

Original article

An algorithm for computing the original units of measure of medieval architecture

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Abstract

Architecture is the expression of the artistic and technological culture of the age in which it is designed and realized. Stylistic features, building techniques and functional aspects change in time. Units of measure are also the expression of an age. In particular, during the Middle Ages these units changed from region to region. The considerable variability in time and space of units of measure led us to draw them a posteriori, by means of an algorithm and some statistical mathematical analyses. For the first time this issue shows the analytical procedure of this algorithm. The algorithm is part of a wider-ranging method in which data coming from historical, historico-architectural, archeological and building-technical analyses, is considered in a repetitive and interdisciplinary way. The method has been tested on some medieval monuments of Southern Italy. The tests have revealed criteria and fields of application for the method itself, of which the aim is to provide information that helps to establish dates for monuments or parts of them.

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1. Research aims

This issue shows a method of computation of the original units of measure of historical architecture, having particular reference to the Medieval age, by means of an algorithm and some statistical analyses.

As many people know, during the Middle Ages, a unitary system of measure did not exist. Every region had different systems, which changed century after century.

Therefore, generally we cannot establish units of measure for medieval architecture, a priori. The problem is more even difficult if the monument underwent many building phases. In this case we can expect to find more units of measure, one for each building phase. For this reason we compute a posteriori original units of measure by using a mathematical algorithm, which is part of a method that considers different kinds of data coming from historical, historico-architectural, archeological and building-technical analyses.

The aim of this algorithm is to obtain its convergence by using an iterative procedure. It is possible if we just select those architectural elements, of which the dimensions are characterized by the same unit of measure. For this reason both the algorithm and the general method are iterative.

The achieved results supply information about the building chronology, only if the method is supported by historical sources, historico-architectural and archeological analyses.

In this issue, the method is applied only to planimetric dimensions.

In the future, it may also be used for architectural elevation and for single building elements, such as squared stone ashlar and clay bricks.

2. Experimental section

2.1. Introduction

Generally, when we study a medieval monument we find a building constructed without using a planning design that outlines prearranged shapes and sizes of its spaces. This fact

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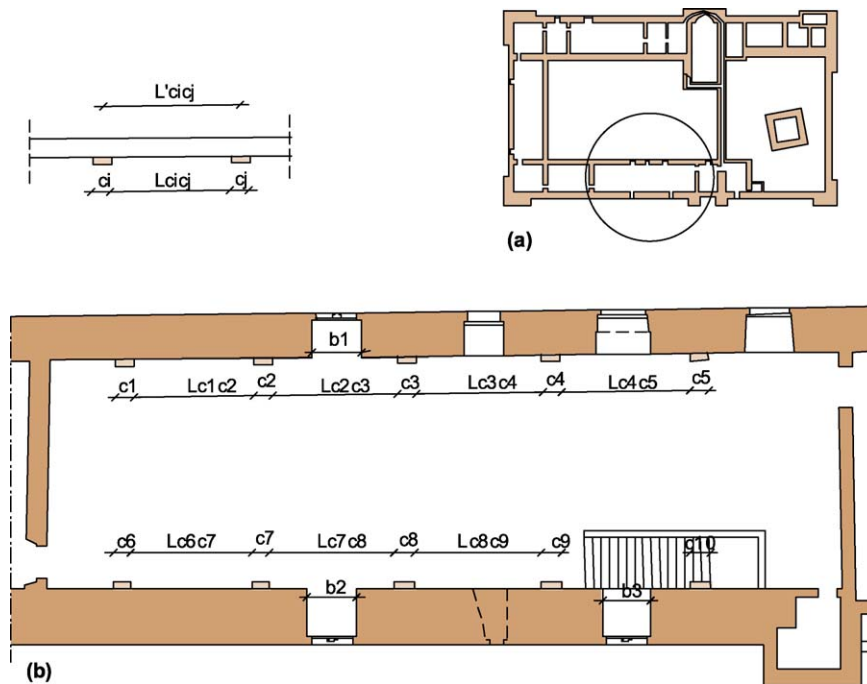


Fig. 1. Castle of Lagopesole. (a) General plan; (b) plan of “Ala dell’Imperatore” and measurements considered.

makes difficult the historical reconstruction, especially if the same monument has undergone a lot of interventions which changed the original features.

In that case, the building chronology would consist of a lot of historical phases. Therefore it would require a finalized archeological approach, in order to reconstruct the building stratigraphy of the architectural elevations [1].

An alternative approach is the metrological and morphological analysis of the plan that represents a cognitive support for archeological and architectural study methodologies.

This approach, applied on some medieval castle and church plans, has shown that for each historical layer, a different unit of measure corresponds.

Therefore an understanding of the units of measure provides some important information about the historical evolution of the same building.

Unfortunately, it is very hard to obtain this information, especially for medieval monuments. Furthermore these units of measure change from one historical period to the next and from region to region. This ‘empasse’ suggested the necessity of drawing them ‘a posteriori’ after basing them on a survey, by means of an iterative procedure based on statistical analyses.

