Analysis of resistive joints for superconducting cables for fusion applications

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Abstract—**In this paper we use an integral formulation of the eddy currents problem in the presence of superconductors to analyse resistive joints for cables of interest for controlled thermonuclear fusion.**

I. INTRODUCTION

Next generation fusion devices (e.g. ITER [1]) will largely benefit from the use of superconducting magnets that are able to carry high average current densities (J ~20 A/mm^2) at high field (13 T) for long operation times (>>1000 s).

To achieve this (large) potential performance gain an effort has been made in Europe as well as in several other countries to manufacture and test Nb_3Sn-based Cable In Counduit Conductors (CICCs) whose technology is most suited for fusion applications. In particular, within the frame of the ITER project, the European Fusion Development Agreement, has coordinated the design, construction and testing of the Toroidal Field Model Coil that has successfully demonstrated the capability of European Industries to built large superconducting magnet (Iop ~ 80 kA).

Since superconducting cables are manufactured in limited lengths (~ hundreds of meters) , a key element in each large magnet design is the joint that electrically connects different cable lengths. Such joining element is of resistive nature (e.g. copper) and it needs to be cooled at cryogenic temperature (~4.5 K) to remove the Joule heat produced as the current flows through the copper without heating the adjacent cable lengths. It follows that a key figure of merit in each joint design is its overall resistance (~ Joule heating).

In this paper we model numerically the basic electromagnetic processes taking place in a typical resistive joint with the main aim of forecasting its resistance and - therefore – its cryogenic load (i.e. power dissipated).

II. MATHEMATICAL AND NUMERICAL FORMULATION

The mathematical formulation of the problem and its numerical counterpart has been already introduced in [2-3]. We consider 3-D non-magnetic domain V_c, made of both superconductors and ordinary conductors, subject to an external magnetic field; in addition, N equipotential electrodes $S_k \subseteq \partial V_c$, connected to an external feeding circuit (φ_k is the potential of the k-th electrode), force a transport current flow in V_c. In this work, we extend the formulation to the case in which the various conducting regions are possibly separated by thin resistive layers Σ, for which we assign the value of $\xi = d\,\eta$ (physical dimensions: Ω m^2), where d is the width and η the resistivity of the layer.

The governing equations in the magneto-quasistatic limit, i.e. the eddy current problem in the time domain, are formulated posing the electric field $\mathbf{E}=-\partial\mathbf{A}/\partial t-\nabla\varphi$, where $\mathbf{A}$ is the magnetic vector potential (with Coulomb gauge):

$$\mathbf{A}(\mathbf{x},\mathbf{J}) = \frac{\mu_0}{4\pi}\int_{V_c} \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} dV + \mathbf{A}_0(\mathbf{x}) \tag{1}$$

and φ is the electric scalar potential, $\mathbf{J}$ is the current density in V_c and $\mathbf{A}_0$ describes the external field. The integral formulation of the field's equation is complemented by the electrical constitutive relationship, represented by the power-law $\mathbf{E}=E_c(J/J_c)^n\mathbf{J}/J$ (E_c, J_c and n are parameters; we notice that n=1 provides the linear case), which is reformulated by the following variational inequality [4]:

$$\textit{find}\ \mathbf{J}\in Q:\ U(\mathbf{J}') - U(\mathbf{J}) \geq (\mathbf{E}, \mathbf{J}'-\mathbf{J}) \quad \forall \mathbf{J}'\in Q \tag{2}$$

$$U(\mathbf{J}) = \int_{V_c} u(\mathbf{x},\mathbf{J}) dV \ , \ u(\mathbf{x},\mathbf{J}) = \int_0^{J(\mathbf{x})} E(j)dj \tag{3}$$

Here, Q is a suitable functional space expressing the properties of $\mathbf{J}$ in V_c and on ∂V_c, and $(\mathbf{a},\mathbf{b})$ is the volume integral $\int_{V_c}\mathbf{a}\cdot\mathbf{b}\,dV$. Making an implicit approximation with a time step Δt for the time derivative and provided the convexity of U, the unique solution of (2) can be found as the minimum with respect to $\mathbf{J}$ of a suitable functional F. In order to take into account the resistive layers separating the conducting regions, we introduce a quadratic component to the functional F presented in [3], in the stream of [5].

The integral formulation allows us to discretize only V_c. We introduce a two-component electric vector potential $\mathbf{T}$ such that $\mathbf{J}=\nabla\times\mathbf{T}$, and expand it in term of edge elements [6]. Suitable linear combinations $\mathbf{M}_h$ of edge elements are in fact chosen as basis function [7], in order to take into account the presence of electrodes and of multiply connected domains, so that $\mathbf{J}=\sum_h I_h \nabla\times\mathbf{M}_h$. The discretization introduced leads to the following discrete functional to be minimized respect to the unknown column array $\underline{I} = \{I_k\}$:

$$F_D(\underline{I}) \triangleq \frac{Y_D(\underline{I}) + S_D(\underline{I})}{\Delta t} + U_D(\underline{I}) + U_{S,D}(\underline{I}) + R_D(\underline{I}) \tag{4}$$

$$Y_D(\underline{I}) = \frac{1}{2}\underline{I}^T \underline{\underline{L}}\,\underline{I} \tag{5}$$

