Assessment of the Accuracy of Electromagnetic Field Calculations for Non Destructive Testing

Raffaele ALBANESE, Raffaele FRESA, Guglielmo RUBINACCI

Consorzio CREATE, Dip. Ing. Elettrica, Via Claudio 21, I-80125 Napoli, Italy
Fax: +39 81 7683171 - E-mail: elettro@disna.dis.unina.it

Abstract. Various stochastic techniques, applied to solve inverse identification problems, require the generation of extensive databases obtained via numerical analysis of the corresponding direct problems. We focus on the problem of the accuracy of the electromagnetic computation. This is a critical issue in the context of the magnetostatic and eddy current NDT techniques, because the quantities of interest are often the difference between two close values. We first discuss the *calibration* of the numerical results, a technique that can be applied to nonlinear problems and reduces to superposition in the linear case. Linear problems possess several properties that can be exploited when performing numerical analysis. Finally, we discuss the question of error estimation. It is well known that the constitutive error approach provides bounds for *global* electromagnetic quantities. We show that upper and lower bounds are also available for *local* quantities in the linear case.

1. Introduction

The accuracy of numerical calculations is a critical issue in electromagnetic NDT because the electric or magnetic fields to be evaluated are often only small perturbations of background signals associated with a reference configuration of the specimen.

When dealing with the problem of the identification of flaws, successful numerical methods have already been proposed, based on integral and differential formulations in terms of the current sources associated to the flaw. However, less attention has been paid to the control of the accuracy of these computations.

In this paper, we discuss several ideas related to the problem of the accuracy in the context of the magnetostatic and eddy current NDT techniques.

Firstly, we recall and adapt a technique useful to reduce the systematic errors in a field computation involving a difference between two very similar quantities.

Secondly, we show how to take advantage of the features of integral formulations in view of the properties of linear systems.

Thirdly, we recall the role of the constitutive error, which leads to the definition of error estimates and in static cases to upper and lower bounds for global quantities.

Finally, we show that for linear but not necessarily homogeneous systems, the existence of such bounds for global functionals allows us to obtain upper and lower bounds even for local field quantities.

2. Numerical Calibration

Inverse problems in Electromagnetic NDE often call for extensive analyses of the corresponding direct problems.

For instance, the common basis for stochastic approaches is the generation of a set of data corresponding to real or simulated experiments [1].

In simulated experiments, the direct formulation is used to evaluate the electromagnetic quantities in a set of locations referring to the specific NDE problem.

The accuracy of numerical calculations is a critical issue in electromagnetic NDT because the electric or magnetic fields to be evaluated are often only small perturbations of background signals associated with a reference configuration of the specimen.

The accuracy of numerical procedures is limited by the discretization errors, which in principle can be reduced by increasing the number of unknowns.

In practice this is limited by the available computer resources in terms of memory size and CPU speed.

The "*calibration of numerical solutions*" [2] evaluates the perturbation field as the difference of two solutions (with and without defect), calculated numerically by using the same mesh.

Let us consider the general boundary value problem:

$$\mathcal{F}[\mathbf{v},\mathbf{p}]=\mathbf{s} \tag{1}$$

where $\mathcal{F}$ is a linear or non linear operator, $\mathbf{p}$ is a set of parameters, $\mathbf{v}$ is the unknown vector field, and $\mathbf{s}$ is the source term.

Let us consider the other boundary value problem:

$$\mathcal{F}[\mathbf{v}_o,\mathbf{p}_o]=\mathbf{s} \tag{2}$$

which differs from (1) only for the different values of the set of parameters.