The method provides useful results for monuments built using a design.

What do we mean by “design” when we study Medieval architecture?

The Medieval constructor either spontaneously built or drew the building plan on the ground. Drawing a plan means assigning dimensions to the length, the height and the thickness of walls and foundations, before beginning construction.

During the Middle Ages, the unit of measure adopted in Southern Italy was the *canna* (2–2.15 m) divided into eight subunits, the *palm* (25–27 cm). In the 13th century, in some monuments of Apulia the *canna* was equal to 2.10 m [2].

Therefore in the case of a monument ‘designed’ or planned directly on the ground, if we compare the measurements drawn from the survey with the original units of measure, such as *palm* or *canna*, in many cases we could obtain multiple values of the measurements. This method allows us to obtain satisfactory results, for the most part, for structures characterized by geometrical plans.

2.2. Architectural examples

In order to test this method we analysed some medieval castles and churches in the Apulia and Basilicata regions. In this issue we show the data and results of applications performed on the castles of Lagopesole and Monte Serico.

The castle of Lagopesole (Fig. 1), near Potenza, is an important example of fortified architecture built between the 12th and 13th centuries. Its architectural spaces, such as courtyards, towers and buildings, are characterized by regular geometrical shapes.

In order to acquire information about the building’s relative chronology the method has been applied on the so-called “Ala dell’Imperatore”, situated on the first floor of the western wing.

The castle of Monte Serico, in north-eastern Basilicata, was founded in the 11th century and was rebuilt between the 12th and 13th centuries.

This castle represents an interesting type of fortified architecture, characterized by a square plan tower, surrounded by

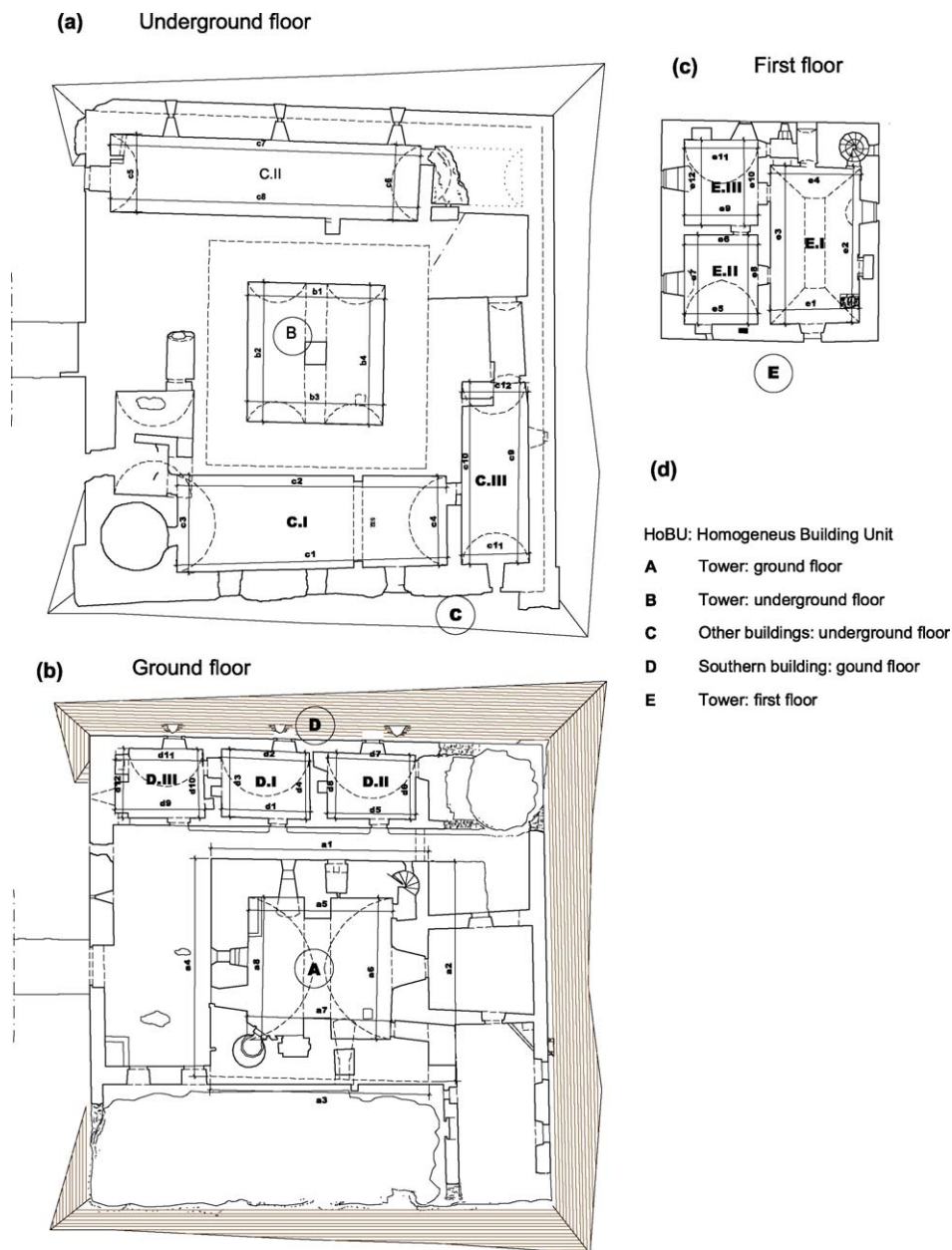


Fig. 2. Castle of Monte Serico. (a) Underground floor plan; (b) ground floor plan; (c) first floor plan and measurements considered; (d) HoBU.

walls and buildings that are concentrically developed from the centre to the outside (Fig. 2).

