BS then decodes s_2 after removing the contribution of $\boldsymbol{s}_1$, and this process repeats sequentially until the weakest user's signal, s_K, is decoded, where all interference has been eliminated.

Therefore, after canceling the interference from users $1, \dots, k-1$ through the SIC process, the signal to interference plus noise ratio (SINR) for decoding user k's signal at the BS is expressed as

$$\gamma_k = \frac{|\boldsymbol{h}^H\boldsymbol{\Theta}\boldsymbol{g}_k|^2 p_k}{\sum_{i=k+1}^{K}|\boldsymbol{h}^H\boldsymbol{\Theta}\boldsymbol{g}_i|^2 p_i + \sigma^2}. \tag{5}$$

Then, since the communication rate for user k is $R_k = \log_2(1+\gamma_k)$, the achievable sum-rate of the system is

$$R_{\text{sum}} = \sum_{k=1}^{K}\log_2(1+\gamma_k). \tag{6}$$

III. Problem formulation

The RIS control system optimizes the phase shift matrix $\boldsymbol{\Theta}$ to maximize the total uplink sum-rate in a NOMA system. Rather than favoring one user, it enhances the overall spectral efficiency across all users by accounting for the SIC decoding order at the BS. Hence, the optimal phase configuration $\boldsymbol{\Theta}^\star$ is obtained as the solution to the following constrained optimization problem

$$\mathcal{P}\left\{\begin{array}{l}\max\limits_{\boldsymbol{\Theta}} \displaystyle\sum_{k=1}^{K}\log_2\left(1 + \frac{|\boldsymbol{h}^H\boldsymbol{\Theta}\boldsymbol{g}_k|^2 p_k}{\displaystyle\sum_{i=k+1}^{K}|\boldsymbol{h}^H\boldsymbol{\Theta}\boldsymbol{g}_i|^2 p_i + \sigma^2}\right) \\ \text{s.t. } |\phi_n| = 1,\ n \in \{1, \dots, N\}\end{array}\right., \tag{7}$$

where $\phi_n = e^{j\theta_n}$, and the unit-modulus constraint is enforced to account for the phase-only contribution of the involved passive RIS.

After some algebraic manipulations [11], Problem (7) can be solved resorting to the following equivalent problem

$$\mathcal{P}_1\left\{\begin{array}{l}\max\limits_{\boldsymbol{\phi}} \displaystyle\sum_{k=1}^{K}\log_2\left(1 + \frac{\boldsymbol{\phi}^\dagger\boldsymbol{Q}_k\boldsymbol{\phi}}{\boldsymbol{\phi}^\dagger\boldsymbol{M}_k\boldsymbol{\phi}}\right) \\ \text{s.t. } \boldsymbol{\phi} \in \mathbb{T}^N\end{array}\right., \tag{8}$$

having denoted with $\boldsymbol{\phi} = \mathbf{diag}(\boldsymbol{\Theta}) = [e^{j\theta_1}, \dots, e^{j\theta_N}]^T \in \mathbb{T}^N$, $\boldsymbol{u}_k = \boldsymbol{h} \odot \boldsymbol{g}_k$, $k = 1, \dots, K$, and $\boldsymbol{Q}_k = p_k\boldsymbol{u}_k\boldsymbol{u}_k^H$. Moreover, $\boldsymbol{M}_k = \sum_{i=k+1}^{K}\boldsymbol{u}_i\boldsymbol{u}_i^H p_i + \frac{\sigma^2}{N}\boldsymbol{I}$ is the interference-plus-noise covariance matrix at user U_k. This matrix captures the aggregate interference from the remaining $K-k$ users whose signals have not yet been cancelled in the SIC decoding order, along with the additive noise.

Problem $\mathcal{P}_1$ is difficult to solve for several reasons. The objective function is non-convex, due to the fractional SINR terms and the unit-modulus constraint on each element of the RIS phase vector. Moreover, because $\boldsymbol{\phi}$ is shared among all users, the objective involves coupled, non-separable terms dependent on the same RIS parameters. This coupling makes joint optimization across users computationally complex and prevents closed-form solutions. Therefore, an appropriate solution method is introduced in the next section.

IV. RIS PHASE CONFIGURATION BASED ON ALTERNATING OPTIMIZATION STRATEGY

To address Problem $\mathcal{P}_1$, an AO strategy [14] is adopted, whereby the RIS phase shifts are iteratively optimized with respect to one user at a time, while the configurations for the remaining users are held fixed. This approach decomposes the original coupled and non-convex problem into a sequence of more tractable sub-problems, enabling iterative refinement of the overall sum-rate. While AO does not guarantee convergence to a global optimum, it achieves a favorable trade-off between computational efficiency and system performance.