The approximate solutions $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{v}}_o$ are computed using weighted residuals:

$$\langle \mathbf{W}_i, \mathcal{F}[\tilde{\mathbf{v}},\mathbf{p}]-\mathbf{s}\rangle = 0, \qquad \langle \mathbf{W}_i, \mathcal{F}[\tilde{\mathbf{v}}_o,\mathbf{p}_o]-\mathbf{s}\rangle = 0 \tag{3}$$

with the same shape and weighting functions $\mathbf{N}_k$ and $\mathbf{W}_k$, i.e., the same mesh:

$$\tilde{\mathbf{v}} = \Sigma\, v_k \mathbf{N}_k, \qquad \tilde{\mathbf{v}}_o = \Sigma\, v_{ok} \mathbf{N}_k \tag{4}$$

If $\|\mathbf{v}-\mathbf{v}_o\|$ is less than the numerical error involved in each computation, the "*numerical calibration*" is indispensable if we want $\tilde{\mathbf{v}} - \tilde{\mathbf{v}}_o$ to be a good evaluation of $\mathbf{v}-\mathbf{v}_o$.

In fact, only using the same shape and weighting functions we ensure that

$$\lim_{\mathbf{p}\to\mathbf{p}_o} \tilde{\mathbf{v}}(\mathbf{p}) = \tilde{\mathbf{v}}_o \tag{5}$$

We analysed two different problems that differ for the presence of a conducting region. As shown by Table 1, the numerical error may be large (about 0.081 mT for the **A** formulation and about 0.008 mT for the **H** formulation), but numerical calibration appreciates B_r-B_{ro} within 0.00003 mT. Notice that $B_r(\mathbf{H})-B_{ro}(\mathbf{A})=0.07087$ mT, yielding a 3000% error because the shape functions do not coincide.

Table 1. Flux density [mT] calculated at a point in perturbed (B_r) and unperturbed (B_{ro}) case

	exact solution	num. solution (**A** formulation)	num. solution (**H** formulation)
B_r	0.87890	0.79732	0.87082
B_{ro}	0.88156	0.79995	0.87346
B_r-B_{ro}	-0.00266	-0.00263	-0.00264

3. Superposition and Reciprocity

Superposition is a particular case of numerical calibration. If $\mathcal{F}$ is linear with respect to $\mathbf{v}$ and $\mathbf{p}$, then the formulation

$$\langle \mathbf{W}_i, \mathcal{F}[\tilde{\mathbf{v}},\mathbf{p}]-\mathbf{s}\rangle = 0, \qquad \langle \mathbf{W}_i, \mathcal{F}[\tilde{\mathbf{v}}_o,\mathbf{p}_o]-\mathbf{s}\rangle = 0 \tag{6}$$

$$\tilde{\mathbf{v}} = \Sigma\, v_k \mathbf{N}_k, \qquad \tilde{\mathbf{v}}_o = \Sigma\, v_{ok} \mathbf{N}_k \tag{7}$$

is equivalent to

$$\langle \mathbf{W}_i, \mathcal{F}[\tilde{\mathbf{v}}-\tilde{\mathbf{v}}_0, \mathbf{p}-\mathbf{p}_0] \rangle = 0 \tag{8}$$

$$\tilde{\mathbf{v}}-\tilde{\mathbf{v}}_0 = \Sigma\, \Delta v_k \mathbf{N}_k \tag{9}$$

The disadvantage of the numerical calibration is that only the truncation error is filtered. So, attention must be paid to round-off error, tolerance in the system inversion and exchange of data.

The advantages are that the problem does not need any particular reformulation of the problem (superposition is generally used by replacing passive region with source terms) and that it works for both linear and nonlinear problems.

Reciprocity too is helpful for linear problems, allowing us to considerably reduce the size of the system matrix to achieve a given accuracy for a problem in Non Destructive Testing [3]:

$$\int_{V_b} \mathbf{E}^a \cdot \mathbf{J}_S^b \, dV = \int_{V_a} \mathbf{E}^b \cdot \mathbf{J}_S^a \, dV \qquad \text{for fields} \tag{10}$$

$$-\frac{\partial \Phi_b^a}{\partial t} I^b = -\frac{\partial \Phi_a^b}{\partial t} I^a \qquad \text{for coils} \tag{11}$$

The differential flux Φ_a^b due to the current flowing in the active coil can be computed as the flux linked with the active probe induced by a suitable pair of opposite currents flowing in the receptive coils.