In elevation, the highest body is the central tower, whereas the heights of the other concentric and peripheral walls decrease in height finishing in a scarp wall footing, forming a truncated pyramidal base. For this reason this architectural type has been defined *pyramidenturm* castle [3].

In Lucera [4], Castelfiorentino and Tertiveri, in northwestern Apulia, and Termoli, in the Molise region, (Fig. 3), we find other examples of this type of architecture which represent an evolution of ‘donjon’, built in the 12th century during the Norman domination in Southern Italy.

In order to discover a common denominator between these castles, comparative metrological analyses have been performed.

In this paper the applications on the castles of Lagopesole and Monte Serico aim only at testing the algorithm, in order to emphasize the potentials and the limits of the method itself.

2.3. Methods

As many people know, before every statistical investigation, it is necessary to clearly determine the “population” of measurements we want to examine. In our case, measurements must refer to the dimensions of architectural spaces, that date back to the same historical age for which we want to determine the unit of measure.

In the case of the Medieval architecture found in Southern Italy, dating back to the 13th century, it is known, with a certain approximation, the “canna” and “palmo” values, in comparison to the present measure system.

ICNOGRAFIA DEI CASTELLI SVEVI A "PYRAMIDENTURM"

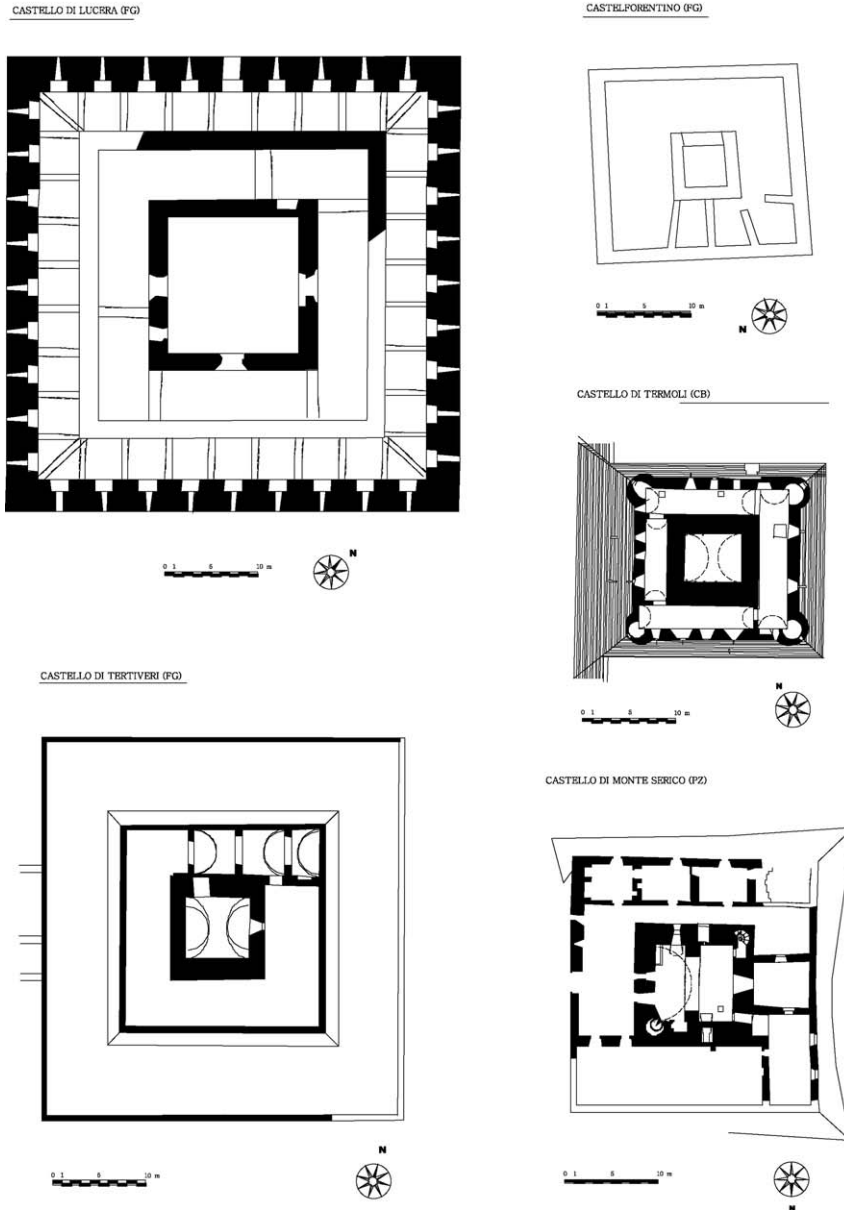


Fig. 3. Plans of castles of Lucera, Tertiveri, Monte Serico, Castelfiorentino, Termoli.