The method proceeds iteratively as described in Algorithm 1. More precisely, for each user k, the algorithm optimizes the individual objective term given by the respective user rate R_k, subject to the unit-modulus constraint $|\phi_n| = 1,\ \forall n$, while treating the interference from other users as fixed. This leads to a *fractional quadratic programming problem* with unit-modulus constraints, which is inherently non-convex, that can be cast as

$$\mathcal{P}_2^{(k)} \begin{cases} \max\limits_{\boldsymbol{\phi}} \dfrac{\boldsymbol{\phi}^\dagger \boldsymbol{Q}_k \boldsymbol{\phi}}{\boldsymbol{\phi}^\dagger \boldsymbol{M}_k \boldsymbol{\phi}} \\ \text{s.t. } \boldsymbol{\phi} \in \mathbb{T}^N \end{cases} . \tag{9}$$

Notice that an optimal solution to Problem $\mathcal{P}_2^{(k)}$, say $\boldsymbol{\phi}_k^\star$, can be obtained using the CGM method [12], [13], [15], described in Subsection IV-A. This solution maximizes the individual utility for user U_k, but may potentially degrade the performance of other users, since $\boldsymbol{\phi}$ is a shared variable. Consequently, the user-specific solutions $\boldsymbol{\phi}_k^\star$ must be aggregated into a unified update $\boldsymbol{\phi}^{(p+1)}$, via a *weighted average*, based on weights $w_k \geq 0$ (with $\sum_k w_k = 1$), that can be selected based on criteria such as user priority, SINR levels, or rate contributions. Finally, to satisfy the unit-modulus constraint, each element of the averaged vector is then projected onto the unit circle. Notice that, the algorithm is repeated until a predefined maximum number of iterations P_{max} is attained.

Algorithm 1 Alternating optimization for RIS-aided NOMA sum-rate maximization

Input: $\boldsymbol{\phi}^{(0)}$ with $|\phi_n^{(0)}| = 1, \forall\, n$
for $p = 0, 1, 2, \ldots, P_{\text{max}}$ **do**
  for each user $k = 1, \ldots, K$ **do**
    $\boldsymbol{\phi}_k^\star = \arg \max\limits_{|\phi_n|=1} R_k(\boldsymbol{\phi})$ starting from $\boldsymbol{\phi}^{(p)}$
  end for
  Compute the weighted average $\boldsymbol{\phi}^{\text{avg}} = \sum_{k=1}^{K} w_k \boldsymbol{\phi}_k^\star$.
  Obtain $\boldsymbol{\phi}^{(p+1)}$ as the project of $\boldsymbol{\phi}^{\text{avg}}$ onto unit modulus via $\phi_n^{(p+1)} = \phi_n^{\text{avg}} / |\phi_n^{\text{avg}}|$, $\forall\, n$.
end for
Output: optimized RIS phase vector $\boldsymbol{\phi}^{(p+1)}$

A. *Phase RIS optimization of user U_k based on CGM*

The generic optimization problem $\mathcal{P}_2^{(k)}$ does not admit a closed-form solution under unit-modulus constraints, due to the existence of multiple local maxima in the feasible set, i.e., $\boldsymbol{\phi} \in \mathbb{T}^N$. For these reasons, the phase-only design problem is addressed by applying the CGM of [12], [13] to the RIS-aided NOMA optimization problem. More precisely, an optimal solution to $\mathcal{P}_2^{(k)}$ in (9) can be obtained with algorithms that leverage first-order derivative information. To this end, let $\nabla\gamma_k(\boldsymbol{\phi})$, $k = 1, \ldots, K$, denote the gradient of the phase-only SINR, whose expression is [12]

$$\nabla\gamma_k(\boldsymbol{\phi}) = \frac{2}{\xi_k} \left(\text{Im}(\alpha_k^* \boldsymbol{u}_k \odot \boldsymbol{\phi}^*) - \gamma_k(\boldsymbol{\phi}) \text{Im}(\boldsymbol{b}_k \odot \boldsymbol{\phi}^*) \right), \tag{10}$$