In this way the unknown of the problem is directly the current density distribution induced in the block by a differential probe, and the numerical superposition of almost equal signals is avoided.

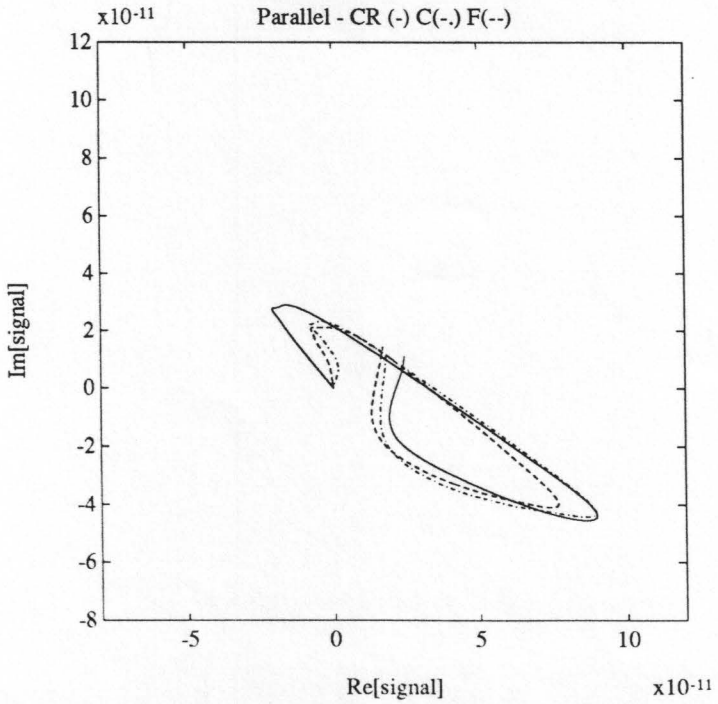

Figure 1. TEAM Workshop Problem 8. Results obtained using reciprocity with a rather coarse mesh (448 elements, 765 nodes, 670 unknowns) in comparison with the results obtained without exploiting reciprocity, using the same mesh and a finer mesh (798 elements, 1320 nodes and 1193 unknowns).

4. Bounds for Global and Local Electromagnetic Quantities

Upper and lower bounds for integral quantities of integral character, like the magnetic stored energy or the ohmic power dissipated in the domain of interest, had been clearly established along with the procedures to obtain them numerically.

However, upper and lower bounds for local quantities would be of paramount interest in several fields of applications like Non Destructive Testing or Nuclear Magnetic Resonance.

We here apply a simple and general procedure we developed for the determination of upper and lower bounds of local field quantities, namely the average value of a field component in an arbitrarily small region.

It is based on the introduction of an auxiliary field, and is the natural extension of the method establishing the bounds of global quantities.

Our procedure has several points in common with the line drawn in 1948 by Greenberg [4], addressed to obtain upper and lower bounds for the solution of the problem of elasticity, characterized by the introduction of subsidiary functions, related to Green functions for homogeneous media.

Our method can be applied to any linear stationary problem for which a form of the virtual work principle is applicable (e.g., magnetostatics). Homogeneity is not required.

4.1. Upper and Lower Bounds for Global Quantities

Upper and lower bounds for elliptic engineering problems were established very long ago, (see for instance [5]). The first application to computational electromagnetics was in 80's [6]. The Ligurian approach for magnetostatics is based on the definition of a local error whose expression in the linear magnetostatic case is:

$$\lambda(\mathbf{B},\mathbf{H}) = (\mathbf{B}-\mu\mathbf{H})^2 / 2\mu = B^2 / 2\mu + \mu H^2 / 2 - \mathbf{B}\cdot\mathbf{H} \geq 0 \qquad (12)$$

whereas canonic equations are enforced by taking $\mathbf{B}=\nabla\times\mathbf{A}$ and $\mathbf{H}=(\mathbf{T}-\nabla\Omega)$ with $\nabla\times\mathbf{T}=\mathbf{J}$.