This method consists of three steps (Fig. 4):

- (1). First, we divide the entire architectural complex in homogenous building units (HoBU), singled out according to the historico-architectural, building and archeological analyses. For example in the Monte Serico test, five HoBU have been singled out.
- (2). Second, for each HoBU, we consider the most significant architectural elements (SAE), of which the dimensions form the population of measurements.
- (3). The third and final step is simple, to perform the statistical analysis of the samples considered. The procedure is iterative.

Let p_0 be the first attempt value of the “palmo” or “canna”, it is compared with the real measurements (x_i) of the SAE considered and the ratios are computed: $r_i = x_i/p_0$; for each

value r_i is associated with the versors $u_i = r_i/[r_i]$ (where $[r_i]$ indicates the whole part of r_i). For n values of u_i ($i = 1, \dots, n$), relative to n dimensions of architectural elements, the averaged value m is computed

$$m = \left(\frac{1}{n} \right) \sum_{i=1}^n u_i \tag{1}$$

and the standard deviation s is

$$s = \left[\sum_{i=1}^n (u_i - m)^2 / (n - 1) \right]^{1/2} \tag{2}$$

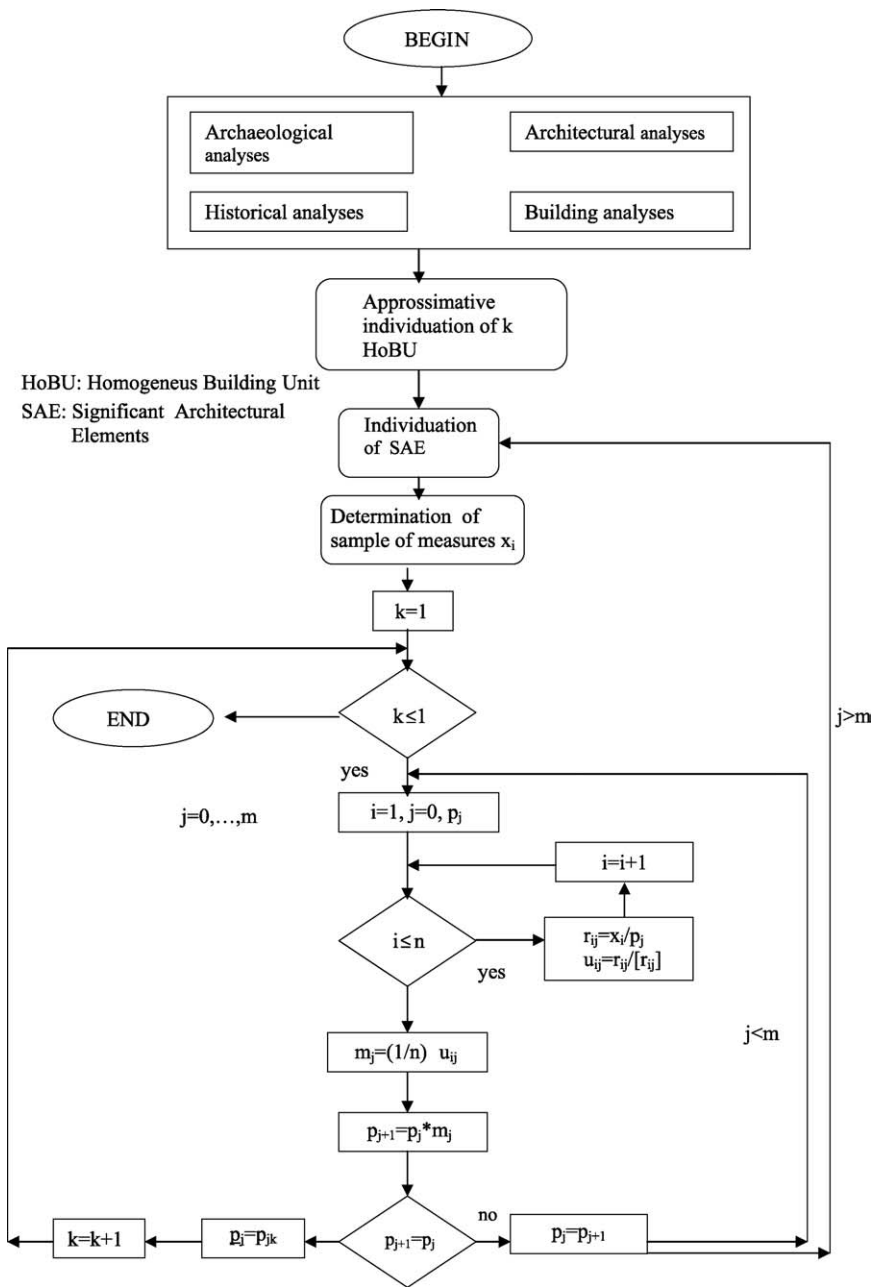


Fig. 4. Method flowchart.

where the $n - 1$ values represent the degrees of freedom associated with the considered data.