where $\alpha_k = \boldsymbol{\phi}^\dagger \boldsymbol{u}_k$, $\boldsymbol{b}_k = \boldsymbol{M}_k \boldsymbol{\phi}$, and $\xi_k = \boldsymbol{\phi}^\dagger \boldsymbol{b}_k$. Notice that, the SINR γ_k in a RIS-aided NOMA system has the same structural form as that of the spatial-only adaptive array radar; hence, interested readers can refer to [12] for all mathematical derivations and proofs. More in details, in [12], the CGM is adapted to perform optimization along geodesic paths on the N-torus (i.e., $e^{jt\textbf{Diag}(\boldsymbol{d})}\boldsymbol{\phi}$, with $\boldsymbol{d}$ the direction and t the step size) rather than along straight lines in Euclidean space (i.e., $\boldsymbol{\phi} + t\boldsymbol{d}$) to ensure that the structure of the constrained problem is maintained during the optimization process. The objective of the CGM [12], [13], [15] summarized in Algorithm 2 is to compute the optimal phase-only vector $\boldsymbol{\phi}_{\text{CGM}}^\star$ which maximizes the SINR under a phase-only constraint, performing an iterative optimization with a predefined number of iterations i_{max}.

As for the computational complexity of Algorithm 1, the dominant cost arises from solving the K RIS phase optimization subproblems at each outer iteration. Specifically, the complexity of one inner iteration is given by $8N^2 + 29N$ Flops [12]. Assuming that i_{max} inner iterations are required for convergence, the complexity of solving one subproblem becomes $\mathcal{O}\big(i_{\text{max}}(8N^2 + 29N)\big)$. Since Algorithm 1 solves K subproblems at each outer iteration and performs at most P_{max} outer iterations, the overall computational complexity is $\mathcal{O}\big(P_{\text{max}} K i_{\text{max}}(8N^2 + 29N)\big)$. For sufficiently large N, the quadratic term dominates, yielding the asymptotic complexity $\mathcal{O}\big(P_{\text{max}} K I_{\text{in}} N^2\big)$.

V. PERFORMANCE ANALYSIS

This section is devoted to assess the performance of the proposed methodology. In the considered setup, the RIS is modeled as a uniform rectangular array (URA) consisting of $N = N_x \times N_y$ reflecting elements. The analysis focuses on investigating the impact of the RIS phase configuration on the overall system sum-rate, while ensuring that the QoS requirements of all users are satisfactorily met. To this end, the performance of the proposed architecture, which employs a passive RIS whose phase shifts are optimized using the AO-CGM algorithm, is evaluated and compared with that of the conventional CGM procedure and an SDR-based approach (as in [16]), both of which aim to maximize the data rate of user U_1 only.

Algorithm 2 Conjugate Gradient-Based Phase Optimization

Input: $\boldsymbol{\phi}_0$, $\boldsymbol{q}_0 = \nabla\gamma_k(\boldsymbol{\phi}_0)$, $\boldsymbol{d}_0 = \boldsymbol{q}_0$
for $i = 0, 1, \ldots, i_{\max}$ **do**
  Computation of optimal step size along the geodesic:
$$t_i^\star = \arg\max_{t_i \geq 0} \left\{ \gamma_k \left(e^{jt_i \mathbf{Diag}(\boldsymbol{d}_i)} \boldsymbol{\phi}_i \right) \right\}$$
  Update of the phase vector: $\boldsymbol{\phi}_{i+1} = e^{jt_i^\star \mathbf{Diag}(\boldsymbol{d}_i)} \boldsymbol{\phi}_i$
  Computation of the gradient: $\boldsymbol{q}_{i+1} = \nabla\gamma_k(\boldsymbol{\phi}_{i+1})$
  Computation of the Polak-Ribière parameter: $\eta_i = \frac{(\boldsymbol{q}_{i+1} - \boldsymbol{q}_i)^T \boldsymbol{q}_{i+1}}{\|\boldsymbol{q}_i\|^2}$
  Update of search direction: $\boldsymbol{d}_{i+1} = \boldsymbol{q}_{i+1} + \eta_i \boldsymbol{d}_i$
end for
Output: Optimized phase vector $\boldsymbol{\phi}^\star$

In the following simulations, unless stated otherwise, the parameters listed in Table I are adopted. For the algorithm implementations, the maximum number of iterations is set to $i_{\max} = 20$ for the CGM and $P_{\max} = 30$ for the AO. Furthermore, to account for the stochastic nature of the communication channels, all performance metrics are averaged over $M_c = 10^3$ independent Monte Carlo trials. Finally, the CGM is initialized with $\boldsymbol{\phi}^0 = \mathbf{1}$, while in the AO, the user weights are chosen to be proportional to their respective channel gains.