This allows for the definition of a global error functional

$$\Lambda(\mathbf{A},\Omega) = \int_V \lambda(\nabla\times\mathbf{A}, \mathbf{T}-\nabla\Omega)\,dV = \Xi_v(\mathbf{A}) + \Theta_v(\Omega) + \Gamma(\mathbf{A},\Omega) \geq 0 \qquad (13)$$

where:

$$\Gamma(\mathbf{A},\Omega) = -\int_V \nabla\times\mathbf{A}\cdot(\mathbf{T}-\nabla\Omega)\,dV \qquad (14)$$

If $\mathbf{A}$ and Ω are selected such as to verify the essential boundary conditions $\Gamma(\mathbf{A}, \Omega)$ can be split, and the error functional can also be split

$$\Lambda(\mathbf{A}, \Omega) = \Xi(\mathbf{A}) + \Theta(\Omega) \geq 0 \qquad (15)$$

We have therefore upper and lower bounds

$$-\Xi(\mathbf{A}) \leq -\Xi(\mathbf{A}_0) \leq \Theta(\Omega_0) \leq \Theta(\Omega), \qquad \forall\ \Omega, \mathbf{A} \qquad (16)$$

For linear media and homogeneous boundary conditions $\Theta(\Omega_0) = -\Xi(\mathbf{A}_0)$ is the magnetic energy. Therefore, as well known, there exist upper and lower bounds for self-inductances.

4.2. Upper and Lower Bounds for Local Quantities

Our method relies on the possibility to determine upper and lower bounds for mutual inductances.

The self-inductance of the series of two circuits is:

$$L = L_1 + L_2 + 2M$$

where L_1 and L_2 are the self inductances of the two circuits separately, and M is their mutual inductance.

The procedure of Section 4.1 provides bounds for L, L_1 and L_2. Therefore, we can get upper and lower bounds for $M=(L-L_1-L_2)/2$:

5. Conclusions

The reliability of the results provided by the numerical analysis of electromagnetic fields for NDE is now satisfactory in the case of *linear problems*. We have also demonstrated that error bounds are available even for local quantities for static problems.

There exist numerical methods and associated codes with which transient and nonlinear eddy current problem are in principle solved. However, transient and nonlinear problems are still challenging. Firstly, we may only have error estimates, *not bounds*. Secondly, although the methods are convergent, present computer capabilities strongly limit the convergence process because of the maximum number of unknowns that can be treated.

Problems with hysteresis and motion are still challenging also from the point of view of the numerical formulations.

However, for the solution of real problems, the main uncertainties remain in the characterization of material properties and in the effects produced by a necessarily approximate solid modelling of structures with complex geometries.

References

[1] L.A.Zadeh, Lectures on Fuzzy Logic and the Calculi of Fuzzy Rules and Fuzzy Graphs, WCNN-94, San Diego, Ca, INNS Press, 1994.

[2] R. Albanese, R. Fresa, and R. Martone, "Accurate Computation of Electromagnetic Fields in the Presence of Conducting and Magnetic Materials", Int. J. Applied Electromagnetic and Mechanics, Vol. 6, 1995, pp. 73-88.

[3] P. Chaussecourte, "A Coil over a Crack: Overview of TEAM Workshop Problem 8 Recent Results", Proc. of 3rd Int. TEAM Workshop, Sorrento 1991, pp. 207-210.

[4] H.J. Greenberg, "The Determination of Upper and Lower Bounds for the Solution of the Dirichlet Problem", Journal of Mathematics and Physics, Vol. 27, No.3, 1948, pp. 161-182.

[5] N. M. Basu, "On an Application of the New Methods of the Calculus of Variations to some Problems in the Theory of Elasticity", Phil. Mag., ser. 7, 10, 1930, pp. 886-896.

[6] J. Rikabi ,C.F. Bryant and E.M. Freeman, "An error-based approach to complementary formulations of static field solutions", Int. J. Num. Meth. Eng., Vol. 26, 1988, pp. 1963-1987.

[7] R. Albanese, R. Fresa, "Upper and Lower Bounds for Local Electromagnetic Quantities", to appear.