The second value of p_1 is therefore obtained by multiplying the averaged value m by p_0 , that is $p_1 = mp_0$. The procedure ends if p_1 converges to p_0 , otherwise we go on iterating until a value p_m very close to p_{m-1} is obtained.

If the convergence is not achieved, we go back to the step 2.

In order to single out the data which produces large mistakes in the statistical algorithm convergence, we usually aggregate them in different ways (see Section 2.4), verifying the relative statistical significance of each measurement (see Section 2.5).

For example, by using this procedure, in the Lagopesole and Monte Serico tests we discovered some measurements which are relative to non-significant architectural elements.

In the end, if the convergence has been obtained we analyse the results. On the contrary we start again from step 2, and if that is not sufficient it is necessary to begin from the step 1.

2.4. Applications and results

In the Lagopesole test, the data analysed regard three different groups of measurements: the widths of some capitals, c_p , the distances between the capitals given above, Lc_{ij} , and the widths of some trilobate mullioned windows having

Table 1
Castle of Lagopesole. Example of numerical data: group of measurements ($c_i + Lc_i + b_i$)

Measurements	x_i	$r_i = x_i/p_0; p_0 = 0.2245$ (m)	$[r_i]$	$u_i = r_i/[r_i]$	$r_i = x_i/p_j; p_j = 0.22557$ (m)	$[r_i]$	$u_i = r_i/[r_i]$
c_1	0.674	3.0022	3	1.0007	2.9956	3	0.9985
c_2	0.688	3.0646	3	1.0215	3.0578	3	1.0193
c_3	0.675	3.0067	3	1.0022	3.0000	3	1.0000
c_4	0.678	3.0200	3	1.0067	3.0133	3	1.0044
c_5	0.670	2.9844	3	0.9948	2.9778	3	0.9926
c_6	0.660	2.9399	3	0.9800	2.9333	3	0.9778
c_7	0.640	2.8508	3	0.9503	2.8444	3	0.9481
c_8	0.690	3.0735	3	1.0245	3.0667	3	1.0222
c_9	0.710	3.1626	3	1.0542	3.1556	3	1.0519
c_{10}	0.700	3.1180	3	1.0393	3.1111	3	1.0370
Lc_1c_2	4.310	22.2316	22	1.0105	19.1556	19	1.0082
Lc_2c_3	4.533	23.2285	23	1.0099	20.1480	20	1.0074
Lc_3c_4	4.521	23.1514	23	1.0066	20.0933	20	1.0047
Lc_4c_5	4.650	23.7149	24	0.9881	20.6667	21	0.9841
Lc_6c_7	4.340	22.2272	22	1.0103	19.2889	19	1.0152
Lc_7c_8	4.530	23.1403	23	1.0061	20.1333	20	1.0067
Lc_8c_9	4.600	23.6080	24	0.9837	20.4444	20	1.0222
b_1	1.780	7.9287	8	0.9911	7.9111	8	0.9889
b_2	1.790	7.9733	8	0.9967	7.9556	8	0.9944
b_3	1.740	7.7506	8	0.9688	7.7333	8	0.9667

p_0 , initial value; p_j , convergence value.

two lights, b_i (Table 1 and Fig. 1). These measurements are relative to two HoBU. The first one is represented by the values c_i and $Lc_i c_j$ and can be traced back to the building of some transversal arches which held the roof covering, in the first half of 13th century [5]. The second one is relative to the mullioned windows mentioned above.

The algorithm supplied the following results.

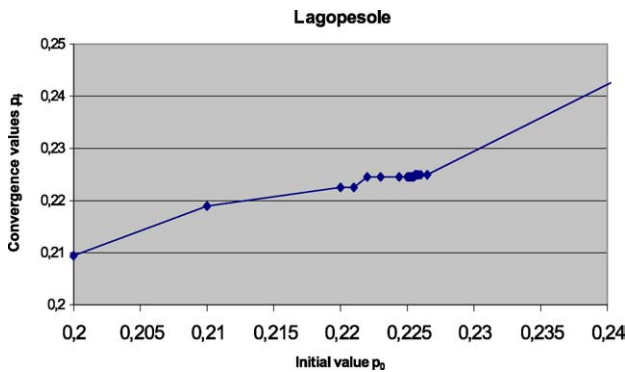


Fig. 5. Castle of Lagopesole. Graph related to convergence values about the group of measurements $c_i + Lc_i c_j + b_i$.