Table I: Simulation Parameters

Parameter	Value	Description
N	8	number of reflecting elements
K	3	number of NOMA users
d_1	70 m	BS to RIS distance
$d_{k,2}$	$\{15, 20, 25\}$ m	RIS to user U_k distance ($k = 1, 2, 3$)
σ^2	-60 dBW	noise power
p_k	1 W	transmit power of user U_k
η_{h_0}	6 dB	channel gain for BS-RIS link
η_{g_k}	$\{6, 5.5, 4.5\}$ dB	channel gain for RIS-U_k link ($k = 1, 2, 3$)
α_1	2	path loss exponent: BS-RIS link
$\alpha_{k,2}$	2.6	path loss exponent: RIS-U_k link

To evaluate the impact of RIS phase correlations on system performance, two distinct scenarios are considered, each corresponding to different structures for the covariance matrices $\boldsymbol{C}_{h_0} \in \mathbb{R}^{N \times N}$ and $\boldsymbol{C}_g \in \mathbb{R}^{N \times N}$.

In the first scenario, an *exponentially decaying* covariance model is adopted, i.e.,

$$[\boldsymbol{C}_{h_0}]_{n,m} = \rho_0^{|n-m|}, \quad 1 \leq n, m \leq N, \tag{11}$$

$$[\boldsymbol{C}_g]_{n,m} = \rho_g^{|n-m|}, \quad 1 \leq n, m \leq N, \tag{12}$$

while, in the second scenario, a *Gaussian-shaped* covariance model is assumed, that is

$$[\boldsymbol{C}_{h_0}]_{n,m} = \rho_0^{|n-m|^2}, \quad 1 \leq n, m \leq N, \tag{13}$$

$$[\boldsymbol{C}_g]_{n,m} = \rho_g^{|n-m|^2}, \quad 1 \leq n, m \leq N, \tag{14}$$

where $\rho_0 = 0.85$ and $\rho_g = 0.95$ are the correlation coefficients associated with the respective channels.

The performance evaluation considers the sum-rate R_{sum} of the NOMA users $U_1, \ldots, U_K$ and the SINR γ_k of each user, highlighting how the AO-CGM approach maximizes the sum-rate while maintaining satisfactory performance across all users[2]. Fig. 1 presents the achievable SINR for each user and the overall sum-rate as a function of the distance $d_{1,2}$ between the RIS and user U_1, assuming exponential correlation. As shown in subplot (d), the AO-CGM method achieves a higher sum-rate than to the CGM and SDR algorithms that target only user U_1. However, subplots (a)–(c) reveal that, while the SINR of user U_1 slightly decreases under AO-CGM relative to CGM alone, the SINRs of users U_2 and U_3 remain significantly higher, improving their signal quality. In contrast, applying CGM exclusively to user U_1 maximizes that user's performance but severely degrades the SINRs of users U_2 and U_3, which drop well below 0 dB. The SDR exhibits similar behavior, also resulting in negative SINR values for users U_2 and U_3. Overall, this trade-off underscores the importance of joint phase optimization strategies, such as AO-CGM, which effectively balance overall system performance with individual user requirements in RIS-assisted NOMA uplink systems. The same analysis performed with the correlation coefficient ρ_g set to 0.85 yielded results that were perfectly superimposable on those shown in Fig. 1. Therefore, they are not reported here in order to avoid overloading the figure with an excessive number of curves.