$$S_D(\underline{I}) = \underline{I}^T\left(\underline{V}_0 - \underline{V}_{0,p}\right) - \underline{I}^T\,\underline{\underline{L}}\,\underline{I}_p{}^T \tag{6}$$

$$R_D(\underline{I}) = \underline{I}^T\,\underline{b} \tag{7}$$

$$U_{S,D}(\underline{I}) = \frac{1}{2}\underline{I}^T\,\underline{\underline{R}}_s\,\underline{I} \tag{8}$$

where $\underline{b}$ is a column vector depending on potentials φ_k and:

$$L_{ij} = \frac{\mu_0}{4\pi}\int_{V_c}\int_{V_c}\frac{\nabla\times\mathbf{M}_i(\mathbf{x})\cdot\nabla\times\mathbf{M}_j(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}dVdV' \tag{9}$$

$$V_{0,i} = \int_{V_c}\mathbf{A}_0\cdot\nabla\times\mathbf{M}_i dV,\quad V_{0,p,i} = \int_{V_c}\mathbf{A}_{0,p}\cdot\nabla\times\mathbf{M}_i dV \tag{10}$$

$$R_{S,ij} = \int_\Sigma \xi\left(\nabla\times\mathbf{M}_i\cdot\hat{\mathbf{n}}\right)\left(\nabla\times\mathbf{M}_j\cdot\hat{\mathbf{n}}\right)dS \tag{11}$$

The functional U_D can be calculated by numerically integrating (3). The quadratic component $U_{S,D}$, which takes into account the resistive layers, does not compromise the uniqueness of the minimum of F_D. The minimum is obtained by the use of a gradient-like method, which uses the numerical approximation of the gradient of the functional F_D.

III. Preliminary Results and Conclusions

The aforementioned formulation has been used for the analysis of a realistic resistive joint of interest for fusion applications. The geometry is reported in Fig. 1a. The cables are 1 m long; the resistive joint is 450 mm long. In the cable cross sections there are 6 petals, twisted around a cooling channel central tube of 10 mm diameter with a twist pitch of 450 mm. The cable outer diameter is 38 mm. The petals are superconducting in the longitudinal direction and they have a resistive interface at their interfaces. In the joint region, around the cable petals there is a copper sleeve (5 mm thick) that is in contact with the petals through a solder that is simulated by a resistive interface. The copper sleeve is in contact with a copper sole, again through a soldered interface. The copper sole is made by 5 identical pieces 90 mm long, insulated between each other to prevent current flow along the longitudinal direction of the joint. Outside the joint, the cable is surrounded by a stainless steel 316 LN jacket of circular cross section with outer diameter of 48 mm. Also here there is a resistive interface at the petals-jacket interface to simulate the petals wrapping.

The mesh used (1968 hexahedral elements) is depicted in Fig. 1b. The difficulty of matching the cable petals with their helicoidally shaped volumes with the copper sole has been solved by assuming step varying volumes for the petals in the joint region. The problem has been preliminarly solved with the simplifying assumption that the superconducting material behaves linearly, with a resistivity equal to 1e-12 Ω m. The resistive interfaces have been assigned a value of ξ=1e-9Ωm^2. We assumed that the voltage distribution over electrodes (shaded in Fig. 1a) is constant and that the jacket is not fed by any voltage (no electrodes contact). We computed the steady state current density distribution after a voltage step; in Fig. 2 some details are shown.

As a future activity, we will study the effects of a correct modelling of the superconducting materials (rather than linear) on the local and global quantities affecting the resistive joint.

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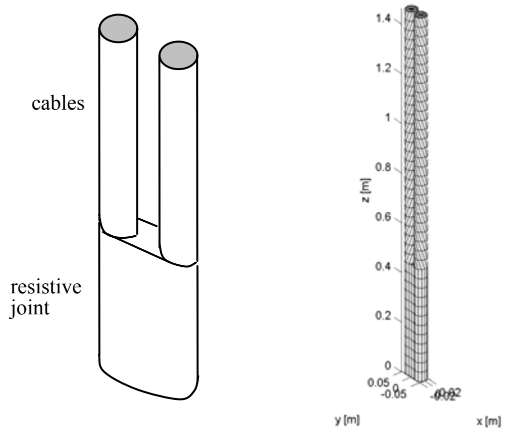
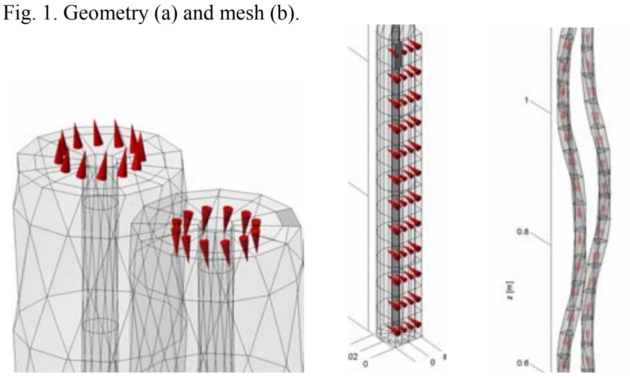


(a) (b)

Fig. 1. Geometry (a) and mesh (b).



(a) (b) (c)

Fig. 2. Details of the current density distribution: (a) near the electrodes A-B; (b) resistive joint; (c) one of the petals of the superconducting cable.

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