Table 2
Castle of Lagopesole. Statistical parameters and convergence values (p_j) of different groups of measurements

Groups of measurements	m	s^2	s	Convergence values of $p_j(m)$
$c_i + Lc_i + b_i$	1.00000000	0.00055	0.02343	0.22557
$c_i + Lc_i$	0.99999999	0.00054	0.02326	0.22633
c_i	0.99999999	0.00087	0.02950	0.22617
$Lc_i + b_i$	1.00000001	0.00027	0.01643	0.22387
$c_i + b_i$	1.00000001	0.00078	0.02806	0.22503
$c_i + Lc_i + b'_i$	1.00000000	0.00054	0.02319	0.22502
$c_i + Lc'_i$	0.99999998	0.00054	0.02315	0.22625
$Lc_i + b_i$	1.00000002	0.00019	0.01413	0.22387

For the entire population of measurements ($c_i + Lc_i c_j + b_i$), the convergence value of the algorithm is equal to 0.225 m (Fig. 5).

Moreover other combinations of groups of measurement have been considered: $c_i + Lc_i c_j + b_p$, $c_i + Lc_i c_j$, $Lc_i c_j + b_p$, $c_i + b_p$, c_i (Table 1).

As seen in Table 2 and Fig. 6, the two groups of measurements $c_i + Lc_i c_j$ and c_i converge towards the same value, equal to 0.226 m. The above-mentioned three mullioned windows (b_i) cause a variation in the convergence value of about 1%, thus the two HoBU we have considered, are of the same unit of measure.

Therefore the arches with capitals and mullioned windows could have been built either in the same historical building phase or in two historically close periods.

In the case of the Monte Serico test, five HoBU have been singled out (Fig. 2 and Table 3). A, B and E are relative to three levels of the central tower. The other two are found in the rooms on the underground floor, C, and in the southern wing on the ground floor, D.

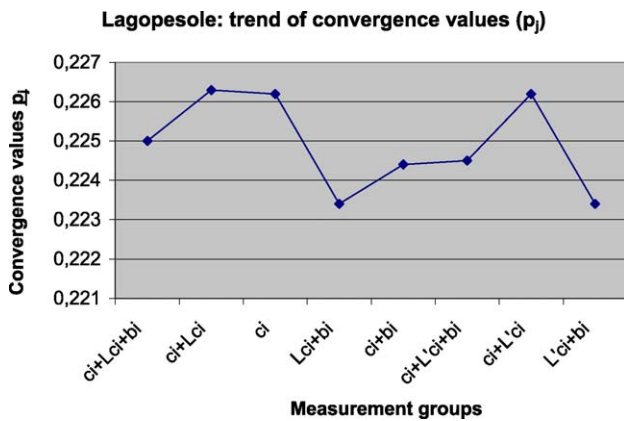


Fig. 6. Castle of Lagopesole. Trend of convergence value p_j of different measurement groups.

Some analyses have been made on combinations of more HoBU for reasons of low number of measurements of each HoBU as well as to know the kind of correlation of the data.

From the analyses performed we found a unit of measure, equal to 2.10 m approximately, for the following HoBU combinations: A + C, B + E, A + B + C and A + B + C + E (Table 4 and Fig. 7).

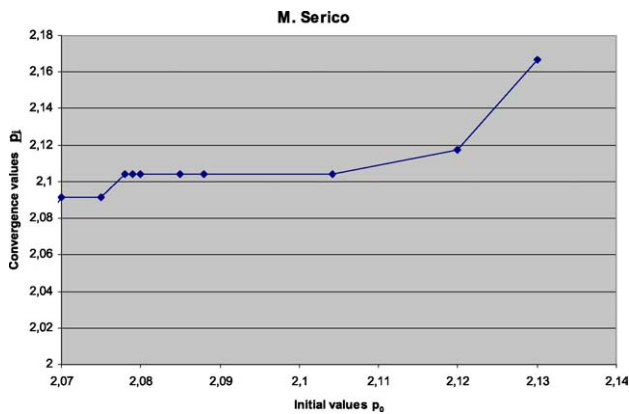


Fig. 7. Castle of Monte Serico. Graph related to convergence values for the group of measurements A + B + C + E.

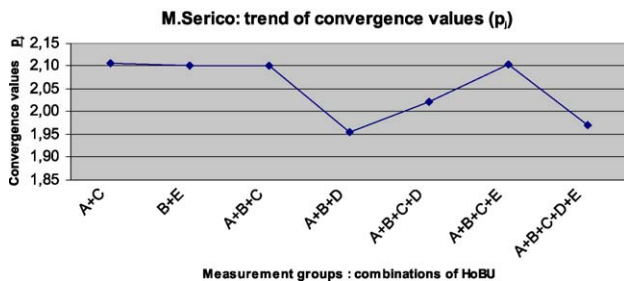


Fig. 8. Castle of Monte Serico. Trend of convergence value p_j of different measurement groups.

Measurements related to the HoBU, D, remarkably reduce the convergence values of p_j (Fig. 8). These results could be the proof that the HoBU relative to the tower and to the other architectural spaces on the underground floor (A, B, C and E) belong to the same historical building period, maybe the

oldest. On the contrary, the HoBU, D, that is, the other structures relative to the southern wing, have a different unit of measure and thus are not coeval to the primary foundation plan.