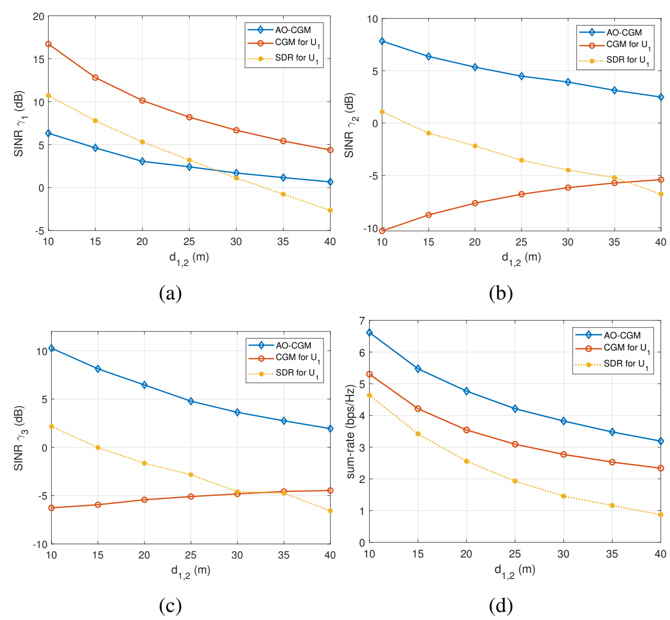
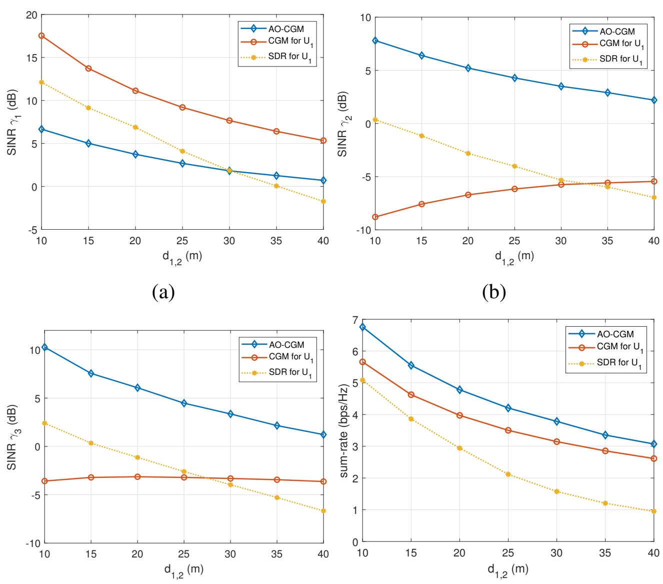


Figure 1: Performance (averaged over $M_c = 1000$ Monte Carlo trials) versus distance $d_{1,2}$ (m) for a RIS-assisted NOMA uplink system under exponential correlation. Subplots show: (a) γ_1, (b) γ_2, (c) γ_3, and (d) sum-rate R_{sum} (bps/Hz).

Fig. 2 shows the achievable SINR for each user and the overall sum-rate versus the distance $d_{1,2}$ for the Gaussian

[2]The convergence of the CGM for phase RIS optimization has been proved in [11].

correlation case. The results are substantially similar to Fig. 1, although the distance for which SDR surpasses CGM reduces. However, similarly to the exponential correlation scenario, the AO-CGM method attains a higher sum-rate than the CGM and SDR algorithms targeting only user U_1. Again, while user γ_1 decreases under AO-CGM, γ_2 and γ_3 remain significantly higher, whereas CGM or SDR applied solely to U_1 severely degrade their SINRs. These results confirm that joint phase optimization effectively balances system-wide performance and individual user requirements under Gaussian correlation as well.



(a) (b)

(c) (d)

Figure 2: Performance (averaged over $M_c = 1000$ Monte Carlo trials) versus distance $d_{1,2}$ (m) for a RIS-assisted NOMA uplink system under Gaussian correlation. Subplots show: (a) γ_1, (b) γ_2, (c) γ_3, and (d) sum-rate R_{sum} (bps/Hz).

Finally, to assess the impact of large-scale RIS, Fig. 3 reports the sum-rate versus the distance $d_{1,2}$ obtained by the proposed algorithm in the same case study as Fig. 1, but for two values of N, viz. 8 and 20. The curves show that the trend remains the same, with the sum-rate that decreases as $d_{1,2}$ increases. However, in the large-RIS regime a higher overall sum-rate is achieved, which is aligned with expectations.

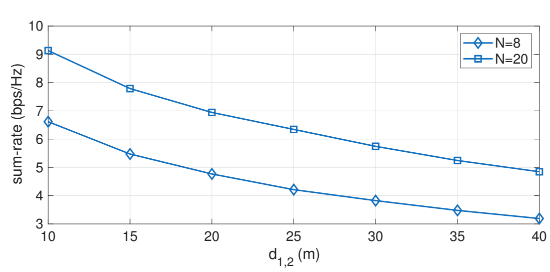


Figure 3: Sum-rate (averaged over $M_c = 1000$ Monte Carlo trials) versus distance $d_{1,2}$ for varying N.

VI. Conclusion

This paper proposes an AO approach combined with a phase-only CGM to maximize the uplink sum-rate in RIS-assisted NOMA systems, assuming correlations between RIS elements. The method efficiently addresses the non-convex phase design problem under unit-modulus constraints by iteratively optimizing user-specific configurations. Simulations demonstrate that the proposed algorithm substantially improves sum-rate performance also in the presence of phase-correlations. Future research could extend the analysis to more complex propagation environments.

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