Other tests have been done on the castle of Lucera and on the cathedral of Acerenza [6].

In the first case, we found only one unit of measure. This is a new evidence of the historical homogeneity of the entire monument, which confirms the data furnished by the sources and the historico-architectural studies [7].

In the second test, other interesting results have been obtained. In this monument, for each HoBU a distinct unit of measure has been found. The result confirms the historical data indicating the presence of two HoBU, the first one, given by the transept apse complex built in the 12th century, and the second one relative to the naves rebuilt after the damage caused by an earthquake in the 17th century.

Furthermore the tests have shown that the main limitation to this method is the non-convergence of the iterative procedure, due to either a bad survey or a wrong choice of representative samples of measurements.

2.5. Verification of homogeneity of data samples

In order to exclude unreliable values that could invalidate the results, the homogeneity of measurements considered has been verified before the statistical data analyses were performed.

The homogeneity of variances has been verified by using both the Student's t distribution and the Test of Bartlett.

In most applications in which sample means are used to estimate population means, the value of σ ; is not known. We can however, obtain an estimate s of σ ; from the sample data that gives us the value of X . If the sample is of size n , the estimate s is based on $(n - 1)$ degrees of freedom. A distribution that enables us to compute confidence limits for μ ; knowing s but not σ ; is required. That distribution is known as Student's t distribution [8].

The 95% confidence limits is given by the following inequality:

$$X - t(0.05; n - 1)(s/\sqrt{n}) \leq \mu \leq X + t(0.05; n - 1)(s/\sqrt{n}) \quad (3)$$

where the quantity t is given by the equation

$$(4) t = (X - \mu)/(s/\sqrt{n})$$

and it is the deviation of the sample mean from the population mean measured in units of the mean's standard error s/\sqrt{n} .

For the two cases analysed the confidence limits have been computed as shown in Table 5.

In order to verify whether mean squares differ significantly, the Bartlett's test was applied. If the all mean squares $s_i^2 (i = 1, \dots, a; \text{ where } a \text{ is the number of measurement groups})$ have a different number of degrees of freedom v_i , the test criterion is:

Table 3
 Castle of Monte Serico. Example of numerical data: group of measurements (A + B + C + D + E)

HoBU	Measures	x_i	$r_i = x_i/p_0; p_0 = 2.110$ (m)	$[r_i]$	$u_i = r_i/[r_i]$	$r_i = x_i/p_j; p_j = 1.97141$ (m)	$[r_i]$	$u_i = r_i/[r_i]$
<i>Tower: ground floor</i>								
External measurements	a_1	13.02	6.1706	6	1.0284	6.6044	7	0.9435
	a_2	13.20	6.2559	6	1.0427	6.6957	7	0.9565
	a_3	13.05	6.1848	6	1.0308	6.6196	7	0.9457
	a_4	13.08	6.1991	6	1.0332	6.6348	7	0.9478
Internal measurements	a_5	8.60	4.0758	4	1.0190	4.3624	4	1.0906
	a_6	8.45	4.0047	4	1.0012	4.2863	4	1.0716
	a_7	8.58	4.0664	4	1.0166	4.3522	4	1.0881
	a_8	8.53	4.0427	4	1.0107	4.3268	4	1.0817
<i>Tower: underground floor</i>								
	b_1	8.17	3.8720	4	0.9680	4.1442	4	1.0361
	b_2	8.38	3.9716	4	0.9929	4.2508	4	1.0627
	b_3	8.14	3.8578	4	0.9645	4.1290	4	1.0323
	b_4	8.37	3.9668	4	0.9917	4.2457	4	1.0614
<i>Other buildings: underground floor</i>								
Room C.I	c_1	16.22	7.6872	8	0.9609	8.2276	8	1.0285
	c_2	16.12	7.6398	8	0.9550	8.1769	8	1.0221
	c_3	5.84	2.7678	3	0.9226	2.9623	3	0.9874
	c_4	5.32	2.5213	3	0.8404	2.6986	3	0.8995
Room C.II	c_5	4.65	2.2038	2	1.1019	2.3587	2	1.1794
	c_6	4.61	2.1848	2	1.0924	2.3384	2	1.1692
	c_7	18.61	8.8199	9	0.9800	9.4399	9	1.0489
Room C.III	c_8	18.38	8.7109	9	0.9679	9.3233	9	1.0359
	c_9	10.78	5.1090	5	1.0218	5.4682	5	1.0936
	c_{10}	10.91	5.1706	5	1.0341	5.5341	6	0.9224
	c_{11}	4.04	1.9147	2	0.9573	2.0493	2	1.0246
	c_{12}	4.03	1.9100	2	0.9550	2.0442	2	1.0221
<i>Southern building: underground floor</i>								
Room D.I	d_1	5.30	2.5118	3	0.8373	2.6884	3	0.8961
	d_2	5.28	2.5024	3	0.8341	2.6783	3	0.8928
	d_3	4.07	1.9289	2	0.9645	2.0645	2	1.0323
	d_4	3.99	1.8910	2	0.9455	2.0239	2	1.0120
Room D.II	d_5	5.31	2.5166	3	0.8389	2.6935	3	0.8978
	d_6	3.84	1.8199	2	0.9100	1.9478	2	0.9739
	d_7	5.23	2.4787	2	1.2393	2.6529	3	0.8843
	d_8	3.96	1.8768	2	0.9384	2.0087	2	1.0044
Room D.III	d_9	5.43	2.5735	3	0.8578	2.7544	3	0.9181
	d_{10}	4.10	1.9431	2	0.9716	2.0797	2	1.0399
	d_{11}	5.23	2.4787	2	1.2393	2.6529	3	0.8843
	d_{12}	4.19	1.9858	2	0.9929	2.1254	2	1.0627
<i>Tower: ground floor</i>								
Room E.I	e_1	5.32	2.5213	3	0.8404	2.6986	3	0.8995
	e_2	9.35	4.4313	4	1.1078	4.7428	5	0.9486
	e_3	9.52	4.5118	5	0.9024	4.8290	5	0.9658
	e_4	5.36	2.5403	3	0.8468	2.7189	3	0.9063
Room E.II	e_5	4.41	2.0900	2	1.0450	2.2370	2	1.1185
	e_6	4.41	2.0900	2	1.0450	2.2370	2	1.1185
	e_7	5.37	2.5450	3	0.8483	2.7239	3	0.9080
Room E.III	e_8	5.37	2.5450	3	0.8483	2.7239	3	0.9080
	e_9	5.15	2.4408	2	1.2204	2.6123	3	0.8708
	e_{10}	4.41	2.0900	2	1.0450	2.2370	2	1.1185
	e_{11}	5.14	2.4360	2	1.2180	2.6073	3	0.8691
	e_{12}	4.41	2.0900	2	1.0450	2.2370	2	1.1185

p_0 , initial value; p_p , convergence value.

Table 4
Castle of Monte Serico. Statistical parameters and convergence values (p_j) of different groups of measurements

Groups of measurements	m	s^2	s	Convergence values of $p_j(m)$
A + B + C + D + E	1.00000061	0.00731	0.08552	1.971411
A + B + C + E	1.00000011	0.00829	0.09107	2.104222
A + B + C + D	0.99999977	0.00597	0.07729	2.021099
A + B + C	1.00000008	0.00299	0.05469	2.100229
A + B + D	1.00000019	0.00533	0.07302	1.956329
A + C	1.00000000	0.00350	0.05913	2.107025
B + E	0.99999988	0.01493	0.12230	2.100718

$$M = \left(\sum_{i=1}^a v_i \right) \ln s^2 - \sum_{i=1}^a v_i \ln s_i^2 \quad (5)$$

$$C = 1 + \left\{ 1/[3(a-1)] \right\} \left(\sum_{i=1}^a 1/v_i - 1/\sum_{i=1}^a v_i \right) \quad (6)$$

$$s^2 = \left(\sum_{i=1}^a v_i s_i^2 \right) / \left(\sum_{i=1}^a v_i \right). \quad (7)$$

Considering the null hypothesis that each s_i^2 is an estimate of the same σ^2 , the quantity M/C is distributed approximately as χ^2 , if x_i are randomly drawn from a normal distribution and the frequency distribution is equal to $(a-1)s^2/\sigma^2$ with $(a-1)$ degrees of freedom.

For the two cases analysed, data differ significantly as shown in Table 6.

In Table 6 we have verified homogeneity of error variances in combining data, so its mean value can be used to estimate the error. In fact the χ^2 value obtained in Lagopesole and Monte Serico cases ($M/C_{Lagopesole} = 8.312$, $M/C_{MonteSerico} = 8.877$) are greater than the χ^2 theoretical values ($\chi^2_{Lagopesole} = 0.997$, $\chi^2_{MonteSerico} = 0.999$).

Table 5
Confidence limits for the cases analysed

Site	Confidence limits
Lagopesole	$0.992 \leq \mu \leq 1.008$
Monte Serico	$0.976 \leq \mu \leq 1.024$

Table 6
Bartlett test. Results from the cases analysed

Site	M/C	$\chi^2(v 0.05)$
Lagopesole	8.312	0.997
Monte Serico	8.877	0.999

3. Conclusions

The tests have shown the potentials and the limits of the method, in order to calculate the original units of measure. The achievement of this result provides information about the building chronology, only if the algorithm is supported by historico-architectural and archeological studies.

Moreover, in order to obtain good results, a correct methodology should be based on the following rules:

- (1). Selection of architectural parts, dating back to the same historical-building age.
- (2). Choice of the measurements relative to significant architectural elements, such as the length of a facade, the load-bearing wall thickness, etc.
- (3). Verification of the homogeneity of variances by using the Bartlett's test functions.

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